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Gravitational Collapse and the Death of a Star

Relativity and nuclear theory together predict the fate of a star which has burned all its nuclear fuel.

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What is the fate of a star when it has consumed all its nuclear fuel and can no longer maintain the nuclear reactions which have sustained it since its birth? This is a question which observational astronomy has done but little to answer. The time required for a star to consume its nuclear fuel is so long (many billions of years in most cases) that only a few stars die in our galaxy per century; and the evolution of a star from the end point of thermonuclear burning to its final dead state is so rapid that its death throes are observable for only a few years.

Despite the paucity of observational data, theoreticians are now able to discuss the deaths of stars in ever increasing detail and with a fair degree of certainty, thanks to recent advances in computer technology and in our understanding of the physics of the atomic nucleus, of elementary particles, and of Einstein's geometrical theory of gravitation (general relativity). The purpose of this article is to review the different types of death and the final resting states of various types of stars, as deduced theoretically, and to point out those few direct astronomical observations which have bearing on the theoretical predictions. Additional observational tests of these predictions, while extremely difficult and perhaps impossible with present-day technology, would provide valuable tests of the gravitation and nuclear theory upon which the predictions rest.

It should be emphasized at the outset that theoretical studies of the deaths of stars are still far from complete. Such complications as stellar rotation, deviations from spherical symmetry, and stellar magnetic fields are largely unstudied and will be ignored here. Rough estimates indicate that these phenomena, when not too pronounced, will probably have little effect on the qualitative picture outlined here; for the most part, only quantitative details are expected to change as theoretical studies become more realistic.

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Evolution to the Dead State

When, after hundreds of millions or billions of years of normal nuclear burning, a star has exhausted its supply of nuclear fuel, it has only one way to replenish the thermal energy that it is radiating: quasi-static gravitational contraction. As it contracts, the star converts its gravitational potential energy into thermal energy and radiates it away, and at the same time it squeezes the material in its core into a smaller and smaller volume and to higher and higher temperatures.

The subsequent fate of the star depends upon how massive it is. For a star of less than ~1.2 solar masses [the "Chandrasekhar limit" (*1*)], quasi-static contraction is halted by rising internal pressure when a central density of $\sim 10^6$ grams per cubic centimeter is reached. The star then settles down into its final resting state, a "white dwarf" configuration (*2*). On the other hand, in a star of more than 1.2 solar masses, the stellar core is squeezed to such high densities during quasi-static contraction that catastrophic nuclear processes occur before rising internal pressure can halt the contraction. These processes cause the star to explode with such violence that its luminosity approaches that of a galaxy for a period of about 100 days [supernova explosion (*3*)].

The precise physical processes which initiate and accompany a supernova explosion are probably different in stars of between 1.2 and about 5 solar masses from those in more massive stars. I shall first describe the mechanism by which the less massive supernovae are produced by tracing out the death of a representative 2-solar-mass star, as predicted by theoretical calculations. Then I shall turn my attention to the more massive supernovae.

A star of 2 solar masses reaches the end point of thermonuclear burning when it has converted all of the hydrogen in its interior to Fe^{56}, the most tightly bound of all nuclei. At this point the star begins to contract quasi-statically, compressing the matter in its center into a smaller and smaller volume.

Now, we know from elementary quantum theory that the smaller the region to which we confine a particle, the larger the particle's zero-point kinetic energy becomes. In particular, when a particle is confined to a region of the order of its Compton wavelength, its zero-point energy becomes of the order of its rest mass; and beyond this point the zero-point energy

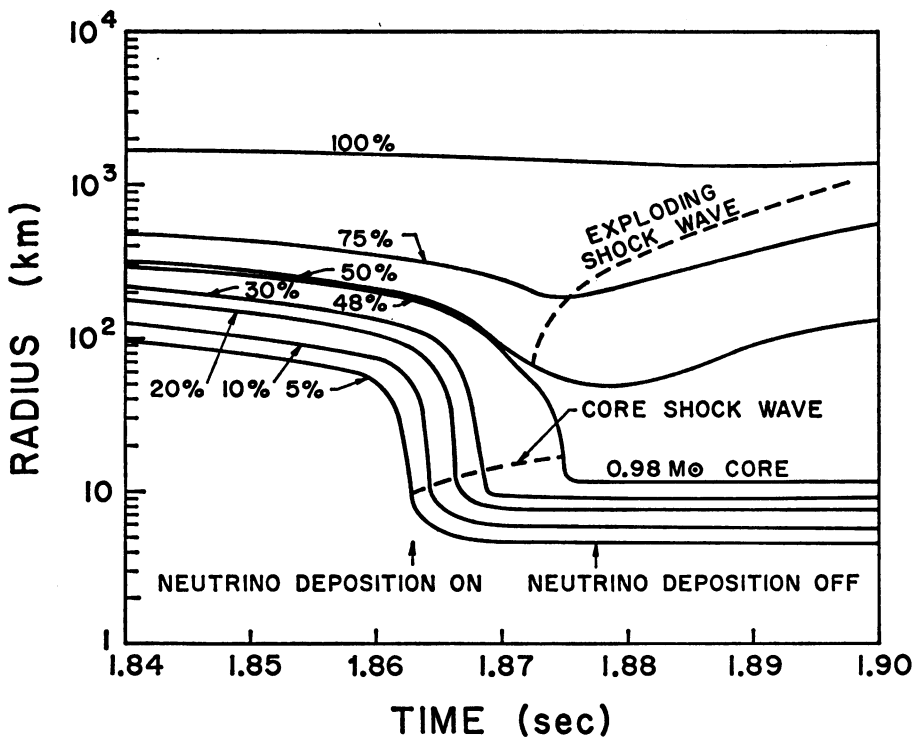

Fig. 1. The dynamics of the collapse and reexplosion of a star of 2 solar masses. Each solid curve represents the radius as a function of time for a spherical shell of matter inside which a certain fraction of the star's mass lies. Each curve is labeled by this "mass-fraction." [Based on calculations by Colgate and White (*4*)]

rises very rapidly with additional compression. Since the Compton wavelength of an electron is 100,000 times that of an Fe^{56} nucleus, electrons will resist being squeezed by the contracting star much sooner than will iron nuclei. As the contraction proceeds, our 2-solar-mass star pushes the zero-point energies of its electrons higher and higher, while the zero-point energies of its iron nuclei remain negligible.

Eventually a point is reached at which the sum of the rest mass of an Fe^{56} nucleus plus the rest mass of an electron plus the electron's rising zero-point energy exceeds the rest mass of a Mn^{56} nucleus. No longer is a free Mn^{56} nucleus unstable against beta decay into electron plus Fe^{56}; rather, electrons and Fe^{56} nuclei are unstable against combining to form Mn^{56} and other neutron-rich nuclei (electron capture); and such combination begins to occur. At this point in the evolution of the star the high-zero-point-energy electrons are providing essentially all the pressure that sustains the weight of the star. When electrons begin to disappear by combining with Fe^{56} nuclei, the sustaining pressure begins to disappear; and the star, which can no longer support itself, begins to collapse catastrophically.

Colgate and White (*4*) have used computers at Lawrence Radiation Laboratory to study the details of this catastrophic collapse (*5*). The details of the collapse of the 2-solar-mass star, as worked out by Colgate and White, are shown in Fig. 1 and are described below.

Supernova Explosions

Catastrophic collapse is initiated by electron capture when the center of the slowly contracting star reaches a density of about 10^{11} grams per cubic centimeter. Within a fraction of a second after initiation of collapse, nearly all the electrons and Fe^{56} nuclei inside the star's core have been transformed into highly neutron-rich nuclei and free neutrons, and the core is in free fall. At 1.86 seconds after initiation the core has acquired a kinetic energy of collapse equivalent to a sizable fraction of its rest mass, and the neutrons in the core have been compressed into regions of the order of their Compton wavelength (density $\sim 10^{14}$ g/cm^3). At this point the zero-point kinetic energy of the neutrons—and with it their zero-point pressure—begins to rise rapidly. The collapsing core is suddenly faced with a huge central pressure, which calls its collapse to a halt and sends a shock wave propagating outward through it. In this core shock front the huge kinetic energy of collapse is converted into heat, and temperatures of over 10 billion degrees are reached. At such high temperatures and densities, elementary particle transformations proceed at a rapid rate, and the heat produced in the core shock front is converted into high-energy neutrinos. The mean free path of the neutrinos is less than 100 meters under these extreme conditions. Hence, instead of escaping freely from the star, the neutrinos diffuse outward, depositing the energy released by the core's collapse in the envelope of the star and thereby raising the envelope to temperatures as high as 200 billion degrees. At these enormous temperatures explosive nuclear burning is initiated in the envelope, with a consequent release of additional thermal energy. Because of the huge thermal energies generated by neutrino deposition and by nuclear burning, the envelope of the star suddenly becomes gravitationally unbound. An exploding shock wave forms and blows the envelope away from the core with speeds approaching the speed of light; and the huge thermal energies of the expanding envelope are converted to radiation so intense that the luminosity of the exploding star approaches that of a galaxy. In addition, nuclear particles are accelerated in the exploding shock wave in such numbers and to such high velocities as to account for a significant fraction of the galactic cosmic rays observed at the earth.

Stars of more than about 5 solar masses undergo supernova explosions similar to that described above. However, the collapse of a massive star's core is initiated not by electron capture, but by the sudden breakup of its Fe^{56} nuclei into He^4 nuclei. This breakup occurs when the temperature in the contracting core has become so high that there are photons present with sufficient energy to disintegrate the Fe^{56} nuclei. The photodisintegration of Fe^{56} reduces the temperature and hence also the pressure in the core of the massive star, and thereby initiates collapse. Once collapse has been initiated, the evolution of a massive supernova is the same as that of supernovae of less than 5 solar masses with one possible exception: Theoretical analyses by W. A. Fowler and

F. Hoyle (with which Colgate and White disagree) (*6*) indicate this, that in a massive star the explosion may be caused, not by the gravitational potential energy released in the core's collapse, but by nuclear energy released in the detonation of O^{16} during the collapse of the star's mantle.

Detailed observations of supernovae (*3*) are in all respects compatible with the above theoretical description of them but do not yet rule out other explanations. A more rigorous test of this description will come from a study of the neutrinos emitted by supernovae—if and when it is technologically possible to detect such neutrinos.

So much for the details of the evolution of a star into its final dead state. Let us now turn to the nature of the final state—to the fate of the core left behind in a supernova explosion and to the final forms of stars which, because their masses are less than the Chandrasekhar limit, never become supernovae.

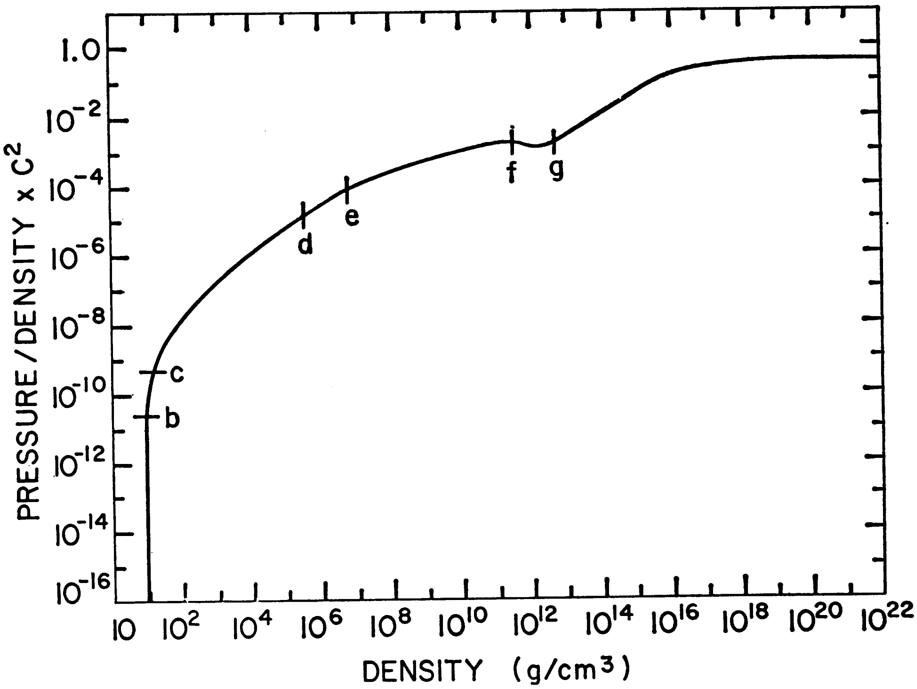

Fig. 2. The Harrison-Wheeler equation of state for matter at the end point of thermonuclear evolution.

Harrison-Wheeler Equation

In order to study the final states of stars, we need an equation of state for the kind of matter from which dead stars are made: matter at the end point of thermonuclear evolution. The equation of state for such matter was calculated in 1958 by B. K. Harrison and J. A. Wheeler (*7, 8*) from a knowledge of the physics of the nucleus. Their calculations were carried to an accuracy as great as present understanding of high-density nuclear physics allows. The resultant "Harrison-Wheeler equation of state" is plotted in Fig. 2.

At low densities (below point *b* of Fig. 2) matter at the end point of thermonuclear evolution is in the form of Fe^{56}, and its pressure is provided by solid-state forces. As the iron is compressed to higher densities (region *b* to *c*), solid-state forces begin to contribute less to the pressure than do orbital electrons of the iron nuclei, which resist being compressed. At point *c* solid-state forces are negligible, and the orbital electrons, which provide all the pressure, constitute a "degenerate Fermi gas," except that they tend to cluster about the iron nuclei. Feynman, Metropolis, and Teller (*9*) have used the Fermi-Thomas statistical model of the atom to correct for this clustering effect, thereby obtaining the pressure-density relation shown by the curve between *c* and *d* in Fig. 2. Between points *d* and *e* the clustering of electrons about Fe^{56} nuclei is negligible, and the electrons form a relativistically degenerate Fermi gas. As the Fermi electron gas and Fe^{56} nuclei are compressed still further (region *e* to *f*), the Fermi energy of the electrons plus the mass of an Fe^{56} nucleus becomes greater than the mass of a Mn^{56} nucleus. Consequently, electrons are squeezed onto the Fe^{56} nuclei to form Mn^{56} and other neutron-rich nuclei (electron capture). As compression becomes greater and greater, the configuration of lowest energy is pushed further and further away from Fe^{56} toward nuclei which are more and more neutron-rich. Eventually neutrons become so numerous that they begin to drip off the nuclei (point *f*), and the material is gradually converted from a mass of neutron-rich nuclei to a dense Fermi gas which is 8/10 neutrons, 1/10 protons, and 1/10 electrons (point *g* and above). At still higher densities the matter at the end point of thermonuclear evolution consists of a mixture of neutrons, protons, electrons, lambda hyperons, and other massive particles which, although highly unstable in the laboratory, are completely stable at extreme densities.

The equation of state is quite well known up to the point at which heavy hyperons become stable ($\sim 10^{15}$ g/cm^3), but totally unknown beyond there. Fortunately, the form of the equation of state in the region of density $\gtrsim 10^{15}$ grams per cubic centimeter is not crucial to my discussion. We can assume for simplicity that in the limit of extreme densities

$$\text{pressure} = (1/3) \times \text{density} \times c^2,$$

where *c* is the speed of light. Large but physically reasonable departures from this limiting form of the equation of state have only small effects on the final states of cold, dead stars.

Uniform Density Approximation

The final, dead state of any star will be a spherical configuration of matter at or near the end point of thermonuclear evolution. How will this matter be distributed inside the star? What will be the star's central density? its radius? its mass? In answering these questions it is useful to introduce the concept of the total number, *A*, of baryons contained in a star.

Baryons are heavy nuclear particles —neutrons, protons, lambda hyperons, and so on. Although baryons can be changed from one form to another (for example, electron + proton $\rightarrow$ neutron + neutrino), the total number of baryons is conserved in any elementary particle transformation.

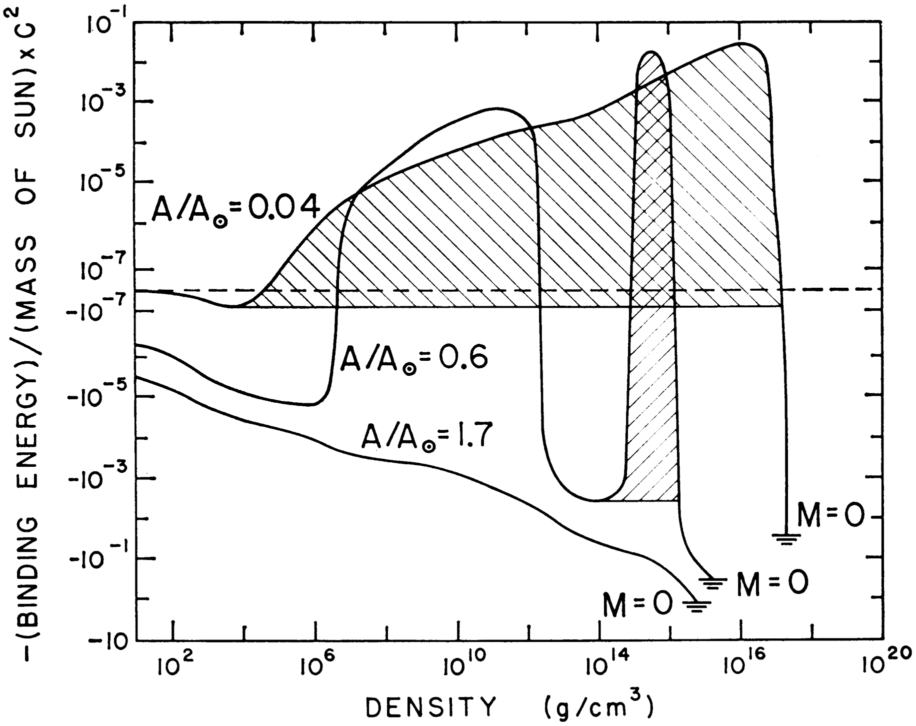

Fig. 3. Spherical configurations of uniform density for the Harrison-Wheeler equation of state: Negative of the binding energy plotted against density. Aside from an additive constant, the quantity plotted on the vertical scale is the total mass-energy of the configuration as measured in units of the mass-energy of the sun. Each curve represents a sequence of uniform density configurations containing a fixed number of baryons, A. ($A_\odot$ is the number of baryons in the sun.) The cross-hatched regions are barriers which separate the configuration of stable equilibrium of highest density from gravitational collapse to zero volume.

For this reason, the total number of baryons inside a star is a measure of the amount of matter which the star contains. By contrast, the mass of a star is not a good measure of its matter content because the mass depends upon the state of binding of the baryons.

An important result valid both in the Newtonian theory of gravitation and in Einstein's theory is that (*8, 10*) in any cold, static star containing a certain number of baryons, the baryons are distributed in such a way as to minimize or maximize the star's total mass-energy [rest mass-energy plus thermal energy—if any—plus internal energy of compression plus (negative) gravitational potential energy]. Hence one way to determine all possible final states for a star containing A baryons is to compare all conceivable distributions of A baryons at the end point of thermonuclear evolution and to pick out those which minimize or maximize the total mass-energy. This would be a difficult task in general, since there are innumerable ways in which the baryons can be distributed throughout a star. Fortunately, however, much can be learned from a comparison of configurations of uniform density.

A comparison of uniform density configurations for matter obeying the Harrison-Wheeler equation of state is made in Fig. 3. On the vertical scale of Fig. 3 is plotted the negative of the binding energy of each configuration —that is, the difference between the total mass-energy of the configuration and the mass-energy it would have if its matter were dispersed to infinite dilution—and on the horizontal scale is plotted the density of the configuration. Each curve represents the uniform density configurations for a star containing a particular number of baryons and can be thought of as a "potential energy curve" for that star.

The qualitative behaviors of the mass-energy curves of Fig. 3 are easily understood in terms of an interplay between negative gravitational potential energy and positive internal energy of compression. Regardless of the number of baryons in a star, as the star is compressed from infinite dilution, its (negative) gravitational energy initially rises more rapidly than its internal energy of compression. Hence, total mass-energy initially decreases. For stars with $A/A_\odot \gtrsim 1.2$ (for example 1.7 in Fig. 3), general relativistic, nonlinear growth of gravitational energy begins so soon that compressional energy can never take over. Hence, for such stars, increasing compaction reduces the total mass-energy monotonically to zero. However, if $A/A_\odot \lesssim 1.2$, rapidly rising pressure causes compressional energy to become more important than gravitational energy before general relativity comes into play. Hence, for such stars total mass-energy reaches a minimum and then rises with increasing compression, until the nonlinear gravitational effects of Einstein's general relativity take over and cause a drop of the total mass-energy to zero. At least this is part of the story. Additional complications arise as a result of a "quirk" in the equation of state for cold matter at the end point of thermonuclear evolution. When a density of $\sim 10^{12}$ grams per cubic centimeter is reached, neutron drip begins to occur in such matter, causing the star's pressure and its energy of compression to rise much less slowly with increasing compaction than at lower densities. (See depression in the curve for the equation of state in Fig. 2.) For a star with $A/A_\odot \gtrsim 0.4$ (for example 0.6 of Fig. 3), negative gravitational energy is rising rapidly enough at this point to dominate the diminished compressional energy and cause total mass-energy to fall temporarily. However, for $A/A_\odot \lesssim 0.4$ (for example, 0.04 of Fig. 3) even the diminished compressional energy at $\sim 10^{12}$ grams per cubic centimeter dominates gravitational energy, and no temporary drop occurs in the total mass-energy.

At each minimum in its mass-energy curve, a star has—in the uniform density approximation—a configuration of stable equilibrium, and at each maximum it has a configuration of unstable equlibrium. Hence, for a star containing 0.04 as many baryons as are in the sun there are two equilibrium configurations: a stable one at the minimum in the mass-energy curve, corresponding to a cold white dwarf star; and an unstable one at the maximum. If a star at the maximum is distended slightly, it will explode; if it is compressed slightly, it will collapse. To what will it collapse? According to general relativity theory, it will collapse to a singularity—that is, into zero volume and to infinite density.

Consider next a star containing 0.6 as many baryons as our sun contains. It has two stable equilibrium configurations (minima of Fig. 3): a cold white dwarf configuration made of a degenerate electron gas and Fe^{56} nuclei, and a neutron star configuration made of a degenerate neutron gas. There are two unstable equilibrium configurations (maxima of Fig. 3); and at the second one, if the star is compressed slightly, it will collapse to zero volume.

Finally consider a cold star at the end point of thermonuclear evolution, which contains more than 1.2 times the number of baryons in the sun—for example, $A/A_\odot = 1.7$ in Fig. 3. Such a star has no equilibrium configurations. There is no way for it to escape gravitational collapse to zero volume.

From Fig. 3, then, we conclude that, if the collapsed core left behind in a supernova explosion is sufficiently massive ($\gtrsim$ 1.2 solar masses), it will not settle down into a cold, dead state. Rather, after it has cooled to near-zero temperature, the core will collapse catastrophically once again, this time to zero volume. We also conclude that a less massive supernova core or a cold star with mass less than the Chandrasekhar limit can be induced to collapse to zero volume if it is compressed sufficiently. These conclusions are so startling that we would like to see them spelled out not only in the approximation of uniform density, but also in the exact theory where the variation of density throughout the star is taken into account.

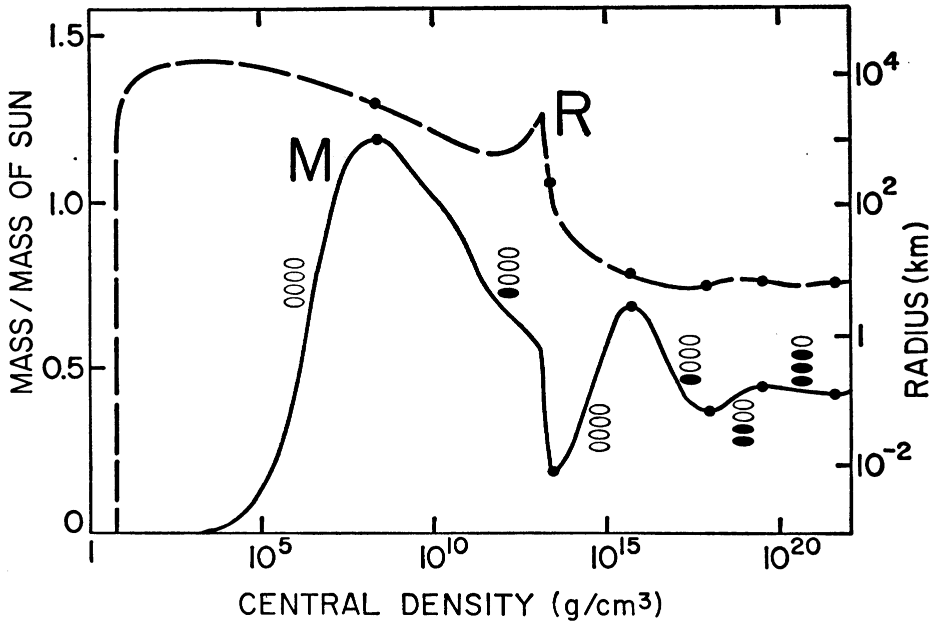

Fig. 4. Harrison-Wakano-Wheeler configurations of hydrostatic equilibrium for cold matter at the end point of thermonuclear evolution. [Based on an extension by B. K. Harrison (*8*) of M. Wakano's original calculations (*7*)]

Harrison-Wakano-Wheeler Configurations

In the exact theory, the variation of density from the center to the surface of an equilibrium star is such as to make its total mass-energy a maximum or a minimum. By analytically performing this extremization, we obtain the general relativity equation of hydrostatic equilibrium

$$\frac{dp}{dr} = -\frac{G(\rho + p/c^2)(m + 4\pi r^3 p/c^2)}{r(r - 2Gm/c^2)}. \quad (1)$$

Here G is Newton's gravitation constant, c is the speed of light, r is radius inside the star, ρ is the mass density, p is the pressure at radius r, and m is the mass-energy inside radius r:

$$m = \int_0^r 4\pi r^2 \rho \, dr. \quad (2)$$

The nonrelativistic (Newtonian) form of the equation of hydrostatic equilibrium, Eq. 1, is obtained by taking the speed of light, c, to be infinite:

$$dp/dr = -G\rho m/r^2. \quad (3)$$

At very high densities and pressures ($\rho \sim p/c^2 \gtrsim 10^{13}$ g/cm³) the general relativity terms in Eq. 1 cause a multiplicative regeneration of pressure: The gravitational force acting on an element of fluid becomes quadratic in its pressure. It is this regeneration of pressure which enables gravitational forces to overwhelm the internal pressure of a star in the relativistic regime, regardless of how high its pressure may be for a given density, and forces excessively dense stars to gravitationally collapse to zero volume.

By integrating Eq. 1 coupled with Eq. 2 for the mass inside radius r and with the Harrison-Wheeler equation of state (Fig. 2), Wakano (*7, 8*) has calculated all possible equilibrium configurations for cold matter at the end point of thermonuclear evolution. Figure 4 shows the masses and radii of these Harrison-Wakano-Wheeler configurations as functions of their central density. Corresponding to each value of the central density there is one and only one HWW equilibrium configuration; and different equilibrium configurations have different masses, radii, and total numbers of baryons, as well as different central densities.

The form of Fig. 4 can be completely understood on physical grounds (*8*), but here I shall only remark that the first oscillation in the mass curve of Fig. 4 is due to electron capture and neutron drip in the matter of which the stars are composed (see Fig. 2 and associated discussion), while subsequent oscillations are due to general relativity (gravitation) effects.

By comparing Fig. 3 (uniform density approximation) with Fig. 4 (exact theory of HWW configurations), we can conclude the following: A star containing 0.6 times as many baryons as our sun will settle down into one of two possible *stable* equilibrium configurations when it dies: a cold white dwarf configuration (first minimum in $A/A_\odot = 0.6$ curve of Fig. 3; configuration at central density 3×10^6 g/cm³ in Fig. 4), or a neutron star configuration (second minimum in $A/A_\odot = 0.6$ curve of Fig. 3; configuration at central density 2×10^{15} g/cm³ in Fig. 4). Alternatively, a star with $A/A_\odot = 0.6$ might attempt to assume one of two possible *unstable*

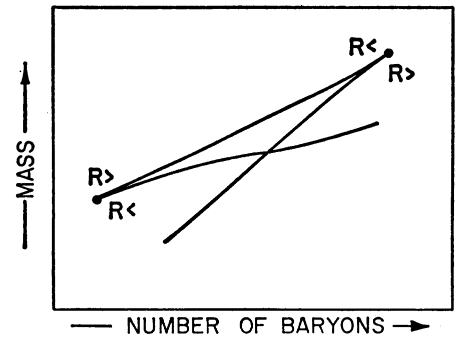

Fig. 5. Harrison-Wakano-Wheeler configurations: A portion of the curve of mass plotted against central density (schematic only). At each cusp the branch of the curve corresponding to configurations of larger radius is labeled "$R>$," and that corresponding to configurations of smaller radius is labeled "$R<$."

equilibrium configurations when it dies: one of central density 3×10^{12} grams per cubic centimeter (first maximum in $A/A_{\odot} = 0.6$ curve of Fig. 3) or one of central density 2×10^{16} grams per cubic centimeter (second maximum of Fig. 3). There are no equilibrium configurations at all for a dead star containing more than ~ 1.2 times the number of baryons in the sun; such a dead star must inevitably collapse to a singularity.

Stability of HWW Configurations

The above comparison of Figs. 3 and 4 enables us to conclude that certain of the HWW equilibrium configurations are stable against gravitational collapse or explosion, whereas others are unstable. However, this method for studying stability has the disadvantage of being nonrigorous (recall that Fig. 3 is based upon the uniform density *approximation*), and it is not applicable to all HWW configurations. To determine precisely which of the HWW configurations are stable and which are unstable it is better to use completely rigorous and beautifully simple criteria formulated by Wheeler (*8*), a description of which I present here:

A star lying at a maximum or a minimum in the curve of mass plotted against central density must have a zero frequency mode of radial oscillation, for such a star can move back and forth on the peak or valley of the mass curve without changing its mass. This mode of oscillation changes stability as we pass from configurations on one side of the peak or valley to configurations on the other.

To determine whether the critical mode of radial oscillation becomes stable or becomes unstable at a particular peak or valley, we consider in Fig. 5 a portion of the curve of mass plotted against total number of baryons for the HWW equilibrium configurations. To each peak or valley in the mass curve of Fig. 4 there corresponds a cusp in Fig. 5. Near a given cusp, configurations on the higher branch of Fig. 5 will be less stable than those on the lower branch; in going from the lower to the higher branch the critical acoustical mode of oscillation goes from stability to instability. Now, the slope of the curve in Fig. 5 is the change in mass, dM, of an equilibrium configuration when a single baryon, dA, is brought in from infinity and gently deposited on the surface of the star; that is the slope is

$$dM/dA = m_b \times (1 + P)$$
$$= m_b \times (1 - GM/c^2R) \quad \text{Newtonian theory}$$
$$= m_b \times (1 - 2GM/c^2R)^{\frac{1}{2}} \quad \text{General relativity theory} \qquad (4)$$

Here m_b is the mass of one baryon, P is the gravitational potential at the surface of the star, G is Newton's gravitation constant, c is the speed of light, and M and R are the total mass and radius of the equilibrium configuration. Near a critical point (cusp of Fig. 5, peak or valley of mass curve in Fig. 4), M changes very little with increasing density, but R changes rapidly. Consequently, at a cusp of Fig. 5 the branch of larger-radius configurations has the larger slope, dM/dA. If the cusp is a point of maximum mass, the configurations with larger radii and larger dM/dA lie on the lower branch of Fig. 5 and are thus more stable than the configurations with smaller radii. If the cusp is a point of minimum mass, the configurations with larger radii lie on the upper branch and are thus less stable than the configurations with smaller radii.

These conclusions are summarized in Table 1. Using this table, we can determine from Fig. 4 the precise number of unstable modes of radial oscillation for each HWW configuration: A sphere of iron the size of a golf ball is stable against collapse or explosion. Since the lowest mode of radial oscillation cannot become unstable until a peak in the mass curve of Fig. 4 is reached, all configurations with central density less than 3×10^8 grams per cubic centimeter (white dwarf stars) are stable. At the first maximum of the mass curve the radius of the star is decreasing. Hence (see Table 1) the first radial mode becomes unstable there. We denote this instability by blackening the lowest oval in Fig. 4. At the first minimum the radius is once again decreasing, so the lowest mode becomes stable again, and we lighten the lowest oval of Fig. 4. Between the first minimum and second maximum lies the regime of stable neutron stars. At the second maximum the radius is again decreasing, so the first mode becomes unstable; at the second minimum the radius is increasing, so a second mode becomes unstable; and so on.

Table 1. Wheeler's criteria for determining the change in stability of the critical mode of radial oscillation at a critical point as central density increases.

Behavior of radius at critical point	Direction of stability change
Maximum mass	
Decreases	Becomes unstable
Increases	Becomes stable
Minimum mass	
Decreases	Becomes stable
Increases	Becomes unstable

It is possible to make this analysis of stability quantitative by means of a variational principle originally formulated by Chandrasekhar (*11*). Meltzer and I (*12*) have used Chandrasekhar's variational principle to determine the values of the frequencies of the lowest three modes of radial oscillation for the HWW configurations. The results of our calculations, shown in Fig. 6, are in perfect agreement with Wheeler's qualitative results.

Thus far I have not discussed stability and instability of HWW configurations against nonradial perturbations. It is generally assumed—though I do not know of any proof—that a cold star can be dynamically unstable against nonradial perturbations only if its lowest radial mode is also unstable. If this is true, then the analysis of purely radial oscillations is sufficient to reveal the absolute stability or instability of all HWW configurations. According to this analysis there are only two regions of stability: The white dwarf region (central density less than 3×10^8

g/cm³) and the neutron star region (central density between 3 × 10^{13} g/cm³ and 6 × 10^{15} g/cm³). The white dwarf region includes stars containing less than ~1.2 times the number of baryons in the sun, while the neutron star region includes stars containing between ~ 0.25 and ~ 0.7 times the number of baryons in the sun (*13*). Any supernova core which contains more than ~ 1.2 times the number of baryons in the sun—and also less massive cores which become sufficiently compressed in the dynamics of supernova collapse—must gravitationally collapse to zero volume. There are no equilibrium configurations for such stars.

Comparison with Observations

After a white dwarf star is formed, it cools off slowly (cooling time, several billion years) by radiating away its thermal energy as light. Astronomical measurements of the masses and radii of radiating white dwarfs are in fairly good agreement with the predictions of Fig. 4 (*14*) and are in excellent agreement with other, more detailed stellar models which take into account rotation and varying chemical composition.

Neutron stars are not as easily observed as white dwarfs. During the first few thousand years after a neutron star is formed it is so hot that essentially all its radiation is in the x-ray region of the spectrum, which is blacked out by the earth's atmosphere (*15*). By the time the star has become cool enough to radiate primarily in the optical region, its luminosity is so low that the star could not be detected with the 200-inch telescope unless it were within a light-year of the earth. Hence, the only hope for detection of neutron stars is with rocket- and satellite-borne x-ray telescopes. Recent observations (*16*) with rocket-borne x-ray telescopes have revealed x-ray sources, some of which may be neutron stars. However, the most promising of the sources—a source in the Crab nebula, which was formed by a supernova explosion observed in A.D. 1054—is now known (*17*) not to be a neutron star.

Finally, what are the prospects for observing a star as it gravitationally collapses to a singularity? Given even the most powerful technology conceivable in the next century, the prospects for observing collapse toward a singularity are nil.

I shall conclude with a brief description of the dynamics of the gravitational collapse to zero volume, which —according to general relativity theory —will be the fate of sufficiently massive supernova cores. In Fig. 7, I use a spacetime diagram to depict the dynamics of the collapse. In this diagram time is plotted vertically and radial distance is plotted horizontally; angular directions are suppressed from the diagram since the collapse is assumed to be spherically symmetric. The time and distance scales chosen are such that radial light rays move along 45-degree lines; but because the gravitational field of the star "warps" the geometry of spacetime, freely moving particles do not move along straight lines in the diagram. The surface of the collapsing star moves through spacetime along the indicated curve, while an observer moving around the collapsing star in an orbit of fixed radius moves along the indicated hyperbola. The collapsing star generates a singularity (jagged hyperbola) in spacetime which swallows the star up; and any other object which falls into this "Schwarzschild singularity" also gets destroyed there.

Suppose that a man on the surface of the collapsing star sends out a series of signals (wavy 45-degree lines in Fig. 7) to the orbiting observer, informing him of the progress of the collapse. As the star gets closer and closer to a certain critical radius, called its "Schwarzschild radius" (intersection of path of surface of star with dotted 45-degree line in Fig. 7), the signals, which are sent at regularly spaced in-

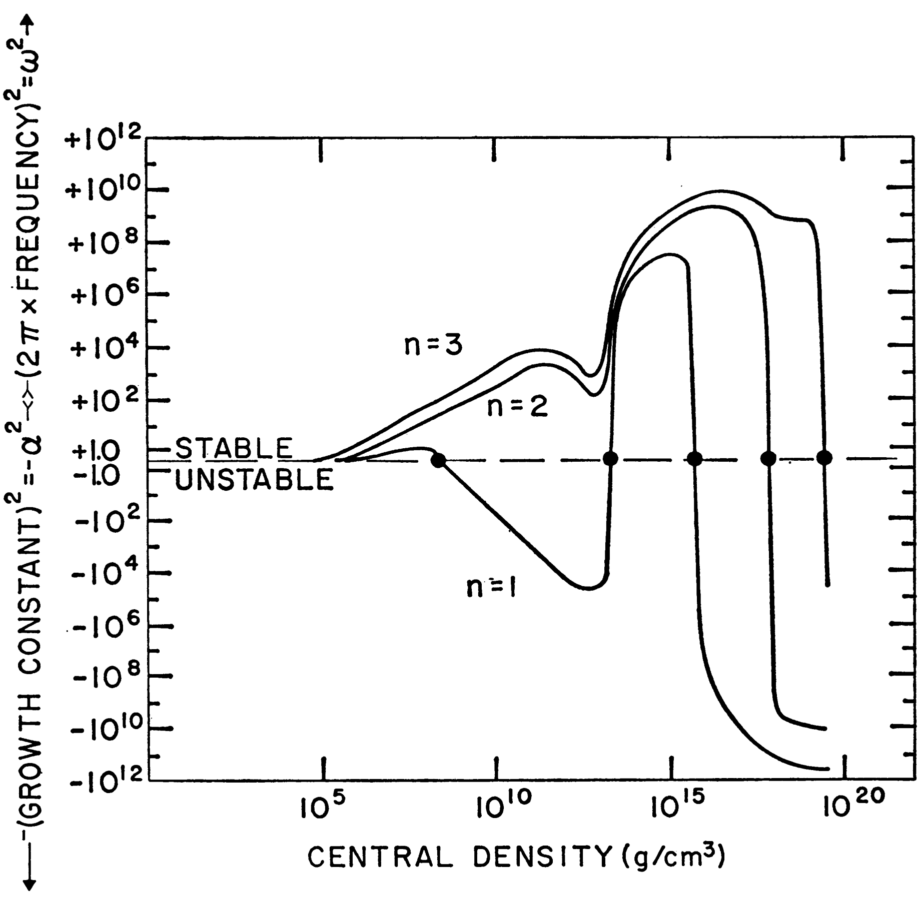

Fig. 6. Frequencies of the lowest three modes of radial oscillation for the Harrison-Wakano-Wheeler configurations. In the stable region the amplitude of radial oscillation varies sinusoidally [amplitude = (function of radius) × cos ωt]; while in the unstable region the amplitude grows exponentially [amplitude = (function of radius) × $e^{\alpha t}$]. In this figure the square of the angular frequency, ω, is plotted against central density in regimes of stability, and the square of the growth constant, α, is plotted against central density in regions of instability. Note the correlation of points of changing stability (black dots) here with maxima and minima in the mass curve of Fig. 4; and note the agreement of the direction of change of stability with the results of Wheeler's analysis (text, Table 1, and blackened ovals of Fig. 4).

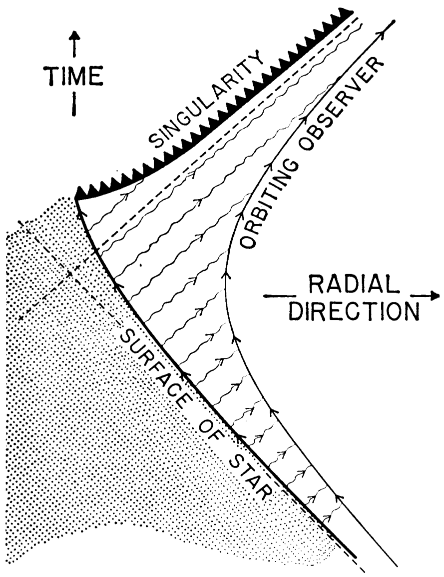

Fig. 7. Spacetime diagram illustrating the dynamics of gravitational collapse to zero volume. The time and radial coordinates are those of Kruskal (*18*), but for the purposes of qualitative exposition they can be thought of as the intuitive time and radial coordinates of Newtonian theory.

tervals, are received by the observer at more and more widely spaced intervals. The observer does not receive a signal emitted just before the Schwarzschild radius is reached, until after an infinite amount of time has elapsed according to his clocks; and he never receives a signal emitted after the Schwarzschild radius is passed. That signal, like the man who sent it, gets caught and crushed out of existence in the singularity.

Hence, to the orbiting observer, the collapsing star appears to slow down as it approaches its Schwarzschild radius; light from the star becomes more and more red-shifted; and clocks on the star appear to run more and more slowly. It takes an infinite time for the star to reach the Schwarzschild radius, and as seen by the orbiting observer, the star never gets beyond there.

But to the man standing on the star as it collapses into oblivion there is nothing at all special about the Schwarzschild radius. He passes right through it and on into the singularity in a fraction of a second. What happens to the man on the star as collapse nears completion? Tidal gravitational forces squeeze him from the sides and stretch him between head and foot. As the singularity approaches, these tidal forces become infinitely strong and the man's body is stretched like a rubber band and simultaneously compressed from the sides to infinite density and zero volume.

At least, this is the picture according to classical general relativity theory. But when a density of $\sim 10^{49}$ grams per cubic centimeter is reached, classical general relativity breaks down. To follow the collapse beyond this point, one must quantize the gravitational field—that is, combine general relativity theory with quantum mechanics—a formidable task which, as yet, is far from completion.

Summary

When a star has burned all its nuclear fuel, it enters a phase of slow, quasi-static gravitational contraction. If the mass of the star is less than the Chandrasekhar limit ($\sim$ 1.2 solar masses) this contraction is brought to a halt by the rising pressure of the star's electrons, which resist being compressed into a small volume; and the star becomes a cold, white dwarf of central density $\sim 10^6$ grams per cubic centimeter and radius $\sim$ 10,000 kilometers. If the star's mass exceeds the Chandrasekhar limit, quasi-static contraction leads to a state of instability against gravitational collapse. Collapse of the star's core proceeds, with the subsequent release of the energy of collapse as neutrinos, which blow away the envelope of the star (supernova explosion). If the supernova core contains less than $\sim$ 1.2 times the number of baryons in the sun and if the core does not become too compressed during supernova collapse, then the core will settle into the form of a cold, white dwarf star (central density $\sim 10^6$ g/cm^3, radius $\sim$ 10,000 km) or a neutron star (central density $\sim 10^{14}$ g/cm^3, radius $\sim$ 10 km). However, if the core contains more than $\sim$ 1.2 times the number of baryons in the sun, or if it becomes sufficiently compressed during supernova collapse, then it will not reach a state of cold, hydrostatic equilibrium; rather, it will gravitationally collapse to zero volume.

This is the story of the death of a star as predicted by a combination of nuclear theory, elementary particle theory, and general relativity theory. Not taken into account in this analysis are such complications as stellar rotation, deviation from spherical symmetry, and effects of magnetic fields. These unaccounted-for factors undoubtedly have considerable effect on the quantitative results reported here; but it is doubtful that, except in extreme cases, they can change the qualitative picture.

References and Notes

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Genetic Transfer in Bacterial Mating

What mechanism insures the orderly transfer of DNA from donor to recipient cells?

Julian D. Gross and Lucien Caro

The genetic material controlling all the essential functions of *Escherichia coli* is organized into a single chromosome which consists, as far as is known, of a continuous double-stranded molecule of DNA, approximately 1100 microns long (*1–3*). Both genetic and microscopic evidence indicate that this chromosome has a circular structure (*3, 4*). Most of the DNA constituting the chromosome is packed into a loosely defined nuclear region less than 0.1 cubic micron in volume. In a fast-growing culture the cells of *E. coli* are 2 to 3 microns long and 0.8 micron in diameter. They contain two to four chromosomes, in various stages of replication. The cells grow by elongating, forming a constriction at the equator, and separating into two daughter cells each containing two chromosomes (*1, 5*). Each chromosome replicates once during each generation, and the products are segregated so that, at division, each daughter cell receives the appropriate number of chromosomes.

Cells of *E. coli* harboring the sex factor F or other similar elements, all of which are constituted entirely or primarily of DNA (*6*), can form a cellular connection with suitable recipient cells. DNA, corresponding to the sex factor, is then efficiently transferred from donor to recipient. In strains in which the sex factor has become associated with the chromosome (Hfr cells), conjugation results in the progressive linear transfer of the entire chromosome, at a rate such that transfer is complete in about 90 minutes (*7*). A striking aspect of this process is that, for any one Hfr strain, the chromosome is transferred in precisely the same sequence from all mating cells. The origin of the sequence is defined by the position at which the F factor had been inserted into the circular bacterial chromosome (*3*).

Various models have been proposed as to how the process of DNA transfer in conjugation may be related to the mechanisms which coordinate chromosome replication and cell growth. In this article we describe these models and discuss experiments which have a bearing on them.

Conjugation in E. coli

The most studied system of conjugation is the one, just mentioned, controlled by the transmissible sex factor F. There are three main mating types: F^-, F^+, and Hfr. F^- cells lack F entirely: they can act only as recipients in matings with donor cells, and they do so with much higher efficiency than either F^+ or Hfr cells. F^+ cells transfer their sex factor with high frequency to F^- cells, converting them to the F^+ type (*8*). The F factor itself is the only genetic material normally transferred in such matings. F^+ cells do occasionally, however, give rise to stable Hfr derivatives capable of transferring the entire bacterial chromosome (*3*).

Genetic experiments, which have been reviewed extensively (*3, 9*), indicate that transfer of the bacterial chromosome by an Hfr is rarely complete. Instead, the majority of the F^- cells receive only a segment of the Hfr chromosome. The frequency of transmission, for any chromosome determinant, decreases with the distance of the determinant from the origin of transfer. The sequence of transfer of genetic markers can be precisely determined by artificially interrupting the mating at various times and assaying for the inheritance, by the F^- cells, of a series of Hfr determinants. Transferred markers are expressed as a result of recombination between the Hfr chromosomal fragment and the F^- chromosome. In interrupted matings, the capacity to act as an Hfr donor is invariably the last character transferred. Hfr cells occasionally revert to the F^+ type or give rise to cells with variant sex factors (F′ factors) capable of transferring, in addition to F itself, a number of genetic markers previously located on one or both sides of the origin of transfer on the circular Hfr chromosome (*10*).

The properties of Hfr cells may be accounted for by postulating that they arise by integrating the F factor into the continuity of the circular bacterial chromosome at any one of a number of possible points. This would be accomplished by a pairing between the sex factor and the chromosome, followed by a reciprocal genetic exchange. The process could be reversed to pro-

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