

Problem Set 5

Due in class, Thursday, 8 February 2018

Reading: See the on-line [syllabus](#) for lecture-by-lecture readings.

Collaboration policy: See the on-line [collaboration policy](#).

Homework Problems: Before starting this set, be sure you have read sections 4.4, 5.1 (covered thoroughly in lecture) and 6.1 of the text, MvdBW.

1. The correlation function and σ_8 . (8pts: 6+2)

The power-spectrum is defined by its shape and its overall amplitude normalization. To set the latter, suppose you assumed that on sufficiently large scales to be still linear, the density fluctuations in the universe were accurately described by the present-day galaxy 2-point correlation function: $\xi(r) = (r/r_0)^\gamma$, with $r_0 = 5h^{-1}\text{Mpc}$ and $\gamma = -1.8$.

- a) Calculate the rms mass fluctuations expected inside spheres of (co-moving) radius $8h^{-1}\text{Mpc}$, the expected value of σ_8 . The currently favored (Planck 2015) value of $\sigma_8 \simeq 0.83$ (defined from the amplitude of the *linearly extrapolated* power spectrum).
- b) How might you reconcile the difference in these estimates, should there be one?

2. First bound objects vs redshift (12 pts: 4+4+4)

Using the graph (Figure 1 below) for our CDM universe of the variance of mass fluctuations, filtered with a spherical top hat of comoving radius R (related to M by $M = 4\pi R^3 \langle \rho_0 \rangle / 3$), as a function of the filter radius:

- a) Show that at $z = 25$, 3σ fluctuations which have just collapsed and virialized have approximately, $R = 0.023 \text{ Mpc}$, masses of $2 \times 10^6 M_\odot$, virial radii of 130 pc , circular velocities of $\sim 8 \text{ km s}^{-1}$, a virial temperature of $\sim 4000 \text{ K}$, and contain about $3 \times 10^5 M_\odot$ of hydrogen.
- b) Again using the variance graph (Fig 1), show that at $z = 11$, 2σ fluctuations which have just collapsed and virialized have approximately, $R = 0.1 \text{ Mpc}$, masses of $1.5 \times 10^8 M_\odot$, virial radii of $\sim 1.5 \text{ kpc}$, circular velocities of $\sim 20 \text{ km s}^{-1}$, and virial temperatures of $\sim 16,000 \text{ K}$.
- c) Again using the variance graph (Fig 1), show that at $z = 3$, 2σ fluctuations which have just collapsed and virialized have approximately, $R = 2.7 \text{ Mpc}$, masses of $3 \times 10^{12} M_\odot$, virial radii of $\sim 10^2 \text{ kpc}$, circular velocities of $\sim 330 \text{ km s}^{-1}$, and virial temperatures of $\sim 4 \times 10^6 \text{ K}$.

3. Mass-Temperature for Clusters of galaxies (10pts: 6+4)

- a) If one observes at redshift z a set of virialized objects at a radius such that the interior matter density is a fixed multiple Δ (> 178) of the critical density of the universe, show that the virial theorem predicts a relation between the virial temperature and the mass M_Δ within that radius

$$M_\Delta \propto T_{\text{vir}}^{3/2} E(z)^{-1} \quad (1)$$

where $E(z) = H(z)/H_0$.

- b) Figure 2 shows the total mass within radii where the density is 500 (top, red) times the critical density and 2500 (bottom, blue) times the critical density versus X-ray temperature, for a sample of clusters of galaxies observed by the Chandra satellite. The *slopes* of the lines are 1.51(11) and 1.58(7) respectively, in excellent agreement with the prediction of part (a). What does the *intercept* of the line tell us?

4. **Faber-Jackson and Tully-Fisher relations for galaxies.** (10pts: 1+(3+3)+3)

In the region of galaxies ($z = 1-6$ and $M \sim 10^9-10^{13} M_\odot$) the plot of $\sigma(R, z) \equiv \langle (\delta M/M)^2 \rangle^{1/2}$ versus R can be approximated as $\sigma(R, z) \propto R^{-s}/(1+z)$.

- a) Estimate the numerical value of s from the plot in figure 1.
- b) Show that if galaxies are virialized ν -sigma perturbations (i.e. they collapsed when their linearly extrapolated overdensity was $1.68/\nu$, with $\nu \sim 1-3$) that
- the collapse redshift z_c is given by $(1+z_c) \propto M^{-s/3}\nu$
 - the mass and circular velocity are related by

$$M \propto v_c^{6/(2-s)} \nu^{-3/(2-s)} .$$

Insert your numerical value of s into this $M(v_c, \nu)$ scaling, and compare to the observed data of figure 3

- c) Elliptical galaxies of a given mass have a higher v_c than spiral galaxies of the same mass. In your simple model of part (b), do you then expect ellipticals to have larger or smaller ν than spirals? What consequence would this have for their predicted clustering?

variance in sphere of radius R

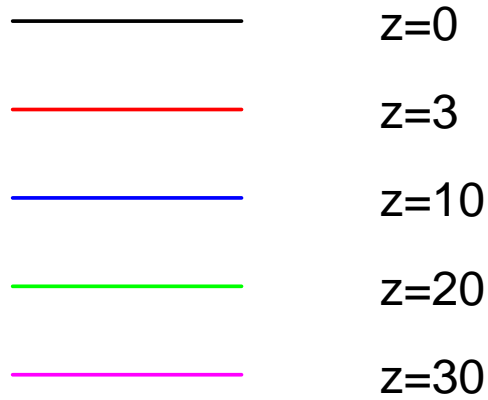
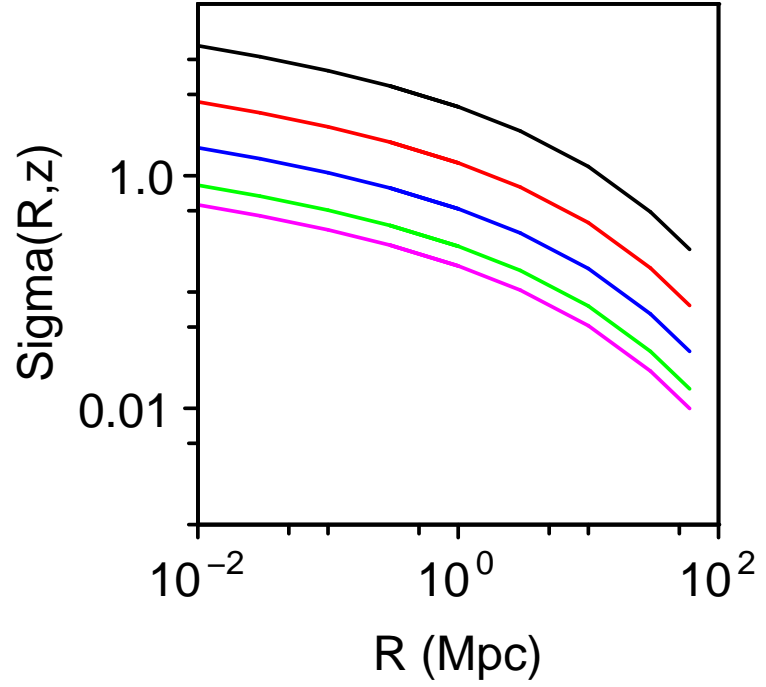


Figure 1: Top hat-filtered variance $\sigma_R = \sqrt{(\delta M/M)^2} = \int_0^\infty \Delta^2(k) W^2(kR) d \ln k$ in the mass enclosed within comoving radius R, for redshifts (top to bottom) 0,3,10,20,30. The Fourier Transform $W(kR)$ of the top-hat window function is defined in section 4.4.4 of MvdBW (eqs 4.271-4.273).

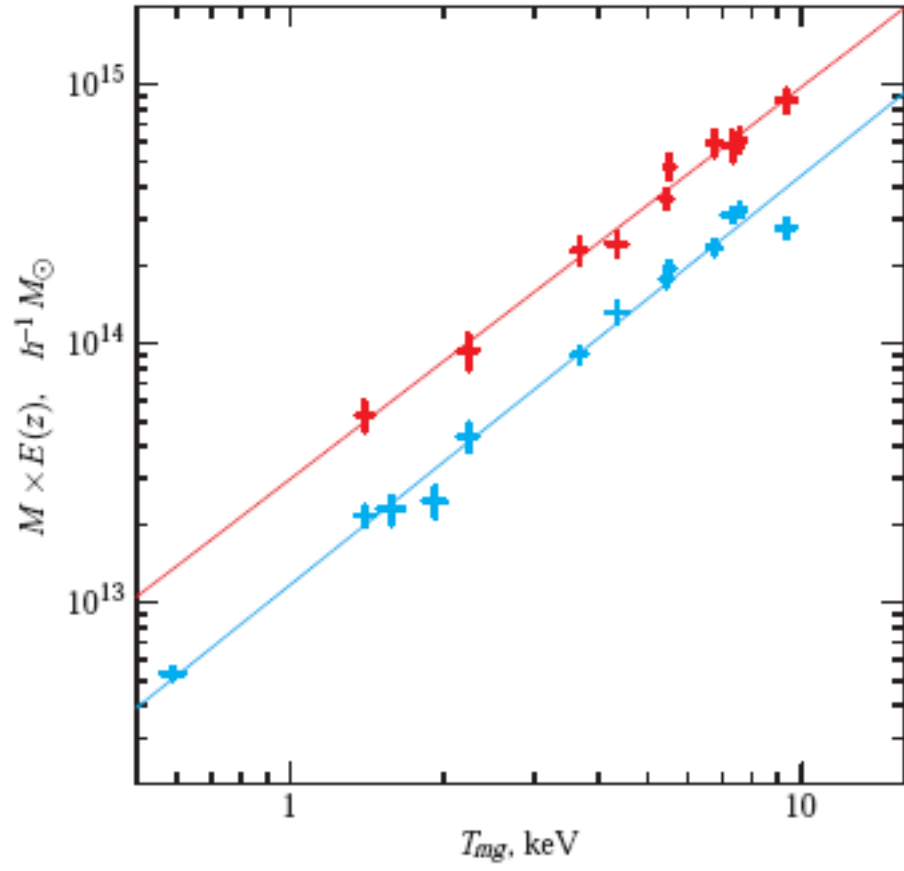


Figure 2: Figure 1. (Figure 19b of Vikhlinin et al astro-ph/0507092)

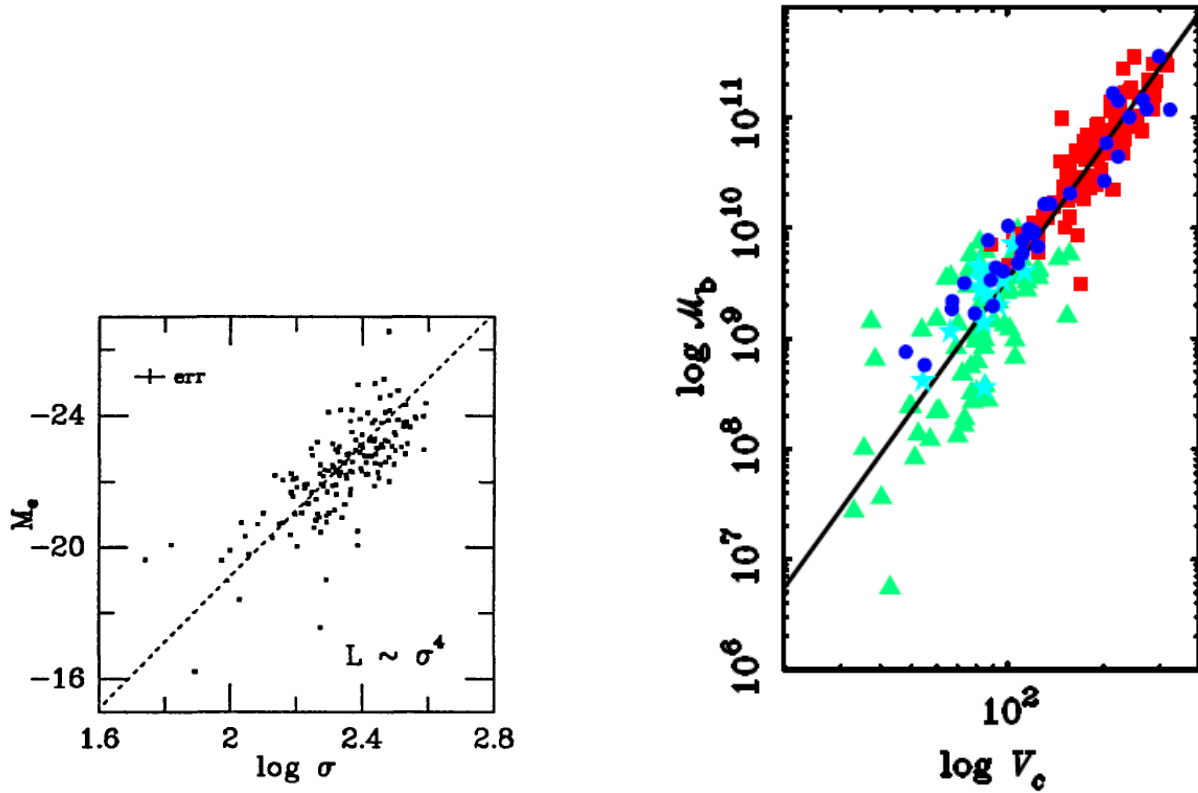


Figure 3: [left panel] Faber-Jackson relation: Magnitude *NOT mass!!* versus log of velocity dispersion (in km/s) $\sigma \sim v_c/\sqrt{2}$ of elliptical galaxies, from Kormendy & Djorgovski 1989 ARAA 27, 235, fig 2. [right panel] Tully-Fisher relation: Total baryon mass (stars plus gas) vs rotation velocity for disk ('spiral') galaxies, from McGaugh 1999 astro-ph/9909452, fig 1.