

Motivation

- Collisions between individual particles and between particles and their containers dictate the behavior of particulate systems.
- For dry systems, the interaction can be described by following the trajectories of the individual particles and by associating a coefficient of restitution to the inelastic energy dissipation.
- For systems immersed in a liquid, the viscous dissipation and added mass can be incorporated with the inelastic losses in an effective coefficient of restitution.
- Such approach can simplify the numerical simulation of liquid–solid systems by modeling the collisions without describing in detail the fluid in the gap.

Normal and oblique particle–wall and particle–particle collisions in a fluid

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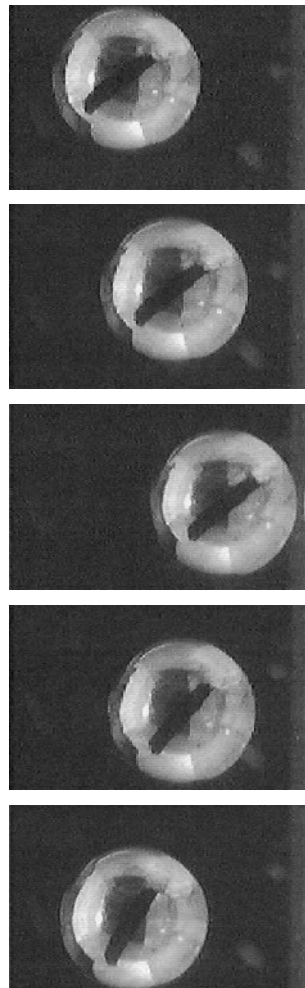
Future work

- Obtain measurements for the full range of incident angles for oblique collisions.
- Ascertain the presence or absence of solid–solid contact in the collisions.
- Numerical simulations of single and multiple particle collisions.

Conclusions

- There exists a critical value for St below which rebound does not occur.
- Surface roughness of the particles affects the repeatability of the experiments.
- The results compare well with both experiments and theoretical calculations in the literature.

Experimental setup and technique

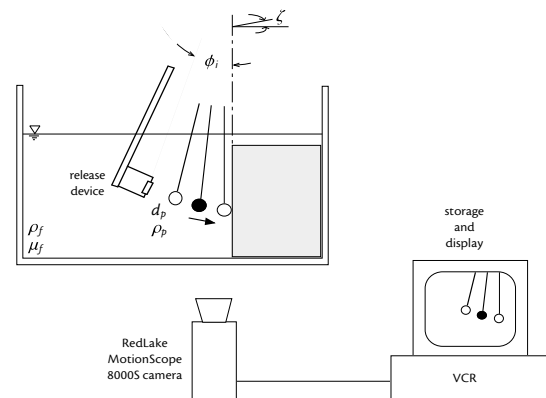


Photographs of a 12.7 mm steel sphere bouncing off a Zerodur block. Notice the spin of the particle after rebound.

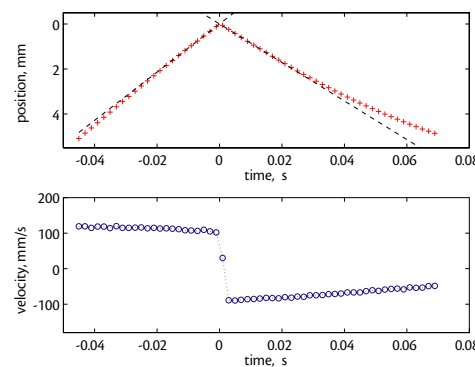
A fine nylon string is attached to a sphere to form a pendulum. A pair of tweezers is used to release the sphere from an initial angle ϕ_i without spin. The wall is positioned such that contact occurs at $\phi = 0$.

For the case of oblique collisions, the wall is rotated about its vertical axis such that its normal forms an angle ζ with the plane of the pendulum.

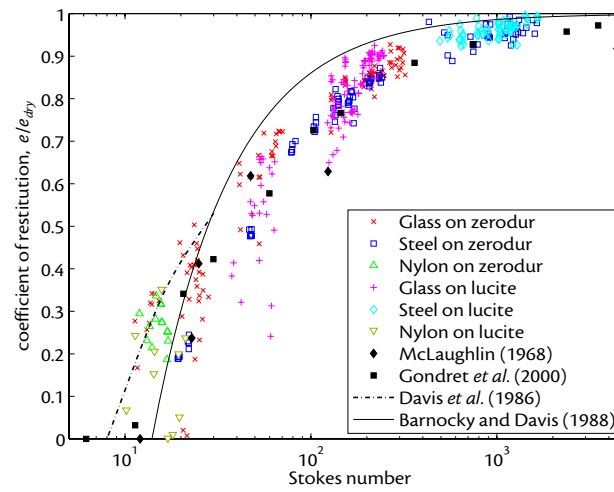
Mixtures of up to 75% glycerol in water with viscosities ranging from 1 to 15 Pa·s were used as surrounding fluids. Trials were also performed in air in order to characterize the dry coefficient of restitution.



Lines are regressed through the 5 data points prior to and after impact. The impact and rebound velocities are defined by the slopes of the fitted lines.



Material	d_p (mm)	ρ_p (kg m ⁻³)	E (GPa)	ν	σ_s (μ m)	λ_p (μ m)
Glass beads	3.0	2540	60	0.23	0.1384	44.70
	4.1	2540	60	0.23	0.0502	41.06
	6.0	2540	60	0.23	0.0721	49.76
Glass sphere	6.35	2540	60	0.23	0.1305	22.59
Steel spheres	4.1–12.7	7780	190	0.27	0.0236	48.04
Nylon sphere	6.35	1140	3	0.40	2.0114	41.86
Delrin sphere	12.7	1400	3	0.35	0.7960	101.49
Zerodur wall	75	2530	91	0.24	--	--
Lucite wall	50	1100	40	0.32	--	--



Theoretical Models

A simple analytical model based on Stokes drag and elastohydrodynamic theory describes the effective coefficient of restitution as

$$e = e_{dry} + \frac{1 + e_{dry}}{St_0} \ln \frac{x_c}{x_0}$$

where x_c is comparable to the surface roughness and x_0 is defined by Davis *et al.* (1986).

If the surface roughnesses are comparable to the minimum distance of approach predicted by elastohydrodynamic theory, a solid–solid contact is to be expected. The contact between roughness elements can explain the variance presented by some of the experimental values.

Oblique collisions can present:

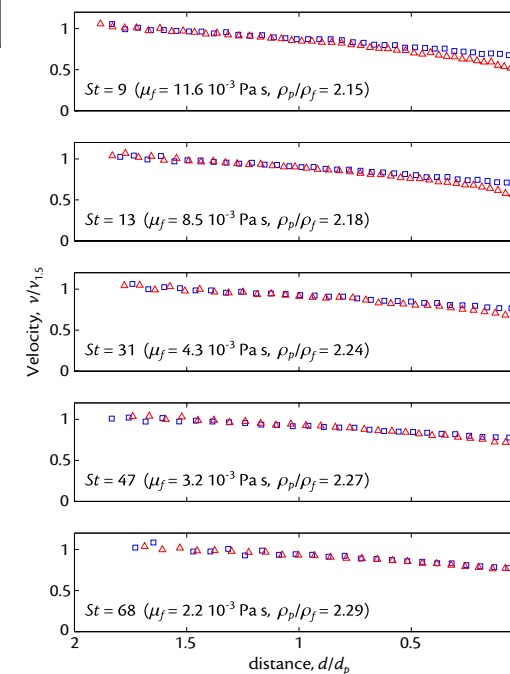
- sliding contact,
- stick–slip contact, or
- rolling contact.

For small values of ζ , enough elastic energy can be stored on the surface of the particles in the tangential direction to cause a backspin upon rebound (Maw *et al.*, 1981). This suggests backspin from oblique collisions as a strong indicator for solid–solid contact.

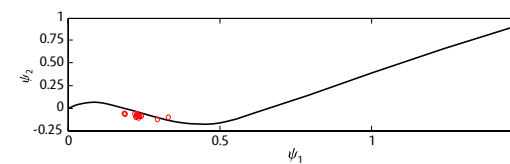
A single curve for the effective coefficient of restitution can be obtained when plotting with respect to the Stokes number, $St = \rho_p v d / 9 \mu$.

There exist four rebound regimes:

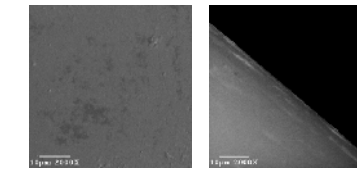
- For St of less than about 10, rebound does not occur.
- For St between 10 and 70, a deceleration due to the presence of the wall is observed.
- For St above 70, there is no apparent deceleration.
- For St of about 2000 and higher, the fluid effects can be neglected.



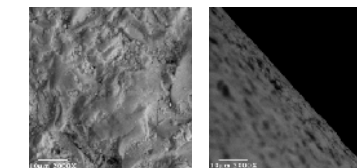
Comparison between the phase plots for a particle approaching a wall (Δ) and the losses due to viscous drag (\square).



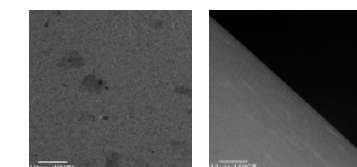
Localized rebound angle as a function of the localized incident angle. The data correspond to a steel sphere bouncing off a Zerodur wall.



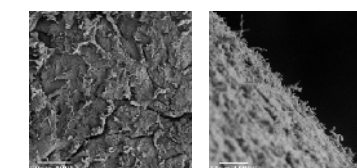
Glass bead, 6.00mm



Glass sphere, 6.35mm



Steel sphere, 6.35mm



Nylon sphere, 6.35mm

Scanning electron micrographs of the particles used.

References

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