Math 130c, Homework 6

All projective spaces are over a field $k$.

1. Let $i : X \hookrightarrow \mathbb{P}^r$ be a hypersurface defined by a homogeneous degree $d$ polynomial. Compute the Hilbert polynomial of $i_* (\mathcal{O}_X)$ (HINT: Construct a short exact sequence $0 \to \mathcal{O}(-d) \to \mathcal{O} \to i_* (\mathcal{O}_X) \to 0$).

2. Let $0 \to \mathcal{F} \to \mathcal{G} \to \mathcal{H} \to 0$ be a short exact sequence of coherent sheaves on $\mathbb{P}^r$.
   (a) Show that if $\mathcal{F}, \mathcal{H}$ are $m$-regular, then $\mathcal{G}$ is also $m$-regular;
   (b) Show that if $\mathcal{F}, \mathcal{G}$ are $m$-regular, then $\mathcal{H}$ is also $m$-regular;
   (c) For $r = 1$, construct an example in which $\mathcal{G}, \mathcal{H}$ are 0-regular, but $\mathcal{F}$ is not 0-regular (HINT: take $\mathcal{G} = \mathcal{O}$ and use a short exact sequence from an exercise in one of the previous homework assignments).

3. Let $F : (Schemes) \to (Sets)$ be the functor which sends a scheme $S$ to the abelian group $\Gamma(S, \mathcal{O}_S)$ (considered as a set). Find a scheme $X$ for which there exists a bijective natural transformation $h_X \to F$. 

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