Source Modeling 1

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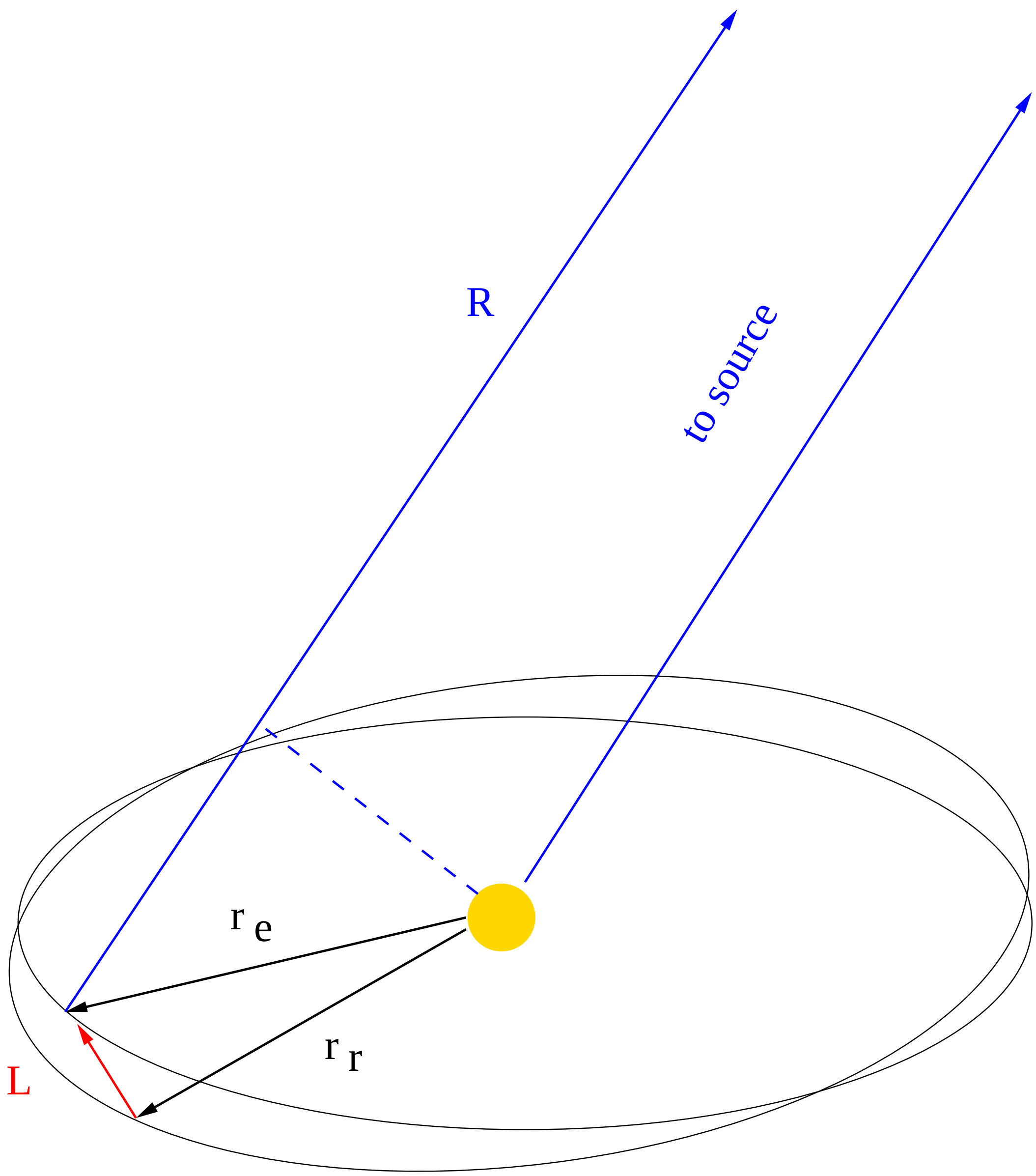
[1]LISA Science/Engineering Workshop #2
[2]http://www.its.caltech.edu/~esp/lisa/101801talk.pdf


R
to source
r e
r r
L

1 LISA observable response to gravitational waves

Response to gravitational waves of one-way Doppler streams (LISA has 12 of these from which all interferometer combinations can be synthesised - cf. Tinto, Estabrook & Armstrong). Only the 6 interspacecraft $y_{ij}(t)$ have significant gravitational wave contributions. The 6 intraspacecraft streams don't. Can be related to optical path, phase or fringe rate as follows:

$$y_{er} \equiv \frac{\nu_e - \nu_r}{\nu_0} = \frac{1}{c}\frac{d[\text{Opt path length}]}{dt} = \frac{1}{2\pi\nu_0}\frac{d\phi}{dt} = \frac{\text{fringe rate}}{\nu_0} \tag{1}$$

Give gravitational wave tensor components $\mathsf{h}^{TT}(t_b)$ at solar system barycenter. Give positions of spacecraft relative to solar system barycenter, $\mathbf{r}_e(t)$ for emitting spacecraft, $\mathbf{r}_r(t)$ for receiving spacecraft (e,r run through 1,2,3 giving the 6 gw-sensitive links).

$$\mathbf{L}(t) = \mathbf{r}_e(t) - \mathbf{r}_r(t) \tag{2}$$

$$y_{er}(t) = \left(1 + \frac{\mathbf{L}\cdot\hat{\mathbf{R}}}{L}\right)\left[\Psi\left(t + \frac{\mathbf{r}_e\cdot\hat{\mathbf{R}}}{c} + \frac{\mathbf{L}\cdot\hat{\mathbf{R}}}{c} - \frac{L}{c}\right) - \Psi\left(t + \frac{\mathbf{r}_e\cdot\hat{\mathbf{R}}}{c}\right)\right] \tag{3}$$

Variable delays $\pm$500s, $\pm$17s, $\pm$0.5s in the last 3 terms in first Ψ.

$$\Psi(t) = \frac{1}{2}\frac{\mathbf{L}\cdot\tilde{\mathsf{h}}^{TT}(t)\cdot\mathbf{L}}{L^2 - (\hat{\mathbf{R}}\cdot\mathbf{L})^2} \tag{4}$$

$$= \frac{1}{2}\left(h_+(t)\cos(2\lambda(t)) + h_\times(t)\sin(2\lambda(t))\right) \tag{5}$$

λ=angle between $\hat{\mathbf{R}}$–$\mathbf{L}$ plane (great circle) and $\hat{\mathbf{R}}$–$\hat{\alpha}$ right-ascension direction with respect to which have (arbitrarily) defined + polarisation for that source position.

Small, but not quite negligible point: the angles in the dot products are as measured in the solar-system barycenter frame. They must be Lorentz-transformed from spacecraft rest frame, giving additional Doppler shift (relative spacecraft motion) *and* aberration of direction (21 arcsec annual source position modulation, 1.4 arcsec annual inter-spacecraft modulation).

2 Source simulations:

At solar system barycenter, $\mathsf{h}^{TT}(t_b)$ from individual sources superpose linearly. This is how theorists (e.g. Kip's talk) usually show waveforms, and compute correlations. (α_i, δ_i are RA and dec of sources)

$$\hat{\mathbf{R}}_i = \cos\delta_i \cos\alpha_i \hat{\mathbf{x}} + \cos\delta_i \sin\alpha_i \hat{\mathbf{y}} + \sin\delta_i \hat{\mathbf{z}} \quad (6)$$

$$h^{TT}_{zz}(t_b) = \sum_i h_{+,i}(t_b)[-\cos^2\delta_i] \quad (7)$$

$$h^{TT}_{yz}(t_b) = h^{TT}_{zy} = \sum_i h_{+,i}(t_b)[\sin\delta_i \cos\delta_i \sin\alpha_i] + h_{\times,i}(t_b)[\cos\alpha_i \cos\delta_i] \quad (8)$$

etc for all 9 h^{TT}_{ij} (5 independent: symmetric, traceless) (9)

But in equation 3, the response is determined not by the sum at constant time, but by a sum of time-shifted and amplitude-modulated waveforms, with the time shifts and amplitude modulations depending on the position of each individual source and the orbits of each spacecraft.

To synthesize data stream, must store $h_+(t_b)$, $h_\times(t_b)$ and direction cosines $\hat{\mathbf{R}}_i$ for each simulated source.

3-years sampled at 0.1Hz, 1 byte amplitudes: 20Mbyte per source.

Number of sources needed to *simulate* data:

- Galactic binaries: 10^8 =2000Tb! But only strongest unconfused $\sim 10^5$ sources need to be stored individually =2Tb. Weak low frequency confused sources can be characterised by noise amplitude as function of direction on sky, for $\sim 10^5$ individual directions.

- Inspiraling compact objects in Galactic Nuclei: $\sim 2 \times 10^4$ =200Gb. Only small fraction individually detectable, but rest produce highly nongaussian noise ($\sim 10^4$ sources of aperiodic bursts with repetition rate ~ 10 per day per source, or ~ 1 per second on sky).

- Merging supermassive black holes: $\sim 1 - 10^2$=20Mb–2Gb.

- String bursts, high-z backgrounds: 'trivial'.

3 Number of template simulations needed to extract astro/physics from data

The end-to-end simulator will also be used in detection and parameter extraction: run source *templates* through simulator to get instrument response. Correlate with measured data. Repeat until find best-fitting source templates to deduce what is in the sky.

Must store or create on the fly templates with fine enough grid to ensure don't lose S/N in correlating with data due to template mismatch.

Bits of data in 3-year LISA data stream (Shannon's theorem):

$$n_{bits} \sim Tf \log_2(S/N) \sim 6 \times 10^5 \tag{10}$$

for $S/N \sim 100$ below 10^{-3}Hz, or $n_{bits} \sim 6 \times 10^6$ for $S/N \sim 100$ below 10^{-2}Hz.

Parameter count:

- Circular weakly chirping binary: α, δ, f, $\dot{f}$, i, ψ, ϕ_0, $|h|$ (or instead of last 4, amplitude and phase at fiducial time of h_+ and $h_\times$) —8 parameters. $\sim 10^{13}$ templates.
- Inspiraling compact object: add spin vector of central body, eccentricity, longitude of peribothron —13 parameters. ? templates.
- Inspiraling comparable mass body: add spin vector of second body —16 parameters. ? templates.

Should be able to find $\sim 10^4$ sources in LISA data:

Large source number, template count imply cleverness needed.

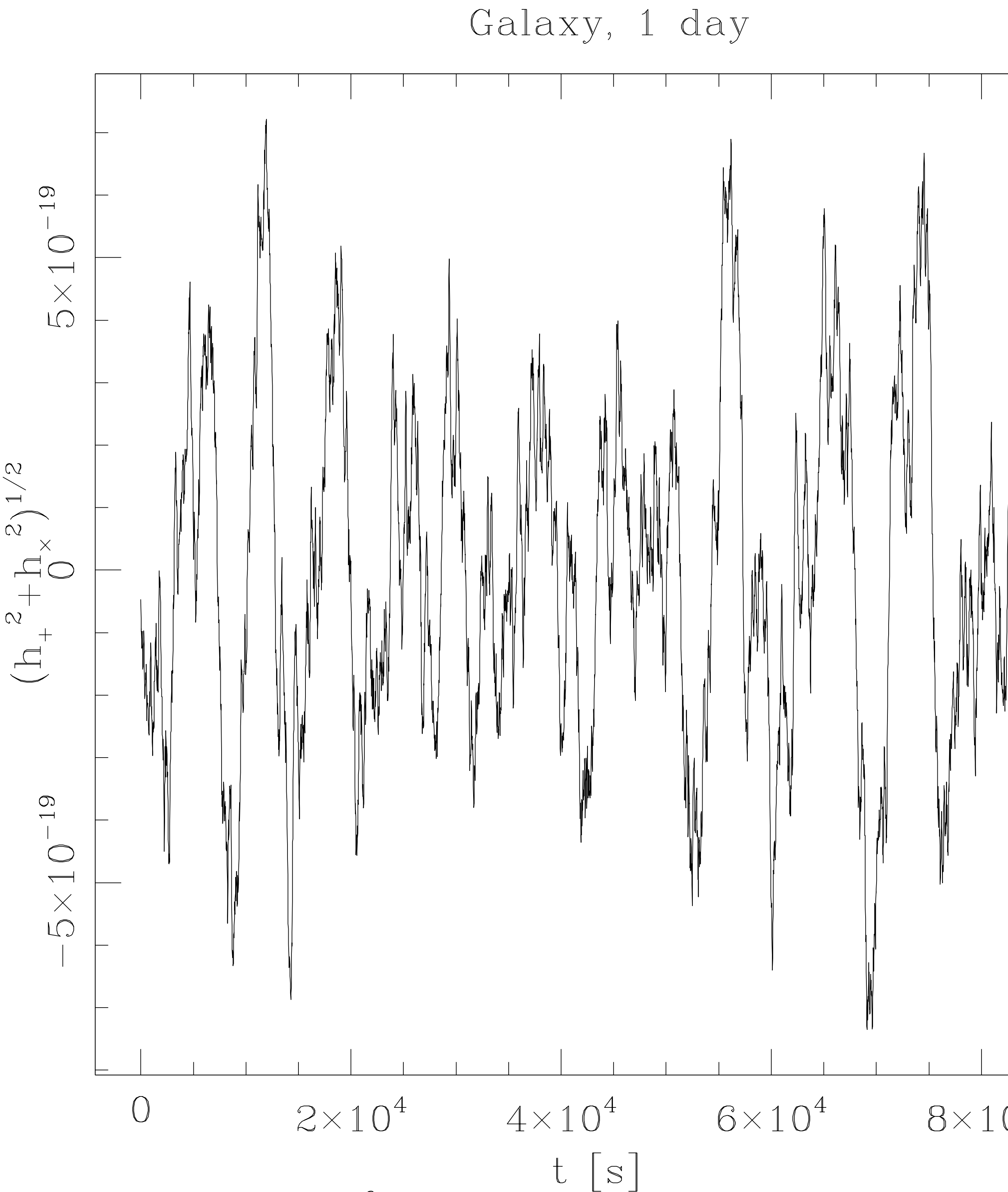
- Store $h_{+,\times}(t)$ not at solar system barycenter, but at center of spacecraft constellation (i.e. with Roemer delay aka Doppler modulation from orbit of center of constellation already included). Then only have to do 17s shifts instead of 500s shifts. Amplitude modulation still required.

- For Galactic binaries (simple in Fourier space), impose Roemer shifts in Fourier space, then FFT to get time-domain (8hr cpu vs 1 century cpu).

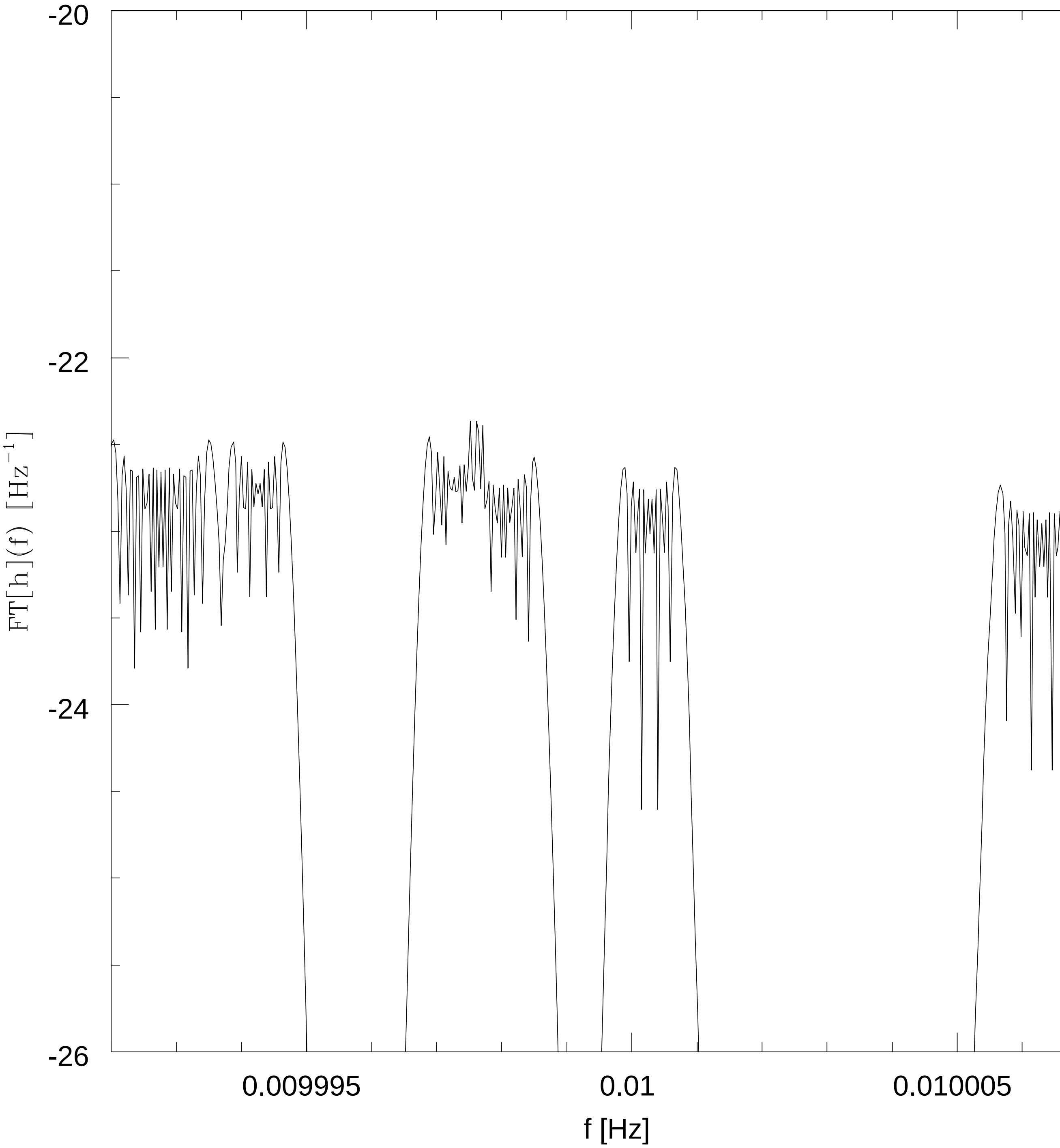
- These two tricks implemented in Galaxy simulation by Elina Brobeck & ESP. [2 graphs, audio]

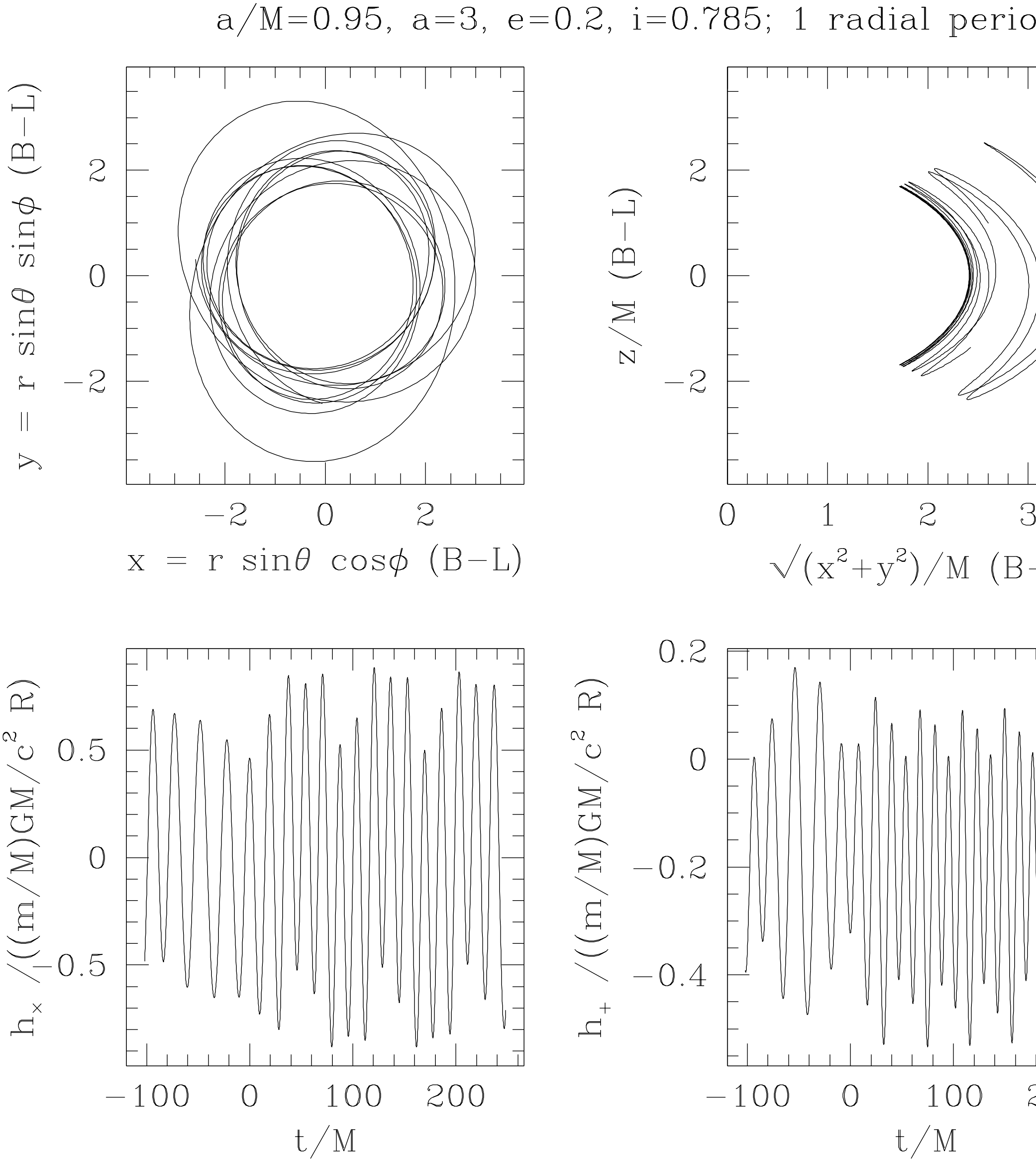
- Do not work for much more complicated waveforms of inspiralling compact objects, which cover all Fourier space. [New result (Mike Hartl & ESP): phase shift due to spin of small body never more than a few radians over entire inspiral. Fears of chaos unfounded.] [2 graphs]

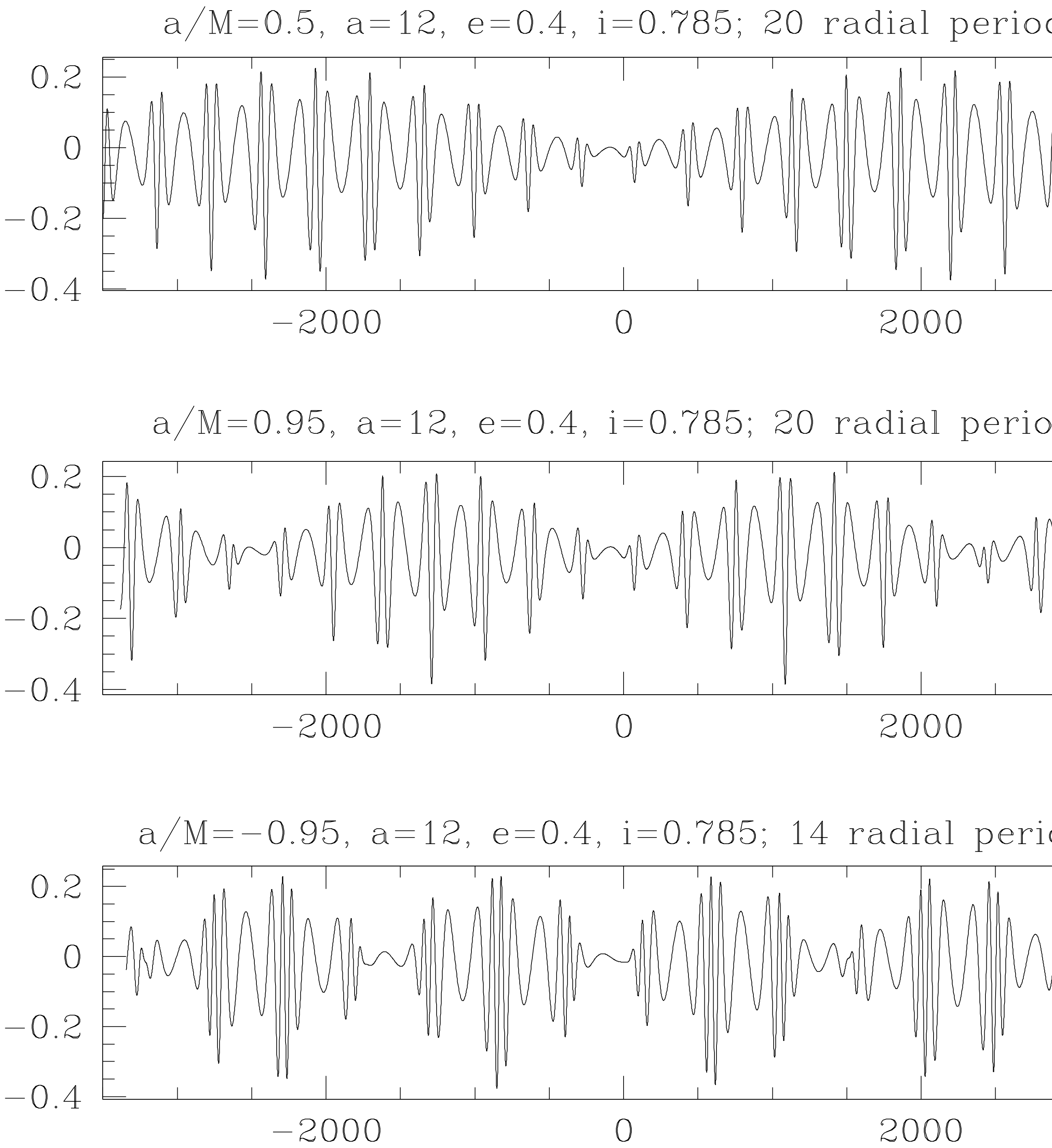
- Because of speed, storage requirements, will need simplified end-to-end simulator which takes as input descriptions of waveforms other than $h_{i,+,\times}(t)$ at some solar system location for each source direction $\hat{\mathbf{R}}$.



Galaxy, 1 day
$(h_+^2+h_\times^2)^{1/2}$
5×10^{-19}
0
-5×10^{-19}
0
2×10^4
4×10^4
6×10^4
t [s]


-20
-22
-24
-26
FT[h](f) [Hz^-1]
0.009995
0.01
0.010005
f [Hz]


a/M=0.95, a=3, e=0.2, i=0.785; 1 radial perio
y = r sinθ sinφ (B−L)
x = r sinθ cosφ (B−L)
z/M (B−L)
√(x²+y²)/M (B-
h× /((m/M)GM/c² R)
h+ /((m/M)GM/c² R)
t/M
t/M



a/M=0.5, a=12, e=0.4, i=0.785; 20 radial perio
0.2
0
−0.2
−0.4
−2000
0
2000
a/M=0.95, a=12, e=0.4, i=0.785; 20 radial perio
0.2
0
−0.2
−0.4
−2000
0
2000
a/M=−0.95, a=12, e=0.4, i=0.785; 14 radial peri
0.2
0
−0.2
−0.4
−2000
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2000