THE RESISTANCE OF MAGNETIC FLUX TO GRAVITATIONAL COLLAPSE*

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I. Introduction and Summary

One of the most significant characteristics of Einstein's general theory of relativity is the extent to which physical singularities pervade solutions to the field equations: Among all matter-filled cosmological models constructed to date within the framework of Einstein's theory (sans cosmological constant), not one is free of singularities both in the remote past and in the remote future.[1] On a smaller scale, general relativity tells us[2] that any non-rotating star, which has reached the end point of thermonuclear evolution and contains more than $A_{crit} \approx 10^{57}$ baryons, must gravitationally collapse to a singularity in a proper time of the order of seconds. Even spherical configurations of cold matter containing much less than 10^{57} baryons cannot escape collapse if they are subjected to sufficient external pressure.

At this point in the development of the theory of gravitational collapse, it is important to determine precisely how inevitable the evolution of singularities is: To what extent can rotation of a massive object or a cosmological model impede or prevent its collapse to a singularity?

The purpose of this paper is to propose a partial answer to the last of these questions: All evidence now available suggests that magnetic and electric field lines resist gravitational collapse; no matter how tightly they are compressed, the gravitational attraction between field lines can never overcome their Maxwell-Faraday repulsion. Let us put this point more precisely:

Principle of Flux Resistance to Gravitational Collapse: In a configuration of electromagnetic fields gravitationally collapsing to a singularity, the total electric and magnetic flux across each 2-surface in the collapsing region must vanish as the singularity is reached—a non-zero flux will stop the collapse. In more mathematical terminology: Let S_2 be an arbitrary 2-surface passing through the region in which collapse is occurring, just before the singularity is reached; and let $\bar{S}_2$ be that portion of S_2 which is in the collapsing region. Then, the principle of flux resistance to gravitational collapse states that

$$\int_{S_2} f^{ij}\, dS_{ij} = 0 = \int_{\bar{S}_2} {}^*f^{ij}\, dS_{ij}, \qquad (1)$$

where ${}^*f^{ij}$ is the dual of the electromagnetic field tensor f_{ij}. This principle is illustrated in Fig. 1.

Comments on the principle: (1) At our present stage of knowledge, the principle of flux resistance to gravitational collapse can only be a conjecture; there is as yet no hard-and-fast proof of its validity within the framework of Einstein's theory. However, considerable evidence for it can be evoked, as we shall see in Section II. (2) As an example, if this principle is valid, then a toroidal bundle of magnetic field lines (geon)[3] of minor radius a and major radius b cannot gravitationally collapse to its guiding line ($a \to 0$, b remain finite), but it might collapse to its center ($a \to 0$ and $b \to 0$ simultaneously). In Section II we will see that the dynamical behavior of a toroidal magnetic geon is in accord with this prediction. (3) As a second

*Expanded version of a paper presented at the Second Texas Symposium on Relativistic Astrophysics, Austin, Texas, December 15-19, 1964. This work was supported in part by the U. S. Air Force Office of Scientific Research.

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[1] For a discussion of this point see, e.g., the contribution of Shepley (1965) to this volume.

[2] For a review of the evidence see Harrison, Thorne, Wakano, and Wheeler (1965).

[3] The concept of a geon was first introduced by J. A. Wheeler and is discussed extensively in Wheeler (1963).

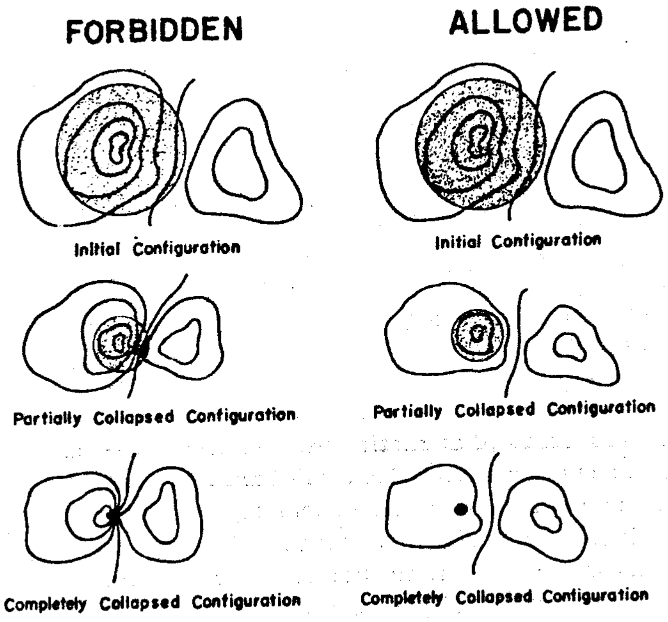

FIG. 1. A schematic illustration of the principle of flux resistance to gravitational collapse. Two possible histories are shown for the gravitational collapse of a system composed of electromagnetic fields. One history is allowed by the principle of flux resistance to gravitational collapse; the other is forbidden. In the figure each history depicts the electromagnetic field on a succession of spacelike hypersurfaces approaching the singularity. The shaded areas represent the region in which the electromagnetic field is gravitationally collapsing, and the solid lines represent magnetic field lines. The electric field is not depicted. In the forbidden mode of collapse, magnetic flux threads the collapsing region at the moment when the singularity is reached, but in the allowed mode, every magnetic field line is either swallowed in its entirety by the singularity or left entirely free—there is no net flux through the singularity.

example, the principle of flux resistance to gravitational collapse does not forbid the collapse of a cloud of electromagnetic radiation or of a radiation-filled universe. This is fortunate, since Tolman (1934) has constructed a radiation-filled cosmological model which both explodes from a singularity and collapses to a singularity. (4) This principle is meant to apply only when electromagnetic fields alone are present. We exclude from attention systems in which particles or neutrinos or other fields contribute to the stress-energy. However, if the principle is valid, then electric and magnetic flux probably also show a partial (or even complete) resistance to collapse in the presence of particulate matter;[4] and magnetic fields might, consequently, play an important role in the collapse of astrophysical objects.[5] (5) The principle asserts that both magnetic and electric flux resist collapse because, in the absence of charges and currents, the electric field and the magnetic field are dynamically equivalent. (6) This principle treats only classical electromagnetic fields interacting with classical gravitational fields in accordance with the laws of general relativity. The quite new phenomena which enter when the electromagnetic field strength reaches the critical value $F_{crit} = mc^2/[e(\hbar/mc)] = 4.4 \times 10^{13}$ gauss $= 1.3 \times 10^{18}$ volt/m, where vacuum polarization effects become important, are not taken into account.

The remainder of this paper is a presentation of evidence supporting the principle of flux resistance to gravitational collapse. We briefly summarize that evidence before presenting it in detail:

A prototype for spherically symmetric gravitational collapse is the collapse of the Einstein-Rosen bridge of the Schwarzschild solution. This collapse is characterized by the pinching-off of the throat of the bridge as its time development is followed. As our first evidence for the principle of flux resistance to gravitational collapse, we review the discovery by Graves and Brill (1960) that, if the Einstein-Rosen bridge is threaded by electric or magnetic flux, then the resistance of that flux to collapse causes the throat to pulsate rather than pinch off.

A second piece of evidence is the existence of many different model electromagnetic universes which do not undergo gravitational collapse, and no known ones that do collapse—except the Tolman universe (cf. comment (3) above) and Lindquist's toroidal magnetic universe (cf. Sec. IIC), whose dynamics are compatible with our conjecture. (Contrast this with matter-filled cosmological

[4] Ginzburg (1964), Ginzburg and Ozernoy (1964), Novikov (1964), and Kardashev (1964) have recently discussed the role of magnetic fields in the gravitational collapse of massive stars. Ginzburg and Ozernoy find that, as a massive magnetic star collapses through its Schwarzschild radius, it pulls its magnetic field lines into its surface and carries them all, in their entirety, into the singularity. This result is what would be expected if magnetic flux resists gravitational collapse in the presence of matter.

[5] Colgate (1965) argues that magnetic field cannot play an important role in the gravitational collapse which initiates supernova explosions.

models, of which there are none known that do not either collapse to a singularity or explode from one.) Of all known model electromagnetic universes, one due to Melvin (1964) is of particular interest. Melvin's universe, which consists of a cylindrically symmetric magnetic field pointing along the axis of symmetry, has been proved to be stable against large as well as small radial perturbations (Melvin 1965; Thorne 1965b); it cannot be induced to evolve a singularity as the result of any finite perturbation.

These first two pieces of evidence for the principle of flux resistance to gravitational collapse are not as satisfying as would be the analysis of more physical electromagnetic systems, around which spacetime is asymptotically flat. Fortunately, the third piece of evidence has what the others lack: It is an analysis of the dynamics of a toroidal bundle of magnetic field lines (geon)[3] residing in asymptotically flat spacetime. This analysis reveals that, in keeping with the principle of flux resistance, a toroidal magnetic geon with magnetic field initially uniform inside the torus cannot collapse to its guiding line (minor radius go to zero, major radius stay finite).[6]

II. Evidence for the Principle of Flux Resistance to Gravitational Collapse

A. Dynamics of the Einstein-Rosen Bridge

The Schwarzschild solution to the vacuum field equations of general relativity has been known for nearly fifty years, but only in the last five years has it been really understood. This is because Schwarzschild's line element

$$ds^2 = (1 - 2m/r)dt^2 - (1 - 2m/r)^{-1}dr^2 - r^2(d\theta^2 + \sin^2\theta\, d\varphi^2) \quad (2)$$

has a coordinate singularity at $r = 2m$ and is incomplete—from any point (r, t) there are spacelike and timelike geodesics to $(r = 2m, t = +\infty)$ or to $(r = 2m, t = -\infty)$ with finite proper length.[7] Our modern understanding of the Schwarzschild solution stems from the work of Kruskal (1960).[8] Kruskal completed the Schwarzschild solution in a manner which exhibits the true nature of the region $r = 2m$. We will not be concerned here with the relationship between the Schwarzschild solution and the Kruskal completion of it, nor with the nature of the region $r = 2m$; rather, we shall confine ourselves to a review of the dynamics of Kruskal's solution.

Kruskal's completion of the Schwarzschild solution is expressed in terms of a new time coordinate, v, and a new radial coordinate, u, in the form

$$ds^2 = f^2(dv^2 - du^2) - r^2(d\theta^2 + \sin\theta\, d\varphi^2). \quad (3)$$

Here f and r are given in terms of u and v by

$$[(r/2m) - 1]\, e^{r/2m} = u^2 - v^2,$$
$$f^2 = (32m^3/r)e^{-r/2m}. \quad (4)$$

The r appearing here is Schwarzschild's r coordinate; and Schwarzschild's t coordinate is related to u and v by

$$\tanh(t/2m) = 2uv/(u^2 + v^2). \quad (5)$$

Fuller and Wheeler (1962) have made clear the dynamical behavior of Kruskal's solution. The spacelike hypersurface $v = 0$ ($t = 0$, $r \geq 2m$ in Schwarzschild coordinates) is a bridge or "wormhole" between two asymptotically flat spaces (see Fig. 2). (It is often called the "Einstein-Rosen bridge," since Einstein and Rosen (1935) discussed it extensively.) One can follow the dynamical evolution of this wormhole in Kruskal's solution by looking at the geometry of a succession of spacelike hypersurfaces, each lying to the future (+v direction) of the preceding one. Such an analysis reveals that the throat of the wormhole undergoes gravitational collapse; it pinches off, disconnecting the asymptotically flat spaces originally linked by the wormhole (see Fig. 3).

Now, suppose that a magnetic field were made to thread the wormhole. The collapse of the wormhole would provide a means for squeezing the magnetic field into a smaller and smaller region. But if the principle of flux resistance to gravitational collapse is correct, the magnetic field should protest against this squeeze; it should actually halt the collapse before the throat pinches off.

[6] For independent evidence that a toroidal magnetic geon may not collapse to its guiding line, see Thorne (1964). For evidence that collapse to the center of the torus should occur for sufficiently massive geons, see Thorne (1964) and Wheeler (1964).

[7] For a discussion of geodesics in the Schwarzschild solution, see Fuller and Wheeler (1962).

[8] See also Fronsdal (1959).

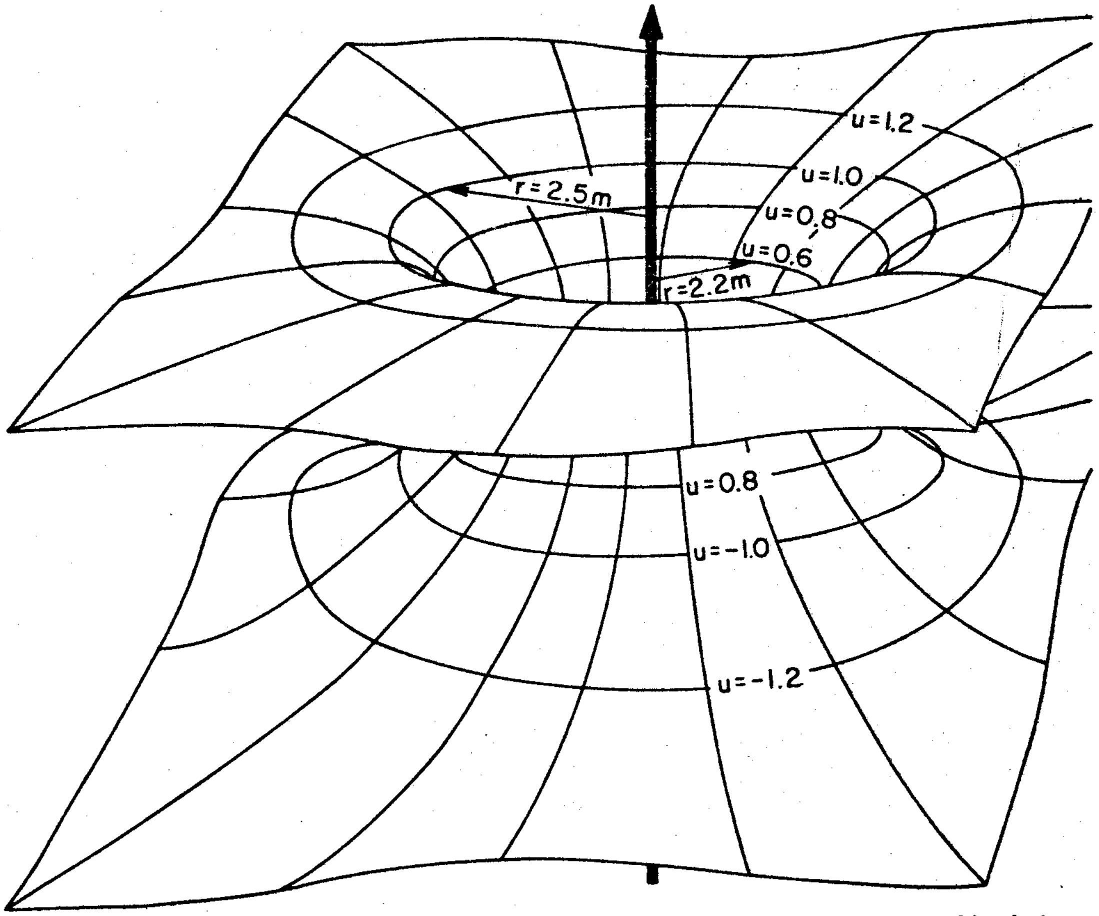

FIG. 2. The 2-surface ($v = 0$, ϕ = constant) of Kruskal's completion of the Schwarzschild solution, as it appears when embedded in 3-dimensional Euclidean space. The hypersurface $v = 0$ is the 3-dimensional analogue of this surface, embedded in a 4-dimensional Euclidean space. The bridge or wormhole connecting the upper and lower asymptotically flat surfaces is often called the "Einstein-Rosen bridge," since Einstein and Rosen (1935) discussed it extensively; however, Weyl (1917) described it much earlier. (This figure was kindly provided by J. A. Wheeler.)

Graves and Brill (1960) have given the solution to the Einstein-Maxwell field equations for a wormhole threaded by electric or magnetic flux. Their solution is a completion of the Reissner-Nordström solution for a "charged, point mass" in the same way as Kruskal's solution is a completion of the Schwarzschild solution for an "uncharged point mass." Graves and Brill find that even a very minute amount of magnetic flux threading the wormhole will cushion its collapse. Rather than pinching off, the throat oscillates in and out between its initial radius r_{max} and a minimum radius

$$r_{min} = \frac{G}{16\pi^2 c^4} \times (\text{Total flux threading the throat})^2 \times \frac{1}{r_{max}}$$

(see Fig. 4). Hence, in this particular example, the (ex post facto) predictions of the principle of flux resistance to gravitational collapse are borne out.

B. Non-Collapsing Model Electromagnetic Universes

The easiest test of the principle of flux resistance to gravitational collapse which can be performed is to search the literature for counterexamples to the principle. The author's search has revealed none.

Nearly all electromagnetic systems with strong gravitational fields which have been studied are non-asymptotically flat at spatial infinity. We call such systems "model electromagnetic universes." There has been much interest in model electromagnetic universes recently (Bertotti 1959; Misra and Radhakrishna 1961; Brill 1964; Melvin 1964, 1965; Thorne 1965a, b; Harrison 1965. Of the model universes recently exhibited, several deserve special mention in connection with the principle of flux resistance to gravitational collapse:

Brill (1964) has given a family of model

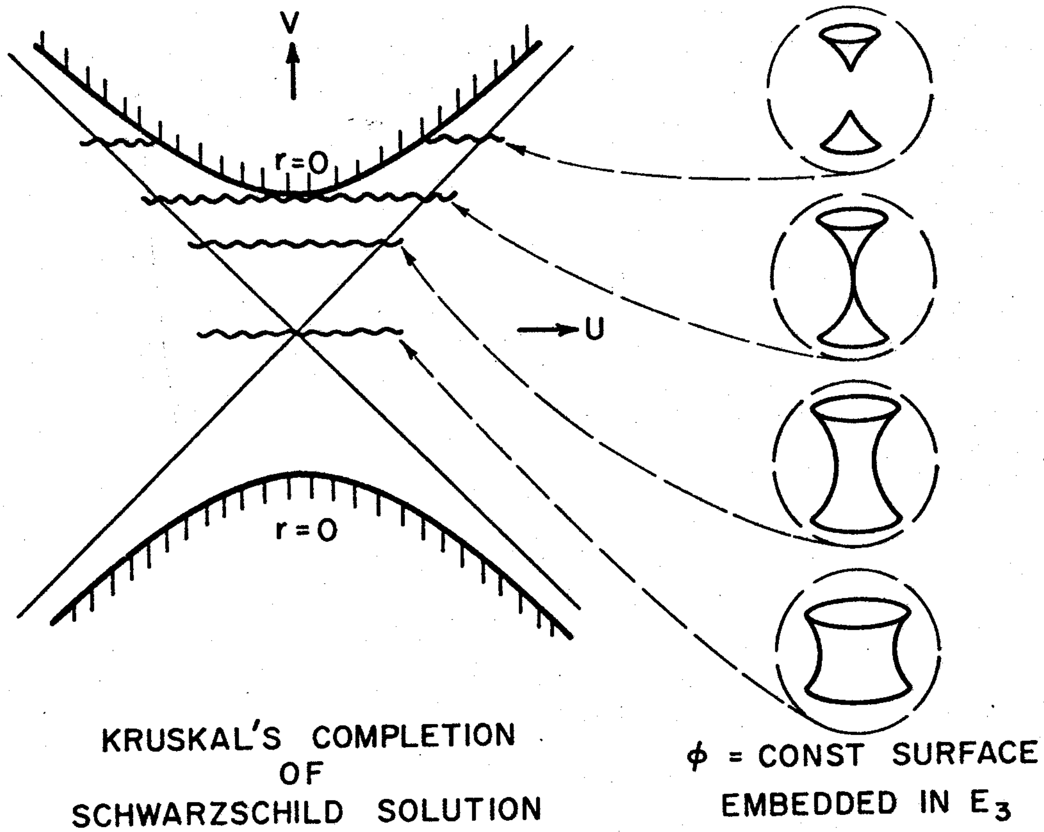

FIG. 3. The dynamics of the throat of the Einstein-Rosen bridge. On the left is a diagram of the Kruskal u-v coordinate plane, showing a succession of spacelike hypersurfaces, each one to the future of the preceding one. On the right is a picture of each of these hypersurfaces embedded in a 3-dimensional Euclidean space. (The angle of rotation, ϕ, is suppressed.) These successive "snapshots" of the throat of the wormhole reveal that it pinches off; it gravitationally collapses to a singularity.

electromagnetic universes which are generalizations of the Taub-NUT vacuum solution to Einstein's equations. Like the Taub-NUT solution, Brill's universes do not possess any physical singularities; however, the dust-filled generalizations of the Taub-NUT solution, which have been given by Behr (1961) and by Shepley (1965), all evolve singularities.

Melvin (1964) has described a static, cylindrically symmetric magnetic universe in which the magnetic field points along the axis of symmetry. This model universe not only does not gravitationally collapse, but no large or small radial perturbation can cause it to collapse (Melvin 1965; Thorne 1965b).

Thorne (1965a) has shown that no cylindrical electromagnetic universe, which is non-singular in a certain canonical coordinate system on some initial spacelike hypersurface, can undergo gravitational collapse.

C. Toroidal Magnetic Geons

In our discussion of evidence supporting the principle of flux resistance to gravitational collapse, we turn now to toroidal magnetic geons residing in asymptotically flat spacetime. We shall consider the family of all geons whose initial configuration has the following properties: (1) It is a momentarily static configuration ("configuration of time-symmetry"). (2) It contains only a magnetic field; the electric field vanishes everywhere. (3) The magnetic field is contained inside a thin-ring torus (torus with minor radius much smaller than major radius), where it is uniform and parallel to the guiding line of the torus. (4) There is no gravitational radiation present anywhere (see Fig. 5).

The members of this family will range from geons with such dilute magnetic field that their dynamics can be treated with great accuracy in the

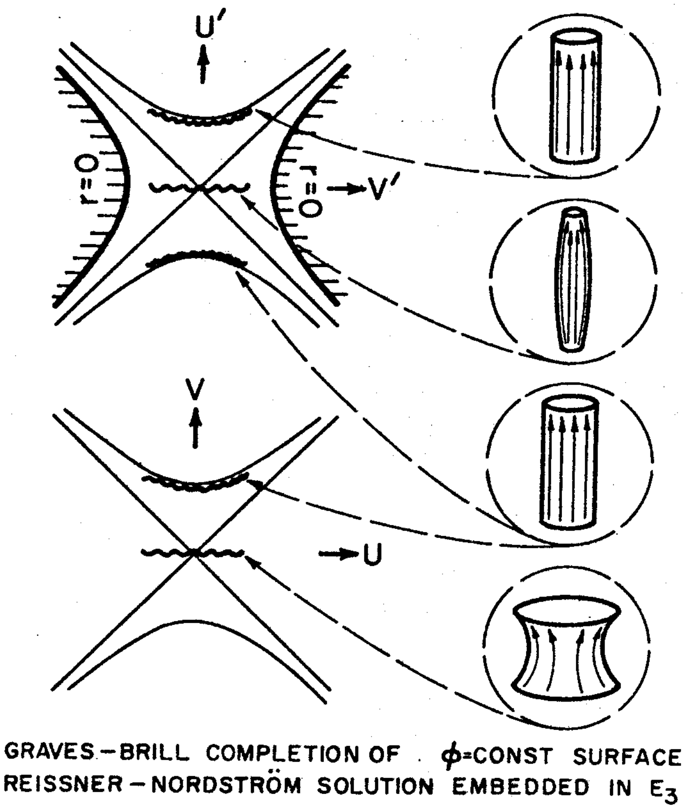

FIG. 4. The dynamics of the throat of Kruskal's wormhole when threaded by a magnetic field. On the left is a diagram of the Graves-Brill (1960) coordinate system which is characterized by an infinite sequence of pairs of coordinate patches identical to the pair shown. A succession of spacelike hypersurfaces, each one to the future of the preceding one, is shown in the coordinate diagram. On the right is a picture of each of these hypersurfaces embedded in a 3-dimensional Euclidean space. (The angle of rotation, ϕ, is suppressed; and the magnetic field lines are represented by arrows.) These successive "snapshots" of the throat of the wormhole reveal that, instead of pinching off, the throat pulsates. What is shown here is one period of the pulsation—from maximum radius to minimum radius, and then back out.

special relativity approximation, to geons with such intense magnetic fields that they wrap space up into closure around the ring of the torus. We shall show, in accordance with the resistance of magnetic flux to gravitational collapse, that none of the geons in this family will undergo collapse to the guiding line of the torus as it evolves in time.

The initial configuration of each of these geons can be constructed from Bertotti's (1959) static cylindrical magnetic universe. Bertotti's universe is described by the line element

$$ds^2 = (1 + B_0^2 z^2)dT^2 - \frac{dz^2}{1 + B_0^2 z^2} - \frac{1}{B_0^2}(d\eta^2 + \sin^2\eta\, d\varphi^2). \qquad (8)$$

(Here, and throughout this paper, we use "geometrized units" in which the speed of light and Newton's gravitational constant are equal to 1.) In Bertotti's universe an observer with world line (z, η, φ) constant sees no electric field, but he sees a uniform magnetic field of strength B_0 pointing along the z-direction. The static surfaces of constant T have the geometry $E_1 \times S_2$; they are closed up in the radial direction (η- direction) but not in the z-direction.

The first step in constructing a toroidal magnetic geon from Bertotti's cylindrical universe is to bend it around into a toroidal universe (give it the geometry $S_1 \times S_2$ rather than $E_1 \times S_2$). Lindquist (1960) has given the prescription for doing this; we follow this discussion closely in the next paragraph.

Introduce new time and longitudinal coordinates, t and μ, defined by

$$B_0 z = \cos(B_0 t)\sinh(B_0 b\mu), \quad \tan B_0 T = \cot(B_0 t)\cosh(B_0 b\mu), \qquad (9)$$

where b is a constant. Thereby transform the line element (8) to read

$$ds^2 = dt^2 - (\cos^2 B_0 t)\, b^2 d\mu^2 - (1/B_0^2)(d\eta^2 + \sin^2\eta\, d\varphi^2). \qquad (10)$$

If μ is now interpreted as an angular coordinate of period 2π, then equation (10) is the line element for a toroidal universe with major circumference $2\pi b \cos(B_0 t)$ and minor circumference $1/B_0$. An observer with world line (μ, η, φ) = constant in this universe sees a uniform magnetic field of strength B_0 pointing in the μ-direction. (The only non-vanishing components of the electromagnetic field tensor are $f_{\eta\varphi} = -f_{\varphi\eta} = 1/B_0 \sin\eta$). The hypersurface t = 0 of the toroidal universe is a hypersurface of time-symmetry. As the universe evolves away from this momentarily static configuration, it gravitationally collapses along the μ-direction (its major circumference, $2\pi b \cos(B_0 t)$, decreases to zero).[9]

[9] None of the invariants of the Riemann tensor are infinite at $t = \pm\pi/2$. Hence, the singularities there can be removed by an appropriate choice of coordinates—e.g., by introducing Bertotti's coordinates (8). However, the singularities are removable only at the expense of destroying the periodicity of the coordinate μ; if we insist that (t, μ, η, φ) and $(t, \mu + 2\pi, \eta, \varphi)$ correspond to one and the same point, then we cannot remove the singularities at $t \pm \pi/2$.

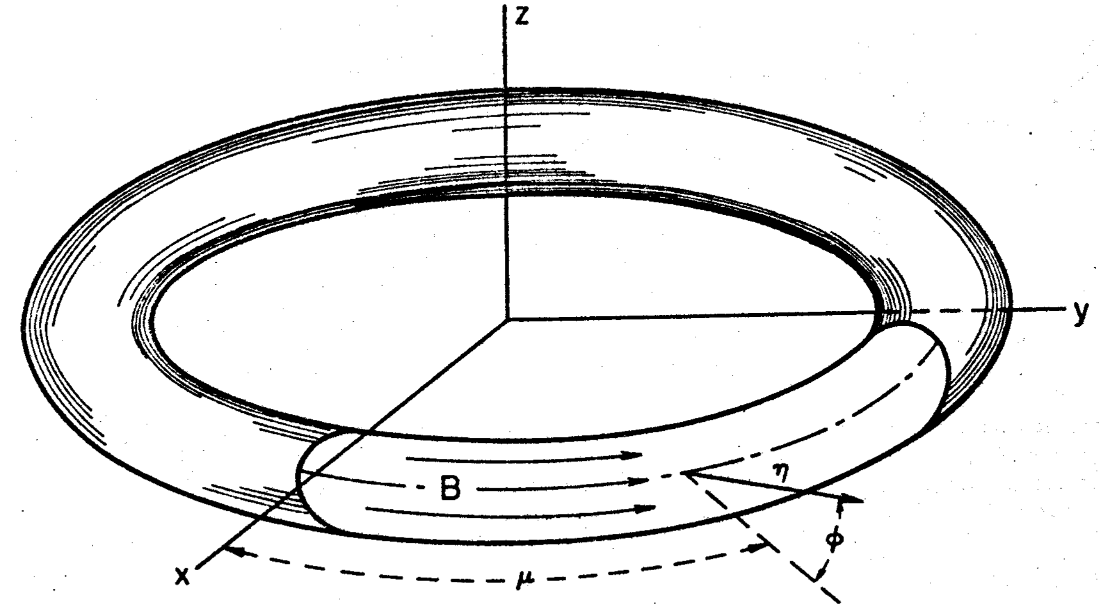

FIG. 5. The initial configuration of the magnetic geon whose subsequent dynamical evolution is studied. A uniform magnetic field threads the ring of the torus. There is no electric field or gravitational radiation in the initial configuration; but as the magnetic-field distribution changes with time it generates them. No currents or charges are present.

Note that this mode of collapse is perfectly compatible with the resistance of flux to gravitational collapse; the length of each closed field line decreases to zero, but the distance between adjacent field lines remains constant.

The next step in constructing a toroidal magnetic geon from Bertotti's universe is to take Lindquist's toroidal form of the universe (10) at the moment of time-symmetry $t = 0$, remove the region $\eta_0 \leq \eta \leq \pi$, and join what is left onto the gravitational field of a static line ring (see Fig. 6). The mathematical details of this procedure will be given elsewhere (Thorne 1965c).

The toroidal geon, which is thereby constructed, is a solution to the initial-value equations of general relativity for a hypersurface of time-symmetry.[10] The geon consists of a torus, which contains a uniform magnetic field of strength B_0. The surface of the torus is at $\eta = \eta_0$; its proper major circumference is $2\pi b$; its proper minor circumference is $2\pi B_0^{-1} \sin \eta_0$; and its proper minor radius is $\eta_0 B_0^{-1}$. If $\eta_0 \ll \pi$, then the geon is so dilute that its subsequent dynamics can be treated in the special relativity approximation; but if $\eta_0 \approx \pi$, then the geon is so massive that it wraps space up around itself almost into closure. For $\eta_0 = \pi$, space is completely closed up around the geon, and we have Lindquist's toroidal magnetic universe. This initial configuration of a toroidal geon contains no gravitational radiation, in the following sense: The dynamical evolution at any point inside or outside the geon is static until information that stresses were not initially balanced at the geon's surface (internal pressure $= B_0^2/8\pi$, external pressure $= 0$) has propagated to the point in question. Spacetime is static outside the geon because the initial external gravitational field is that of a static ring mass. It is locally static inside the geon because the dynamical evolution is initially that of the toroidal universe (10), which can be put into a static form by introducing Bertotti's coordinates locally. (Bertotti's coordinates cannot be introduced globally if we insist that μ be an angular coordinate of period 2π.)

Let each of these geons be followed as it evolves away from its initial configuration. Will gravitational collapse to the guiding line occur, in violation of the principle of flux resistance to gravitational collapse? No! The way in which the toroidal universe evolution (10), which involves no motion in the radial (η) direction, will be modified is this: Explosion away from the guiding line, rather than collapse to the guiding line, will be induced by the lack of balance of the magnetic pressure at the geon's surface.[11]

[10] For a discussion of the time-symmetric initial value equations, see Brill(1959).

[11] For geons with surfaces at $\eta_0 < \pi/2$, one can alternatively prove the impossibility of collapse to the guiding line by means of C-energy arguments, similar to those used by Thorne (1965a) to rule out the collapse of certain cylindrical electromagnetic universes.

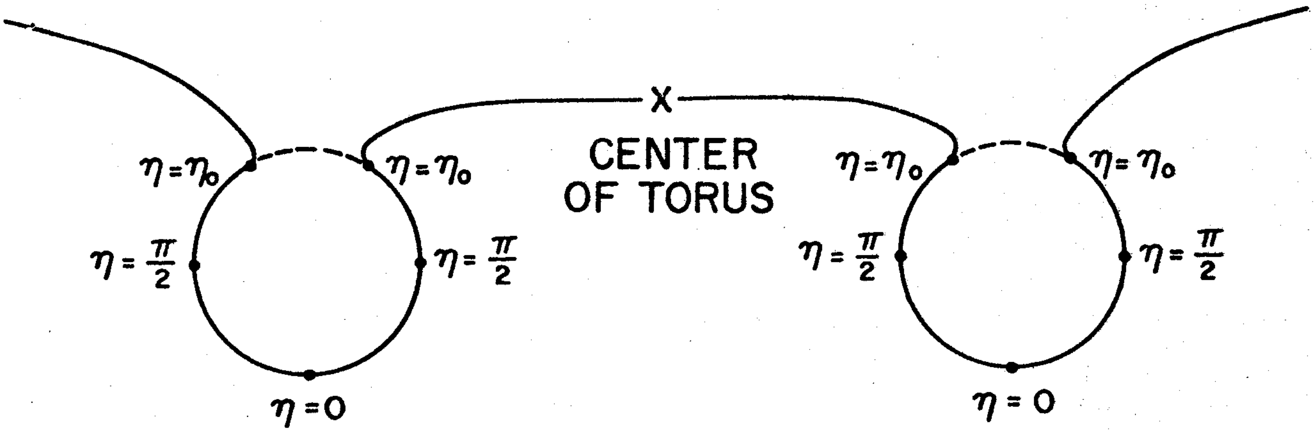

FIG. 6. A schematic illustration of the initial configuration of a magnetic geon constructed from the hypersurface t = 0 of Lindquist's toroidal form (eq. [10]) of Bertotti's universe. A cross-section through the geon is shown. The geon is constructed by (1) removing the region $\eta_0 \leq \eta \leq \pi$ from the toroidal universe; (2) joining the remaining configuration ($\eta < \eta_0$) onto the gravitational field of a static-line torus lying in asymptotically flat space.

III. Conclusions

We have suggested in this paper that, in vacuo, electric and magnetic flux resist gravitational collapse; and we have given a number of examples which support this viewpoint. A concerted effort should be made to prove or disprove this conjecture, not only because it would give us deeper insight into the nature of gravitational collapse in general relativity theory, but also for the following reason: If this conjecture is correct, and if flux also partially or completely resists collapse when matter is present, then magnetic fields could play an important role in astrophysical processes based on gravitational collapse.

REFERENCES

B. Bertotti, 1959, Phys. Rev., 116, 1531.

C. Behr, 1961, Mathematische Diplom-Arbeit, University of Hamburg (unpublished).

D. R. Brill, 1959, Ann. Phys., 7, 466.

_____. 1964, Phys. Rev., 133, B845.

S. Colgate, 1965, in Proceedings of the Second International Symposium on Relativistic Astrophysics.

A. Einstein, and N. Rosen, Phys. Rev., 48, 73.

R. W. Fuller, and J. A. Wheeler, 1962, Phys. Rev., 128, 919.

C. Fronsdal, Phys. Rev., 116, 778.

V. L. Ginzburg, 1964, Doklady Akad. Nauk SSSR, 156, 43 (translation: Sov. Phys.—Doklady, 9, 329 [1964]).

V. L. Ginzburg, and L. M. Ozernoi, (1964), Zh. Eksp. i. Teoret. Fiz., 47, 1030 (translation: Sov. Phys.—J.E.T.P. [to be published]).

J. C. Graves, and D. R. Brill, 1960, Phys. Rev., 120, 1507.

B. K. Harrison, 1965, paper submitted for publication.

B. K. Harrison, K. S. Thorne, M. Wakano, and J. A. Wheeler, 1965, Gravitation Theory and Gravitational Collapse (Chicago University of Chicago Press).

N. Kardashev, (1964), Astr. Zh., 41, xxx (translation: Soviet Astr.—A. J. [in press]).

M. D. Kruskal, 1960, Phys. Rev., 119, 1743.

R. Lindquist, (1960), "The Two Body Problem in Geometrodynamics," unpublished Ph. D. thesis, Princeton University.

M. A. Melvin, 1964, Phys. Letters, 8, 65.

_____. 1965, paper submitted for publication.

M. Misra, and L. Rhadakrishna, 1962, Proc. Nat. Inst. Sci. India, 28A, 632.

I. Novikov 1964, Astr. Tsirk., No. 290.

L. Shepley, 1965, chapter in Proceedings of the Second International Symposium on Relativistic Astrophysics.

K. S. Thorne, in Quasi-stellar Sources and Gravitational Collapse, ed. I. Robinson, A. Schild, and E. Schucking (Chicago: University of Chicago Press).

_____. 1965a, Phys. Rev., to be published.

_____. 1965b, submitted to Phys. Rev.

_____. 1965c, in preparation.

R. C. Tolman, 1934, Relativity, Thermodynamics, and Cosmology (Oxford: Clarendon Press).

H. Weyl, 1917, Ann. d. Phys., 54, 117.

J. A. Wheeler, 1963, Geometrodynamics (New York: Academic Press).

_____. 1964, in Relativity, Groups, and Topology, ed. C. and B. DeWitt (New York: Gordon and Breach).