California Institute of Technology

AY127 Problem Set 1, Winter 2018

Instructor: Sterl Phinney & Chuck Steidel
TA: Guochao (Jason) Sun*

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Problem 1

§ Detailed Solution[1]:
Use the definition of redshift $\lambda_{\mathrm{obs}}/\lambda_{\mathrm{em}} = 1 + z$. Examples of lines which can be used to calculate the redshift are given in Table 1 and shown in Figure 1. You can form a matrix or try and identify individual absorption lines. Consideration of either mean, median or mode of the $(1 + z)$ values should give you an answer close to the actual value, $z \approx 0.14$.

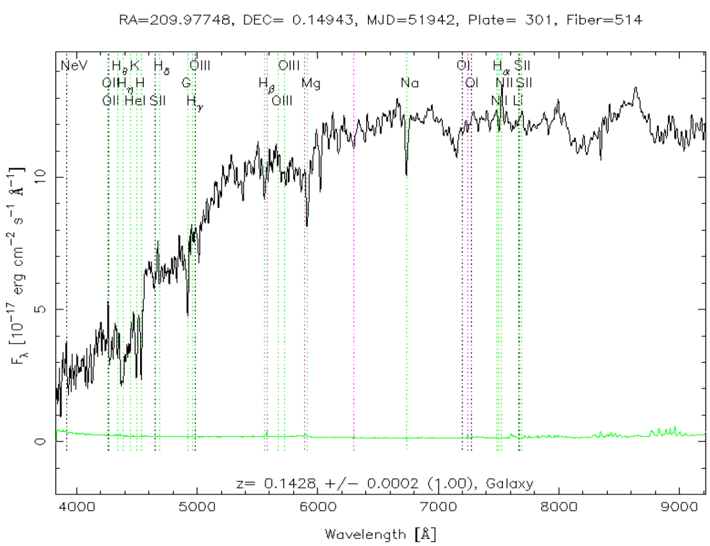

Figure 1: The given spectrum is that of galaxy SDSS J135954.59+000857.9 (http://cas.sdss.org/dr3/en/tools/explore/obj.asp?id=587722983363838126). Some spectroscopic lines which might be considered for the calculation of redshift are marked. Additional exercises based on this data can be found on the SDSS webpage (http://cas.sdss.org/dr3/en/proj/advanced/galaxies/spectra.asp).

*gsun@astro.caltech.edu

[1]Courtesy of E. S. Phinney, Kunal Mooley and J. Sebastian Pineda

ion	$\lambda_{\rm em}$ [Å]	$\lambda_{\rm obs}$ [Å]
Ca II, K	3934	4496
Ca II, H	3968	4535
Fe I, G	4308	4923
Hβ	4861	5555
Hα	6563	7500

Table 1: Expected and observed wavelengths of some absorption lines

Problem 2

§ **Detailed Solution**[2]**:**

(a) Perusing the database you can find several images of NGC2639. The most useful is the HST image reproduced below. From the image you can guess that the galaxy is probably an early non-barred spiral galaxy of Hubble Type Sa.

(b) In order to estimate the inclination angle, we assume the isophotes of the spiral galaxy would be very nearly circular if viewed face-on. When inclined, the projected isophotes will then appear elliptical, with an axis ratio independent of the isophote chosen to measure it. We can always rotate the galaxy in the plane of the sky, so we'll assume we are looking at the galaxy at the paralactic angle of 135, to place the long axis horizontally. Along the long axis the distance from the center to the edge of the galaxy corresponds to the actual radius of the disk, R, and along the short axis the observed projection is the length r (see HST image, Figure 2). In Figure 3 we demonstrate the associated geometry for observing the galaxy. From the geometry you can see that the lengths are related by the inclination angle according to

$$r = R\cos\Theta \tag{1}$$

Measuring r and R to a given isophote (level of surface brightness) on a common length scale (you can just use a ruler placed over the screen) and taking the ratio gives a value of $\cos\Theta \approx 0.42$ and an inclination $\Theta \approx 65^\circ$.

(c) We assume that the stars and gas of the spiral galaxy are orbiting in circles about the center and look at the radial velocity curve of NGC 2639 to figure out the recessional velocity. We then use that to get the circular velocity about the center while accounting for the factor due to the inclination.

i. We can get the recessional velocity by noting that the measured radial velocities on each side of the nucleus are symmetric about the mean motion of the galaxy. Taking the max velocities in each wing as approximately 3500 km/s and 2900 km/s, we can compute a recessional velocity of $v_r = (3500+2900)/2 = 3200\,\mathrm{km/s}$.

ii. The observed radial velocities are a combination of the mean motion and the circular motion as

$$v_{obs} = v_r + v_c\sin\Theta \tag{2}$$

The maximum circular velocity is about 300 km/s relative the mean motion of the galaxy so we compute a corrected velocity of $v \approx 300/\sin 65 = 331\,\mathrm{km/s}$.

iii. If we take NGC 2639 and the Sun to be cosmic observers and we note that the galaxy is in the local universe we can compute the distance using

$$d = v_r/H_0 \tag{3}$$

Plugging in our value and using a reasonable value of the Hubble constant, $H_0 = 73\,\mathrm{km/s/Mpc}$ gives $d \approx 44\,\mathrm{Mpc}$.

(d) From the SDSS data we have apparent magnitudes of $u, g, r, i, z = 14.05, 12.2, 11.29, 10.92, 10.55$. We can relate the magnitudes to the luminosity and distance,

$$m - M_\odot = -2.5\log_{10}\frac{L(10\mathrm{pc})^2}{L_\odot d^2} \tag{4}$$

Rearranging and plugging in with the g-band magnitudes and using our distance value from earlier we get a luminosity of $L_g = 2.85\times10^{10}L_{\odot,g}$.

[2] Courtesy of E. S. Phinney, Kunal Mooley and J. Sebastian Pineda

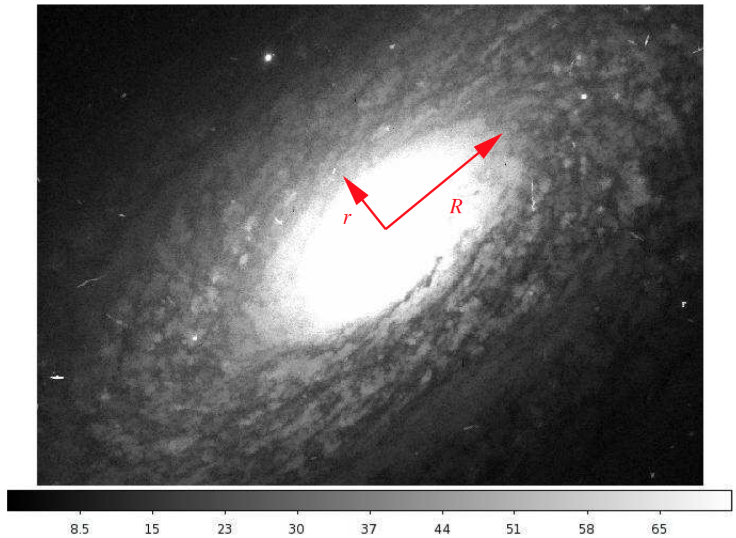

Figure 2: HST image of NGC2639 with arrows overlaid showing the principal axes of the projected ellipse to an isophote.

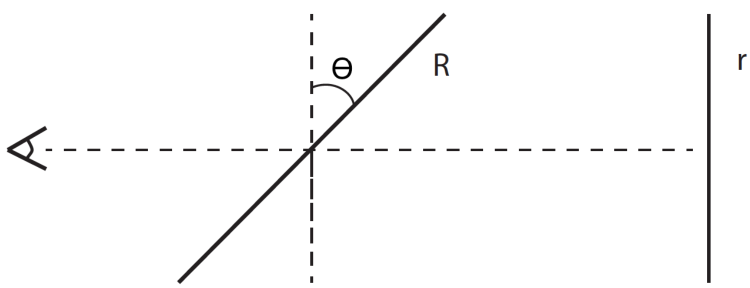

Figure 3: Diagram of geometry for projected ellipse of galaxy with observer on the left. R is the actual radius of the galaxy with inclination angle Θ and r is the observed distance of the short axis.

Problem 3

§ Detailed Solution:

(a) The question is basically asking about a standard of rest in the CMB reference frame, which requires you to determine the velocity to some given precision (150 km/s in this case). It's helpful to note that the CMB is a convenient reference frame in cosmology, because any observers at rest with respect to Hubble flow would observe the same CMB spectrum at a given cosmic epoch, even though they are moving away from each other due to cosmic expansion. A moving observer in the CMB's rest frame then would measure a Doppler shift that gives rise to the dipole anisotropy of the CMB (about one part in a thousand), and its motion is described by the so-called peculiar velocity, v_p. Consequently, the net velocity that one would determine from spectroscopy can be written as $\boxed{cz = v_p + H_0 r}$, where $H_0 r$ gives the velocity of Hubble flow, and we would like to determine v_p to 150 km/s, i.e. $\boxed{\delta v_p < 150\,\mathrm{km/s}}$. Here, for an order-of-magnitude estimation, we could treat H_0 as some well-known quantity and also neglect that the uncertainty in cz. As a result, δv_p would be solely determined by the quality of our standard candles, namely their intrinsic scatter, which is 0.35 dex in luminosity or just 0.35 in absolute magnitude.

Differentiating the distance modulus $m - M = 5\log_{10} r + 5$, we have

$$\delta M = 5\log_{10}(e)\delta r/r = 0.35 \tag{5}$$

Thus the uncertainty in the peculiar velocity due to the intrinsic scatter of standard candles is simply

$$\delta v_p = H_0 \delta r = [0.2/\log_{10}(e)]\delta M H_0 r \tag{6}$$

for a single object. From the central limit theorem, it's easy to know that if N objects are measured, we would have

$$\delta v_p = [0.2/\log_{10}(e)]\delta M H_0 r/\sqrt{N} \lesssim 150\,\mathrm{km/s}, \tag{7}$$

which implies a lower limit on the number of objects

$$\boxed{N \gtrsim 115} \tag{8}$$

(b) For a hand-wavy estimation of the uncertainty in H_0, we could assume that the peculiar velocities of our standard candles are negligible, which would greatly simplify our calculation. In such a case, we have $r = cz/H_0$ and the distance modulus can be expressed as

$$m - M = 5\log_{10}(cz/H_0) + 5. \tag{9}$$

Differentiating both sides (and treating m and cz as constants), we can obtain the following expression if N objects are measured

$$\delta H_0/H_0 = \delta M/[5\log_{10}(e)\sqrt{N}], \tag{10}$$

where $\delta M = 0.35$. Solving for $\delta H_0/H_0 \lesssim 0.1$, we can get a lower limit on the number of objects

$$\boxed{N \gtrsim 3} \tag{11}$$

Problem 4

§ Detailed Solution[3]**:**

(a) The universe is composed of Hydrogen, ${}^1_1$H, and Helium, ${}^4_2$He, and we want to consider the fusion of these elements into iron, ${}^{56}_{26}$Fe. To determine the energy release we must know the binding energies of these nuclei. ${}^{56}_{26}$Fe has a binding energy of 8.8 MeV per nucleon and ${}^4_2$He has a binding energy of 7.1 MeV. We consider the fusion of the Hydrogen separately from the Helium. We don't concern ourselves with the intermediary reactions, just the final products, and account just for the number of nucleon necessary.

$$56\ {}^1_1\mathrm{H} \rightarrow {}^{56}_{26}\mathrm{Fe}$$
$$14\ {}^4_2\mathrm{He} \rightarrow {}^{56}_{26}\mathrm{Fe}$$

To determine the energy released we compare the binding energies of the left and right-hand sides. For Hydrogen there is no binding energy, so there is $\epsilon_1 = 56 \times 8.8\,\mathrm{MeV} = 492.8\,\mathrm{MeV}$ released per Fe produced. For the Helium reaction there is $\epsilon_2 = (56 \times 8.8 - 14 \times 7.1 \times 4)\,\mathrm{MeV} = 95.2\,\mathrm{MeV}$ released per Fe produced. Now we need to know how much Fe can be produced from $\rho_\mathrm{b} = 4.2 \times 10^{-31}\,\mathrm{g/cm^3}$. We are given the mass fractions, so the number density of Hydrogen, $n_\mathrm{H} = 0.75\rho_\mathrm{b}/m_\mathrm{H}$, where $m_\mathrm{H} = 1.67 \times 10^{-24}\,\mathrm{g}$ is the mass of Hydrogen. Similarly the number density of Helium is $n_\mathrm{He} = 0.25\rho_\mathrm{b}/m_\mathrm{He}$, where $m_\mathrm{He} = 6.65 \times 10^{-24}\,\mathrm{g}$ is the mass of Helium. Using the reaction equations we can write down an expression the total energy density released, if all the baryons were converted to iron,

$$u = \epsilon_1 n_\mathrm{H}/56 + \epsilon_2 n_\mathrm{He}/14 = aT^4 \tag{12}$$

where the numerical factors account for the number of nucleons needed in each reaction and the last equality gives the temperature of the energy since we take it to have thermalized into blackbody radiation. The constant a is the radiation constant. Solving for temperature gives us $\boxed{T = 4.4\,\mathrm{K}}$.

(b) We can use Wien's displacement law for the peak wavelength of the blackbody function (taking the derivative of the blackbody function with reset to wavelength and setting it equal to zero will give the same result).

$$\lambda_\mathrm{peak} T = 0.29\,\mathrm{cm} \cdot \mathrm{K} \tag{13}$$

Plugging in our temperature from (a) gives $\boxed{\lambda_\mathrm{peak} = 0.066\,\mathrm{cm}}$, namely sub-millimeter wavelengths — these are typically radio waves.

(c) The problem gives the luminosity per volume as $L = 3 \times 10^8\, L_\odot \mathrm{Mpc}^{-3}$. We consider this to be the rate at which the energy density computed as part of problem (a) is produced since the source of the stellar luminosity is nuclear fusion. We can get the time it would take be dividing the energy density u by this rate

$$\boxed{t = \frac{u}{L} = \frac{aT^4}{L} \approx 2256\,\mathrm{Gyr}}. \tag{14}$$

This time is much greater than the current age of the universe, 13.7 Gyr.

(d) Our result suggests that it would take a time much longer than the current age of the universe to use up all of the baryons at the current rate. However, for a universe filled uniformly with stars in which the light is produced via nuclear energy production, the stuff producing light would eventually run out. Thus, if the universe stretches in infinitely back in time, the baryon density should have been higher in the past than it is now. This contradicts the notion of a static universe inherent to Olber's paradox. There is also a limit to how dense it could have been in the past. The results suggest that the universe is not likely static and is finite in time, making it possible that not all the light has had enough time to reach the Earth.

[3] Courtesy of E. S. Phinney and J. Sebastian Pineda