Problem 2

A rudimentary transmission can be made by forcing two uniform cylindrical wheels with frictional coefficient $\mu$ together. The wheels have masses $m_1$ and $m_2$, and both have radius $R$. Initially, wheel 1 is rotating with angular velocity $\omega_1$ and wheel 2 is at rest. The wheels are being forced together with a constant force $F$, uniformly distributed across each wheel's face.

(a) 3 points When the wheels are first brought together, what is the magnitude of the torque that wheel 1 applies to wheel 2 via friction? Hint: split wheel 1 up into infinitesimal concentric rings of radius $r$ and width $dr$, and calculate the torque exerted by them.

If you could not answer part (a) use $\tau = \mu FR$ in what follows.

(b) 2 point What is the final angular velocity, $\omega_f$, of the two wheels (in terms of constants and $\omega_1$)?

(c) 3 point How long does it take to reach $\omega_f$?

(d) 2 point What was the change in energy of the system as a fraction of the initial energy? Where did this energy go?