Reading: See the on-line syllabus for lecture-by-lecture readings. For making sure you have the equations right, the following may be useful: http://arxiv.org/abs/astro-ph/9905116. For checking your answers, this one may be useful: http://www.astro.ucla.edu/~wright/CosmoCalc.html. In the homeworks you should, however derive and solve the equations yourself, so you are sure you understand how they work.

Collaboration policy: See the on-line collaboration policy.

Homework Problems:

1. Algernon Astronomer surveys a square region of sky 1/2 degree on a side with a narrow-band filter, hunting Ly-alpha emitting galaxies. The passband of the amazing filter is 5500Å–5600Å with constant transmission in-band, and zero transmission out-of-band. His survey discovers 5 galaxies, one exactly in the middle of the square at redshift \(z = 3.564\), and one at each corner, at \(z = 3.524, 3.523, 3.564\) and \(z = 3.604\).

   a) Calculate the comoving distances (in cMpc) between the central galaxy and each of the four corner galaxies.

   b) Calculate the total comoving volume Algernon surveyed, and therefore the space density of Algernon’s galaxies (in galaxies per cubic comoving Megaparsecs, i.e., galaxies cMpc\(^{-3}\)).

   c) Algernon’s collaborator, Annie Astrophysicist, models the spectral energy distribution of his central galaxy, and determines that the oldest stars in that galaxy are 2.5 Gyr old. Is there any conflict between her calculation and that of the cosmological parameters given on page 24 of Serjeant’s text (with \(H_0 = 70.5\) km s\(^{-1}\) Mpc\(^{-1}\))? 

      i. If there is, what change in \(H_0\) (keeping other parameters fixed) would be required to solve the conflict?

      ii. If there is a conflict, what, if any, change in \(\Omega_{m0}\) (keeping \(H_0 = 70.5\) km s\(^{-1}\) Mpc\(^{-1}\), and keeping the universe flat, \(\Omega_{\Lambda,0} = 1 - \Omega_{m0}\)) could solve the conflict?

      iii. Will Annie’s paper have any trouble with an alert referee?

2. Thermal dust emission from small grains has a spectrum \(F_\nu \propto \nu^3\) for frequencies below the thermal peak, i.e. for \(\nu < kT/h \sim 3\) THz. A dusty starburst galaxy at redshift \(z = 0.05\) is observed to have \(F_\nu = 20\) mJy at \(\nu = 240\) GHz (1.25 mm wavelength).

What \(F_\nu\) would you measure at the same \(\nu = 240\) GHz from starburst galaxies identical to that one, but at redshifts \(z = 1, 3, 9\)?

   Explain any counter-intuitive features of your results.

3. The galaxy 3C 179, at redshift \(z = 0.846\), is observed to have knots of radio emission moving away from its center. The separation of these knots increases by 0.19 milliarcsec per year. For the cosmological parameters given on page 24 of Serjeant’s text (with
\( H_0 = 70.5 \text{ km s}^{-1}\text{Mpc}^{-1} \), calculate the speed at which the knots are moving out. Be sure the speed you give is that which would be measured in the frame of cosmological observers near 3C 179. Compare the speed to well-known speed limits.

4. The quasar Q0636+6801 shows two absorption lines due to Si\(^{++}\) (rest wavelength 1206\(\AA\)) in galaxies along the line of sight between earth and the quasar. The lines are at observed wavelengths 4711\(\AA\) and 4836\(\AA\). Compute

a) the distance between the two galaxies in comoving Mpc (assume both are moving with the cosmological flow), and

b) the redshift which an observer on the lower redshift galaxy would have measured for the higher one at the time the lower galaxy absorbed the light from the quasar now reaching earth.

c) Show that for small redshift differences, the answers to (a) and (b) can be simply related to the Hubble constant \( H(z) \) at the earlier epoch, and give a simple expression for \( H(z) \). Compute an approximate answer to (a) using this expression and the result of (b).

5. If objects distributed at random in the universe have a constant comoving number density \( n \) and constant cross-section \( \sigma \) for absorbing light, how many absorbers per unit redshift \( dN/dz \) does one expect on an average line of sight? Describe the variation of \( dN/dz \) with \( z \).