Observation of vortex-lattice melting by NMR spin-lattice relaxation in the mixed state

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For anisotropic layered superconductors a NMR technique is proposed to study the structure of the moving vortex state in the presence of transport current flowing in the layers. The internal field in an anisotropic superconductor will have a component perpendicular to an applied field which is tilted away from the c axis. The moving ordered vortex lattice produces an alternating magnetic field which causes nutation of nuclear spins and contributes to the decay of the spin-echo amplitude. This decay rate will display an array of peaks as a function of frequency. In the liquid phase this alternating field contributes to the longitudinal relaxation rate \( W_I \), which has a single peak.

Recently NMR has been used intensively to study the vortex state in type-II superconductors at equilibrium.\(^1\)\(^-\)\(^3\) Vortex fluctuations provide a reservoir for spin-lattice relaxation \( W_I = 1/T_I \) and measurements of \( W_I \) provide information on the spectral density of such fluctuations weighted by their interaction with a magnetic field. Randomly fluctuating magnetic fields associated with vortex fluctuations also give rise to dephasing of the precessing nuclear magnetization affecting both \( 1/T_2 \) and the NMR linewidth. Here we propose a NMR method to study the spatial structure of a nonequilibrium, moving vortex phase in anisotropic superconductors in the presence of transport current. The method is based on a transformation of the spatial structure of the vortex state in a reference frame moving with vortices into time variations of the magnetic field viewed in the laboratory frame. The temporal characteristics of this alternating field correspond to the spatial characteristics of the vortex state: This alternating field is periodic for an ordered vortex lattice moving under the effect of transport current and random for the moving liquid vortex state. Measurements of the reduction of the spin-echo amplitude due to this alternating magnetic field provides information about the vortex state structure.

The organization of the paper is as follows. First we will present quantitative results for the spin-echo signal in the case of a perfectly ordered moving lattice. Then we will analyze three possible states of moving vortices in the presence of a transport current above the depinning line, (i) disordered solid, (ii) crystal, and (iii) liquid, and show how our proposed NMR method can distinguish these structures.

The proposal to probe the motion of the vortex lattice by NMR measurements was made by Delrieu.\(^4\) He noticed that the motion of vortices narrows the inhomogeneous static NMR line shape. These proposed measurements were successfully carried out in charge-density-wave (CDW) systems\(^5\) and, with less important results, in superconductors. For spin-density-wave (SDW) systems another method was proposed which is based on the suppression of the spin-echo signal by an additional dephasing of nuclear spins by the alternating internal field of a moving SDW.\(^6\)

A key feature of our method is that in the anisotropic superconductors, if the applied field is tilted away from the c axis, the spatially inhomogeneous field of a vortex lattice at rest and the alternating field of a moving lattice will have a component perpendicular to the applied field. The character of this alternating field depends on the vortex lattice: For an ordered lattice, it will have various Fourier components corresponding to the reciprocal lattice vectors of the lattice. These alternating fields will cause nutation of the nuclear magnetization. The rate of nutation as a function of NMR frequency \( \omega \) and vortex lattice velocity \( v \) has a series of peaks in the ordered lattice phase corresponding to the reciprocal lattice vectors of the vortex lattice. In contrast, in the liquid state, the time variations of the field of the vortices will be stochastic; these fields will enable relaxation of the nuclear spin temperature toward that of the vortex lattice. Here only one peak (at low \( \omega \)) is present in the relaxation rate.

To proceed with a quantitative description we note first that the magnetic field inside a superconductor in the ordered mixed state has inhomogeneous periodic component: It is maximum near the vortex cores and weaker in the space between vortices. In isotropic superconductors this field is aligned with the applied field \( H_0 \) and is given by

\[
h(\mathbf{r}) = \sum_{\mathbf{G}} h(\mathbf{G}) \exp(i\mathbf{G}\mathbf{r}), \quad h(\mathbf{G}) = \frac{B/2\pi^2}{1 + \lambda_{ab}^2 G^2},
\]

where \( \mathbf{G} \) are reciprocal vectors of the vortex lattice, \( B \) is the internal field oriented parallel to the applied field \( H_0 \), and \( \lambda_{ab} \) is the London penetration length for currents in the \( ab \) plane.\(^7\) At large applied fields, well above the lower critical field, the spatial modulation of \( B \) is \( \approx \Phi_0/4\pi^2 \lambda_{ab}^2 \).

In highly anisotropic layered superconductors such as the
high-temperature superconducting (HTS) materials \( h(r) \) is oriented practically perpendicular to the layers because interlayer screening currents are very weak in comparison with currents along layers. Here \( h_{\perp}(r) \) is given by Eq. (1).

If the lattice is moving, the inhomogeneous component of the field gives rise to a periodic alternating field

\[
\mathbf{h}(r, t) = \sum \mathbf{h(G)} \exp[i \mathbf{G}(r - vt)],
\]

with frequencies \( \omega(G) = \mathbf{G} \cdot \mathbf{v} \). In an anisotropic superconductor this alternating field has a component perpendicular to \( \mathbf{B} \) which causes mutation of the nuclear magnetization in the plane formed by \( \mathbf{B} \) and \( \mathbf{h} \times \mathbf{B} \) in spin-echo experiments. Its effect is strong when the NMR frequency \( \omega = \gamma_n B \) is close to \( \omega(G) \); \( \gamma_n \) is the gyromagnetic ratio.

Let us denote by \( \alpha \) the angle between \( \mathbf{B} \) and the \( c \) axis. We choose the coordinate system with \( z \) along the \( c \) axis, \( y \) along the projection of \( \mathbf{B} \) on the \( ab \) plane, \( z' \) along \( \mathbf{B} \), \( y' \) lies in the \( e, H_0 \) plane perpendicular to \( H_0 \), and \( x = z' \) is perpendicular to \( y, z \); see Fig. 1. We consider first a perfectly ordered moving lattice. Then the spatial periodicity of the static lattice will produce an alternating field for the nuclear spins. The component of this field perpendicular to the applied field, \( h_{\perp} = h \sin \alpha \), will cause mutation of the nuclear spins in the \( (x, z) \) plane with a frequency determined by the strength of the varying field, \( \Omega(r) = \gamma_n h(G) \sin \alpha \cos(Gr) \), if \( \omega(G) = \gamma_n B \).

Note that \( \Omega \) depends on the coordinates of nuclei inside the vortex unit cell. In a spin-echo experiment, applied rf pulses produce rotations of the magnetization \( \mathbf{M} \) about the rf field. An initial rotation of \( 90^\circ \) leaves \( \mathbf{M} \) in the \( x-y' \) plane. Inhomogeneities in the local magnetic field cause different nuclear spins to precess at different rates, causing a loss of phase coherence. A subsequent \( 180^\circ \) pulse inverts the spins and reverses this process. Coherence of the nuclear spins is regained, producing a so-called spin echo. In the present case, the application of a pulse of current in the superconductor will cause the vortex lattice to move and thus produce an alternating field similar to the rf field applied by the NMR coil. If this field were homogeneous throughout the sample and were inserted between the \( 90^\circ \) and \( 180^\circ \) pulses, it would modulate the amplitude of the resulting spin echo. The period of this modulation would be proportional to the length of the current pulse and to the strength of the time-varying field arising from the vortex lattice.

The suppression of the echo due to dephased mutation will display peaks as a function of the NMR frequency at \( \omega = \mathbf{G} \cdot \mathbf{v} \). For a perfectly ordered lattice the widths of these peaks will be given by \( \gamma_n h(G) \sin \alpha \). Finally the frequency of mutation of the magnetization vector in the ordered lattice at \( \gamma_n B = \mathbf{G} \) is

\[
N(\omega) \approx \gamma_n h(G) \sin \alpha.
\]

For a minimum reciprocal vector \( G_0 \approx 2\pi(B/\Phi_0)^{1/2} \), resonance occurs at velocity \( v_\perp \approx \gamma_n(B/\Phi_0)^{1/2}/2\pi \). For \( ^{205}\text{TI} \) nuclei with \( \gamma_n = 2\pi(2.45 \times 10^3)/(\text{G s}) \) this resonance velocity is \( \approx 100 \text{ cm/s} \) in a \( 1 \text{ T} \) field. We can estimate \( h(G) \sin \alpha \), with the help of Eq. (1), to be \( \approx \Phi_0/4\pi^3 \lambda_{ab}^2 \approx 5 \text{ G for } \lambda_{ab} = 1700 \text{ Å} \). This value provides a very high mutation rate \( N \approx 10^8 \text{ s}^{-1} \) at resonance velocities. Note that this is much quicker than the relaxation rate, \( W_1 \leq 10 \text{ s}^{-1} \), due to thermal motion of vortices below 30 K, and the latter rate would be typical out of resonance conditions. Thus peaks at resonance conditions should be seen very clearly.

To express \( v \) in terms of the applied current, we suppose that when a current with density \( j \) flows in the \( ab \) plane along the \( y \) axis, motion of vortices along the \( x \) axis in the \( ab \) plane results. At high currents, in the flux-flow regime, their velocity \( v_\perp \) is determined by the equation for viscous motion, \( v_\perp = \Phi_0 j/c \), where \( \eta \) is the viscosity coefficient per unit length of vortex [in Bi (2:2:1:2) for \( T < 70 \text{ K} \) the viscosity coefficient is \( \eta \leq 10^{-7} \text{ g/cm} \text{s} \) (Ref. 9)]. Thus at a given field \( B \) the resonance current is \( j_\perp \approx (c\gamma_n/2\pi)(B/\Phi_0)^{1/2}; \) it increases as \( \sqrt{B} \) with \( B \). The distance between peaks is approximately \( \omega \). Thus, for a perfect moving lattice, reciprocal lattice vectors are converted into peaks in the mutation rate \( N(\omega) \).

We discuss now how this simple picture changes in a real material where the structure of the vortex lattice is affected by pinning centers and thermal disorder. The irreversibility line, \( T_{irr}(B) \), separates the phase diagram of a mixed state in the \( (B, T) \) plane into two regions: the disordered solid state (glass) for \( T < T_{irr}(B) \) and the melted phase \( T > T_{irr}(B) \). Let us consider first the glass phase. Here, in the absence of transport current, vortices are pinned, and the long-range order of the static vortex lattice is destroyed. If a small current less than a certain critical value \( j_c \), is applied, the lattice remains pinned. For a pinned vortex lattice, \( j < j_c \), weak vortex motion occurs via the thermal activation of the vortex bundles over pinning barriers and the effect of this motion on the nuclear spin magnetization may be neglected.

If a large current is applied, \( j > j_c \), the vortices become unpinned and display viscous motion although their motion is still affected by pinning. At low temperatures \( j_c \) and, correspondingly, \( v \) are high and a moving vortex state is ordered (see below). Near \( T_{irr} \), the critical current is low because \( j_c(T) \to 0 \) as \( T \to T_{irr} \). Here a moving disordered solid state is realized in the interval of currents \( j_c < j < j_c \), while for \( j > j_c \), the vortex lattice displays long-range order; for more details see Ref. 14. The position of line \( j_c(T) \) remains unknown. The phase diagram in the plane \( (j, T) \) is shown in Fig. 2.
where \( c_{44} \approx (B\Phi_0/(16\pi^2\lambda_{ab}^2Q^2))\ln(a_0/s) \) and \( c_{66} = B\Phi_0/(8\pi\lambda_{ab})^2 \) are tilt and shear moduli (see Ref. 11).

Notice that \( u_p \) is finite at \( v > 0 \) \((v \propto j)\) contrary to the result\(^{12}\) at \( v = 0 \). We estimate \( u_p \) using the condition \( j > j_c \) and by expressing the disorder parameter \( \gamma_U \) determined from the critical current \( j_c(0) \) at \( T = 0 \) in the framework of the collective pinning approach for highly anisotropic superconductors.\(^{15}\) In the single pancake regime of strong pinning at temperatures well below \( T_{irr} \), we get \( \gamma_U(0) = (3\sqrt{3}\Phi_0/j_c(0)/4c)^2 \) and \( u_p \ll a_0 \) for \( j > j_c \).

We conclude that the vortex lattice is ordered in this regime.

Near \( T_{irr} \), thermal displacements weaken pinning, and the two-dimensional regime of collective pinning is realized. Here \( \gamma_U(T) = (j_c(T)/j_c(0))^{2/3}\sqrt{3}j_c(0)\Phi_0c_{66}\xi_{ab}/4c \) and \( u_p \simeq a_0 \) for moderate currents, \( j \sim j_c(T) \).

Nevertheless, the correlation length in the \( ab \) plane, \( R_{ab}(j) > R_{ab}(0) \approx (a_0/12\pi\lambda_{ab})\xi_{ab}BC(j_c(0))^{1/2}j_c(0)/j_c(T) \), is large as compared with \( a_0 \). It means that long-range order of vortex state is absent but correlations along layers remain strong. For larger currents \( j \) the effect of disorder weakens and the ordered moving lattice is established when \( u_p \) becomes much smaller than \( a_0 \).

The suppression of the spin-echo intensity by the alternating field of a moving ordered lattice should be sharply peaked as a function of frequency. These will be reduced by Debye-Waller factors \( \exp[-(u_p^2 + u_{th}^2)G_0^2a_0^2] \), where \( n \) is the number of peaks. Measurement of the height of these peaks will provide information on displacements of vortices. Similarly, in the moving disordered solid state, the peaks in \( N(\omega) \) will still be present because \( R_{ab} \gg a_0 \), but they will be broadened by disorder. Their width will provide information on the correlation length \( R_{ab} \) of the moving vortex disordered state.

Next we consider the moving liquid vortex state above the irreversibility line, \( T > T_{irr}(B) \). Here the time-dependent component of the magnetic field, \( h_z(t) \sin \alpha \) (perpendicular to \( H_0 \)), is a random function, and we can use the fluctuation-dissipation theorem. Thus its effect on relaxation of NMR spin-echo signals can be characterized by the spin-vortex lattice relaxation rate \( W_1 \) for nuclei at position \( r, z \):

\[
W_1(\omega) = \frac{1}{2}n^2n^2 \alpha \int dt \langle h_z(r, z, t)h_z(r, z, 0) \rangle \exp(i\omega t),
\]

where \( \langle \cdots \rangle \) means thermodynamic and disorder averaging.

The field \( h_z \) may be expressed via the structure factor of the vortices. Moving vortices induce a time-varying magnetic field\(^3\)

\[
h_z(r, z, t) = \frac{\Phi_0}{4\pi^2} \int_{\infty}^{\infty} dq dq \int_{-\infty}^{+\infty} \frac{dk}{1 + \lambda_{ab}^2(k^2 + Q^2)} \exp\{ik[r - r_{nu}(t)] + iq(z - ns)\},
\]

where \( Q^2 = 2s^2(1 - \cos sq) \). With the help of Eqs. (7), (8) we get the relaxation rate averaged over nuclear positions in the vertex unit cell:
\[ \tilde{W}_1(\omega) = \frac{\tau^2 B \Phi_0 \omega}{2\pi^3} \int_0^{2\pi/s} dq \int dk \frac{S(k, q, \omega)}{\left[1 + \lambda_{ab}^2 k^2 + Q^2 q^2 \right]^2} \left( k^2 + Q^2 q^2 \right)^2, \]

where \( S(k, q, \omega) \) is the Fourier component of the structure factor (\( N \) is the number of pancakes):

\[ S(k, q, t) = \frac{1}{N} \sum_{n, n', \nu'} \{ \exp[-ik(r_{n\nu}(t) - r_{n'\nu'}(0)) - iq(n - n')s] \}. \]

We assume that pancakes in different layers are uncorrelated and the correlation function inside a layer in the moving frame is \( S(k, t) = S(k, 0) \exp(-\delta t) \). For small \( \delta \) we get

\[ \tilde{W}_1(\omega) = \frac{\tau^2 B \Phi_0 \omega}{16\pi^2\lambda_{ab} v^2} \times \int_1^\infty dx \frac{xS(\omega x/v)}{\left[1 + \lambda_{ab}^2 x^2 + v_x^2 x^2/3 + v_y^2 x^2/3 \right]^{1/2}}. \]

Assuming that the function \( S(k, 0) \) has a broad peak near \( k \approx G_0 \) and smooth behavior at \( k \to 0 \), we obtain the maximum \( \tilde{W}_1 \) at very low frequency, \( \omega \approx \omega_p = v/\lambda_{ab} \), due to superconducting screening. This screening strongly suppresses the ac magnetic field at frequencies higher than \( v/\lambda_{ab} \), and the broad structural peak in \( S(k, 0) \) becomes invisible. We get

\[ \tilde{W}_1(\omega_p) \approx \frac{\tau^2 B \Phi_0 \omega_p}{16\pi^2\lambda_{ab}^2 v}. \]

It is smaller than \( N(\omega) \) at \( \omega = G_0 v \) in the solid phase by the factor \( \tau_{NS}B_0/v \approx 0.01-0.1 \) for fields 0.1-1 T. Thus in the liquid phase one weak peak should exist at low frequency in contrast to the solid phases where a series of peaks can be observed at much higher frequencies.

In conclusion, we propose a NMR technique to study the structure of moving vortex state in anisotropic superconductors. It is uniquely capable of providing information about the effects of pinning on the moving vortex lattice; in particular one can distinguish and study the ordered and liquid phases. In the moving crystal and disordered solid state we expect a series of narrow peaks corresponding to the Bragg peaks, whereas in the liquid state only one low-frequency weak peak should exist.

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