Recent Progress toward Sub-Quantum-Noise-Limited Gravitational-wave Interferometry

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Outline

- Quantum noise limits in GW ifos
- Quantum optics primer
- Sub-quantum noise limited ifos
  - Injecting squeezed vacuum
  - Setting requirements – the wishlist
- Generating squeezed states
  - Nonlinear optical media – “crystal”
  - Radiation pressure coupling – “ponderomotive”
- Macroscopic quantum measurement – the spinoff

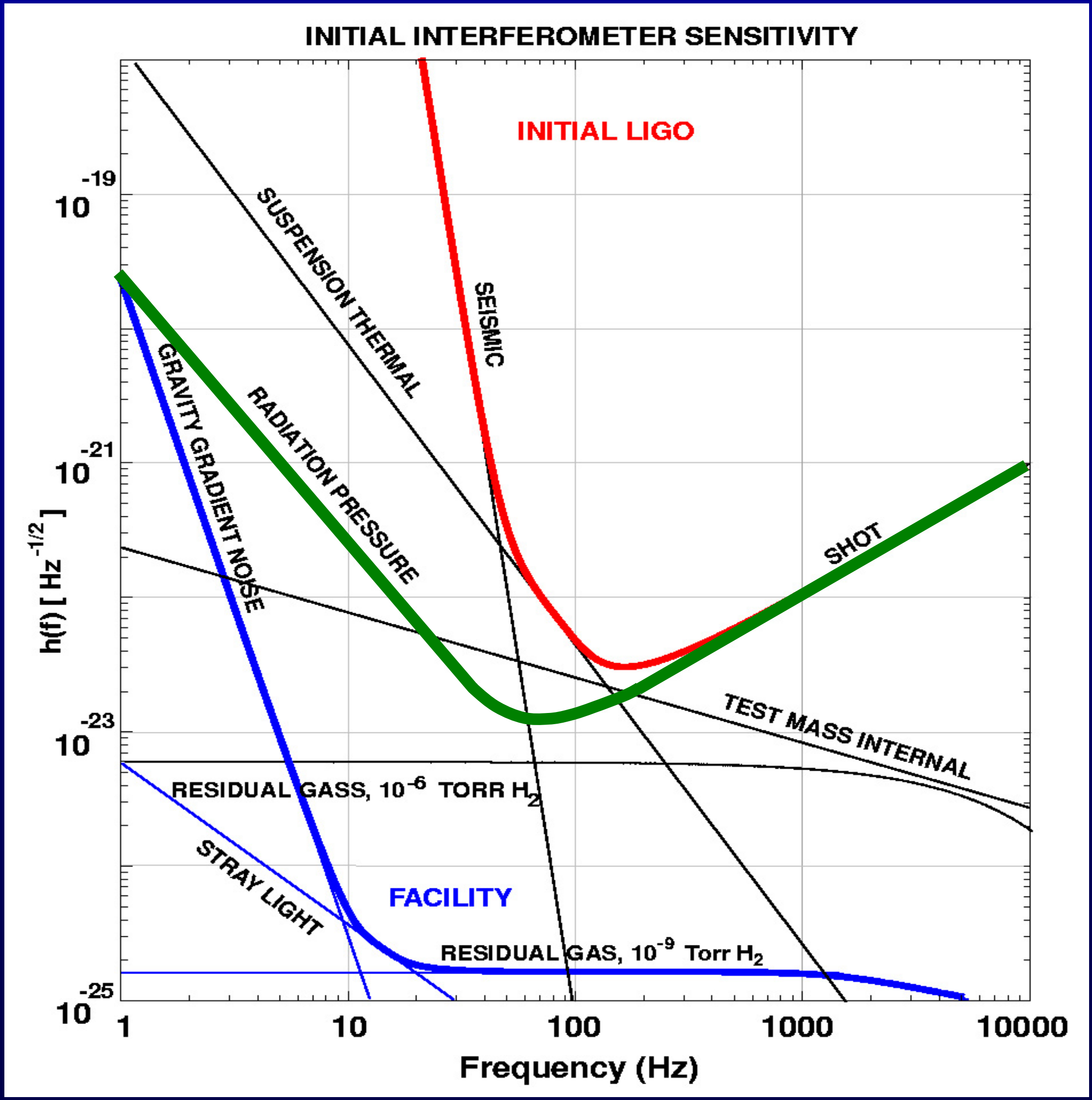
Our quantum tale begins...

Initial LIGO

Input laser power
~ 6 W

Circulating power
~ 20 kW

Mirror mass
10 kg



Limiting Noise Sources: Optical Noise

- Shot Noise
  - Uncertainty in number of photons detected ⇨
  - Higher circulating power P_{bs} ⇨ low optical losses
  - Frequency dependence ⇨ light (GW signal) storage time in the interferometer

$$\boxed{h(f) \propto \sqrt{\frac{1}{P_{bs}}}}$$

- Radiation Pressure Noise
  - Photons impart momentum to cavity mirrors Fluctuations in number of photons ⇨
  - Lower power, P_{bs}
  - Frequency dependence ⇨ response of mass to forces

$$\boxed{h(f) \propto \sqrt{\frac{P_{bs}}{M^2 f^4}}}$$

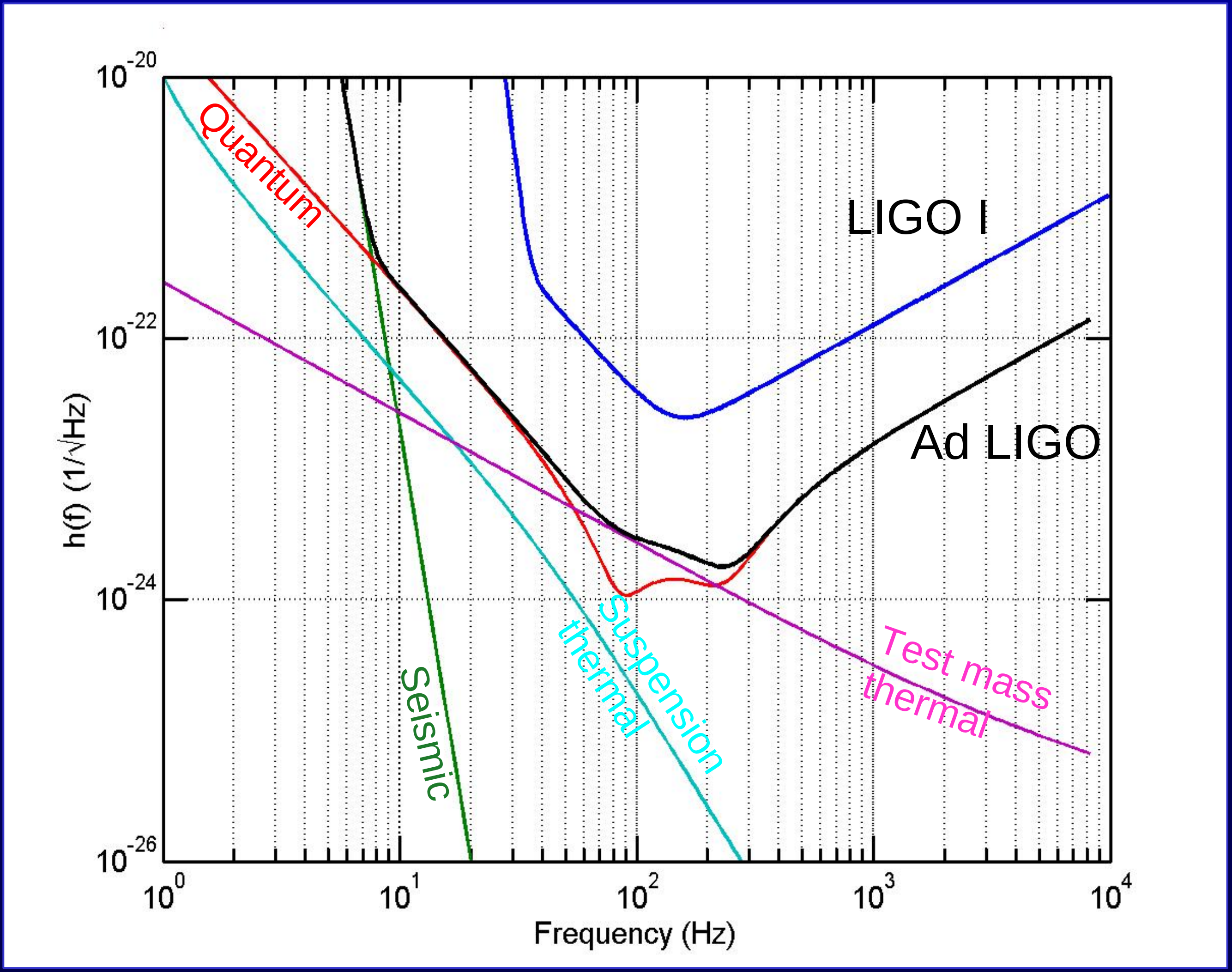
➔ Optimal input power depends on frequency

A Quantum Limited Interferometer

Input laser
power
> 100 W

Circulating
power
> 0.5 MW

Mirror mass
40 kg



Quantum Optics Primer

Classical Optics - Quantum Optics

- Classical optics
  - Amplitude
  - Phase
  - Power

$$\mathrm{E} = Ee^{i\phi}$$

$$\mathrm{P} = \mathrm{E}^{*}\mathrm{E}$$

  - Phasor representation
    - Length → amplitude
    - Angle → phase

- Quantum optics
  - Annihilation operator
  - Creation operator
  - Photon number

$$\hat{a}$$

$$\hat{a}^{\dagger}$$

$$n = \hat{a}^{\dagger}\hat{a}$$

  - $\hat{a}$ and $\hat{a}^{\dagger}$ are not Hermitian
    → NOT observables

Defining Observables

- Define a Hermitian operator pair

$$\hat{X}^{+} = \hat{a}^{\dagger} + \hat{a},$$
$$\hat{X}^{-} = i\left(\hat{a} - \hat{a}^{\dagger}\right)$$

- $\hat{X}^{+}$ is the amplitude quadrature operator
- $\hat{X}^{-}$ is the phase quadrature operator

- Linearization
  - DC term + fluctuations

$$\hat{a} \approx \alpha + \delta\hat{a}(\omega)$$
$$\approx \alpha + \frac{1}{2}(\delta X^{+}(\omega) + i\delta X^{-}(\omega))$$

  - Fluctuations in amplitude and phase quadratures

$$\delta X^{+} = \delta\hat{a} + \delta\hat{a}^{\dagger}$$
$$\delta X^{-} = i(\delta\hat{a}^{\dagger} - \delta\hat{a})$$

Heisenberg Uncertainty Principle

- No measurement can be completely deterministic in two non-commuting observables.
- E.g. Position and momentum of a particle

$$\Delta x \Delta p \geq \hbar / 2$$

- Similarly for the electromagnetic field

$$\Delta \hat{X}^{+} \Delta \hat{X}^{-} \geq 1$$

Correlations → noise reduction

- In the presence of correlations
  - Output = Shot + Radiation pressure + Signal

$$\boxed{S_{total} = S_{SN} + S_{RPN} + 2S_{corr}}$$

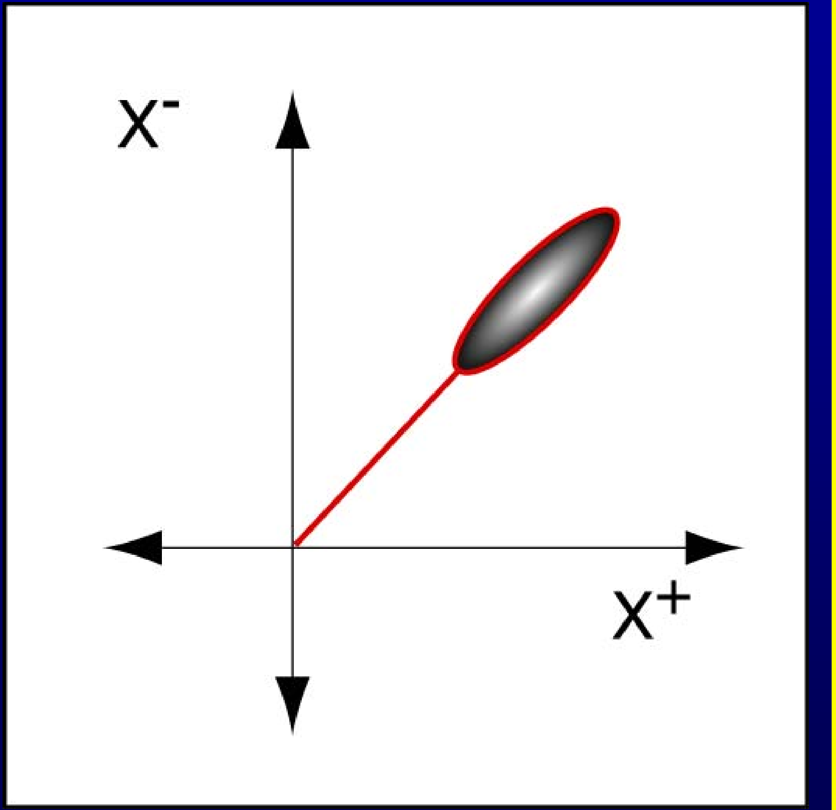
  - Heisenberg uncertainty principle (in spectral domain)

$$\boxed{S_{shot} S_{RP} \leq \frac{S_{SQL}^{2}}{4} \quad \text{when} \quad \left| S_{corr} \right| \neq 0}$$

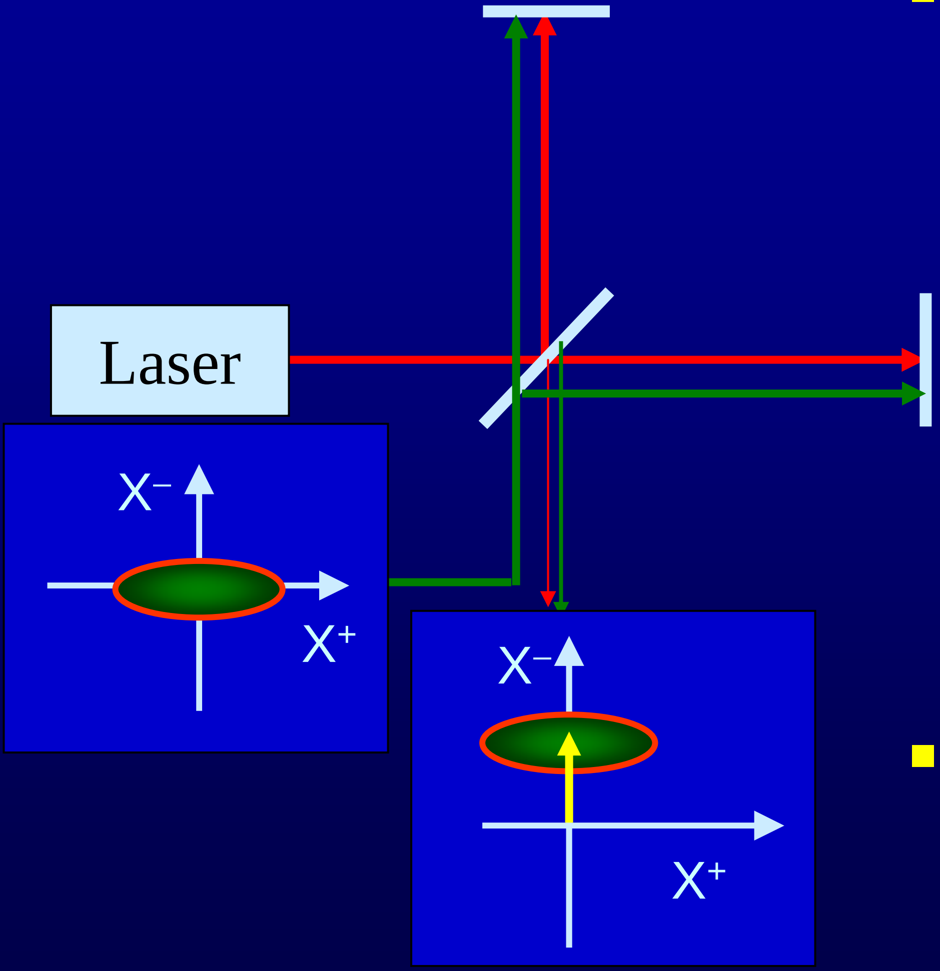
- How to correlate quadratures?
  - Make noise in each quadrature not independent of the other
  - (Nonlinear) coupling process needed
  - Squeezed states of light and vacuum

Some quantum states of light

- Analogous to the phasor diagram
- Stick → dc term
- Ball → fluctuations
- Common states
    - Coherent state
    - Vacuum state
    - Amplitude squeezed state
    - Phase squeezed state

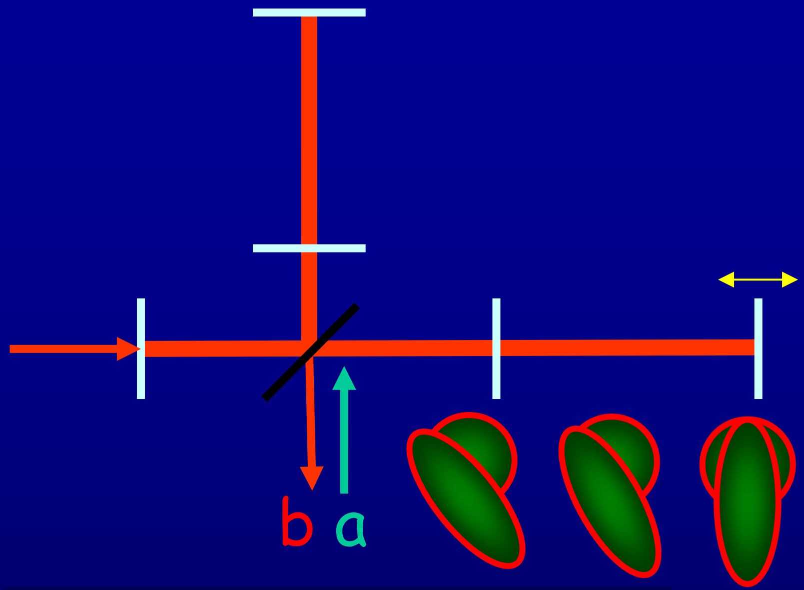


Squeezed input vacuum state in Michelson Interferometer



- Consider GW signal in the phase quadrature
  - Not true for all interferometer configurations
  - Detuned signal recycled interferometer → GW signal in both quadratures
- Orient squeezed state to reduce noise in phase quadrature

Back Action Produces Squeezing



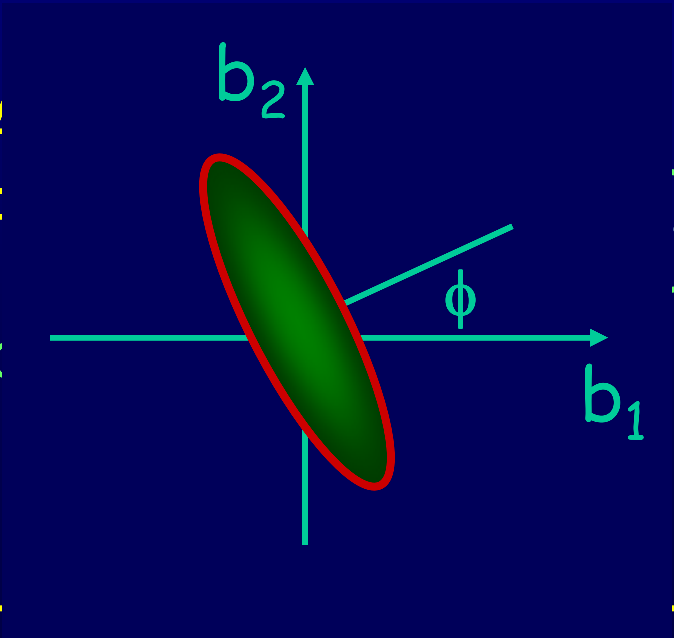
- Vacuum state enters anti-symmetric port
- Amplitude fluctuations of input state drive mirror position
- Mirror motion imposes those amplitude fluctuations onto phase of output field

A

F $a_1 + a_2 + h$

Rad

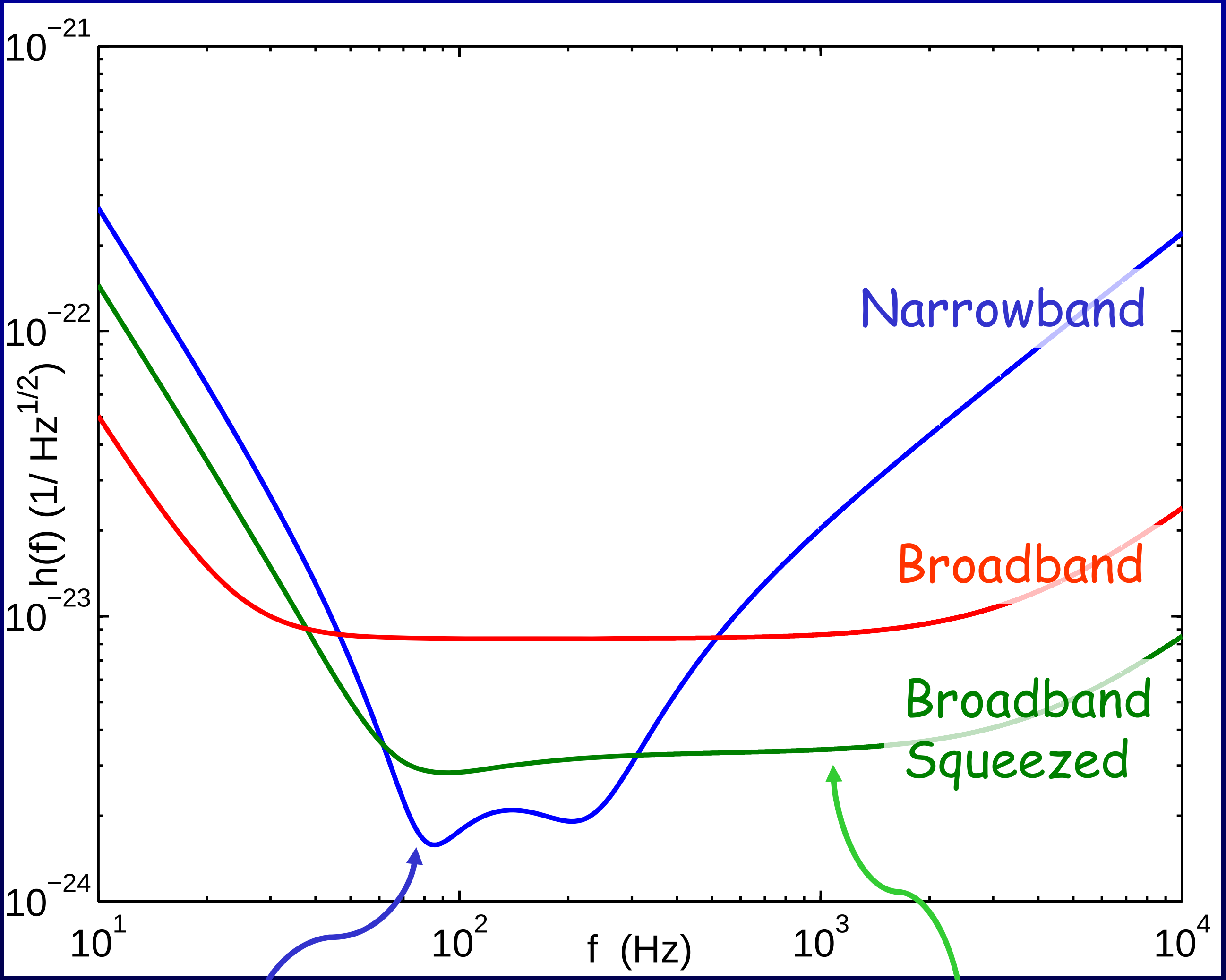
Shot Noise



Squeezing produced by back-action force of fluctuating radiation pressure on mirrors

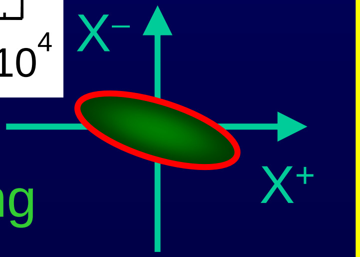
Sub-Quantum Interferometers

Sub-quantum-limited interferometer



Quantum correlations

Input squeezing



Squeezing – the ubiquitous fix?

- All interferometer configurations can benefit from squeezing
    - Shot noise limit only improved by more power (yikes!) or squeezing (eek!)
    - Reduction in shot noise by squeezing can allow for reduction in circulating power (for the same sensitivity) – important for power-handling

Squeezed vacuum states for GW detectors

- Requirements
  - Squeezing at low frequencies (within GW band)
  - Frequency-dependent squeeze angle
  - Increased levels of squeezing
  - Long-term stable operation
- Generation methods
  - Non-linear optical media ($\chi^{(2)}$ and $\chi^{(3)}$ non-linearites) ← crystal-based squeezing
  - Radiation pressure effects in interferometers ← ponderomotive squeezing

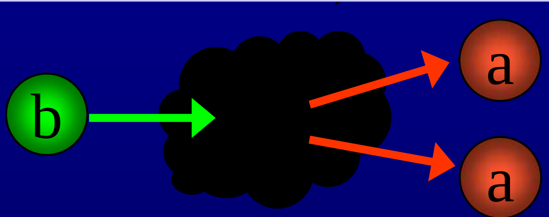
Squeezing using nonlinear optical media

Basic principle

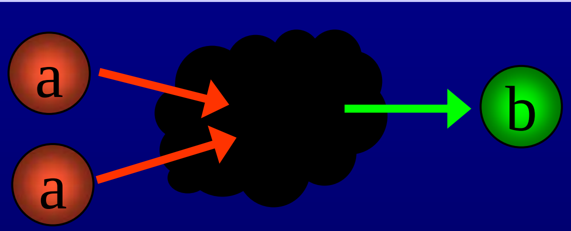
$E_a = \hbar\omega_0$

$E_b = 2\hbar\omega_0$

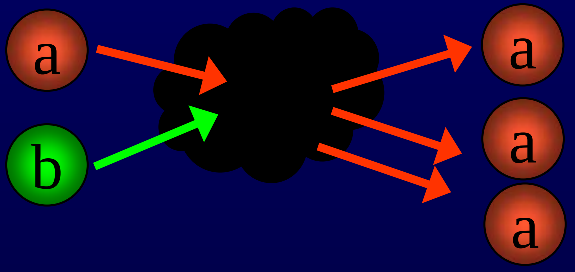
$$\hat{H} = i\hbar\kappa\left(\hat{a}^\dagger\hat{a}^\dagger\hat{b} - \hat{a}\hat{a}\hat{b}^\dagger\right)$$



Parametric oscillation



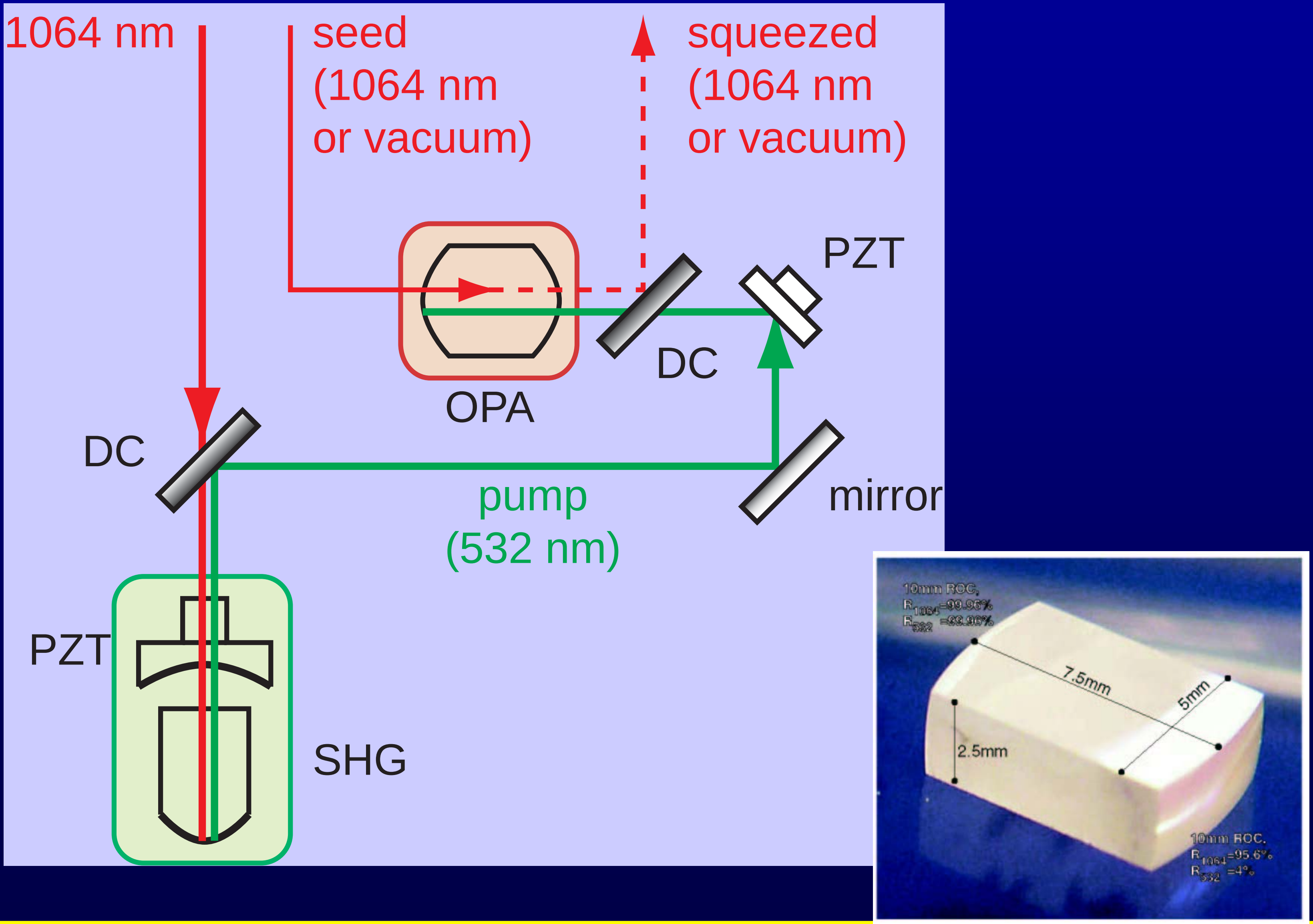
Second harmonic generation



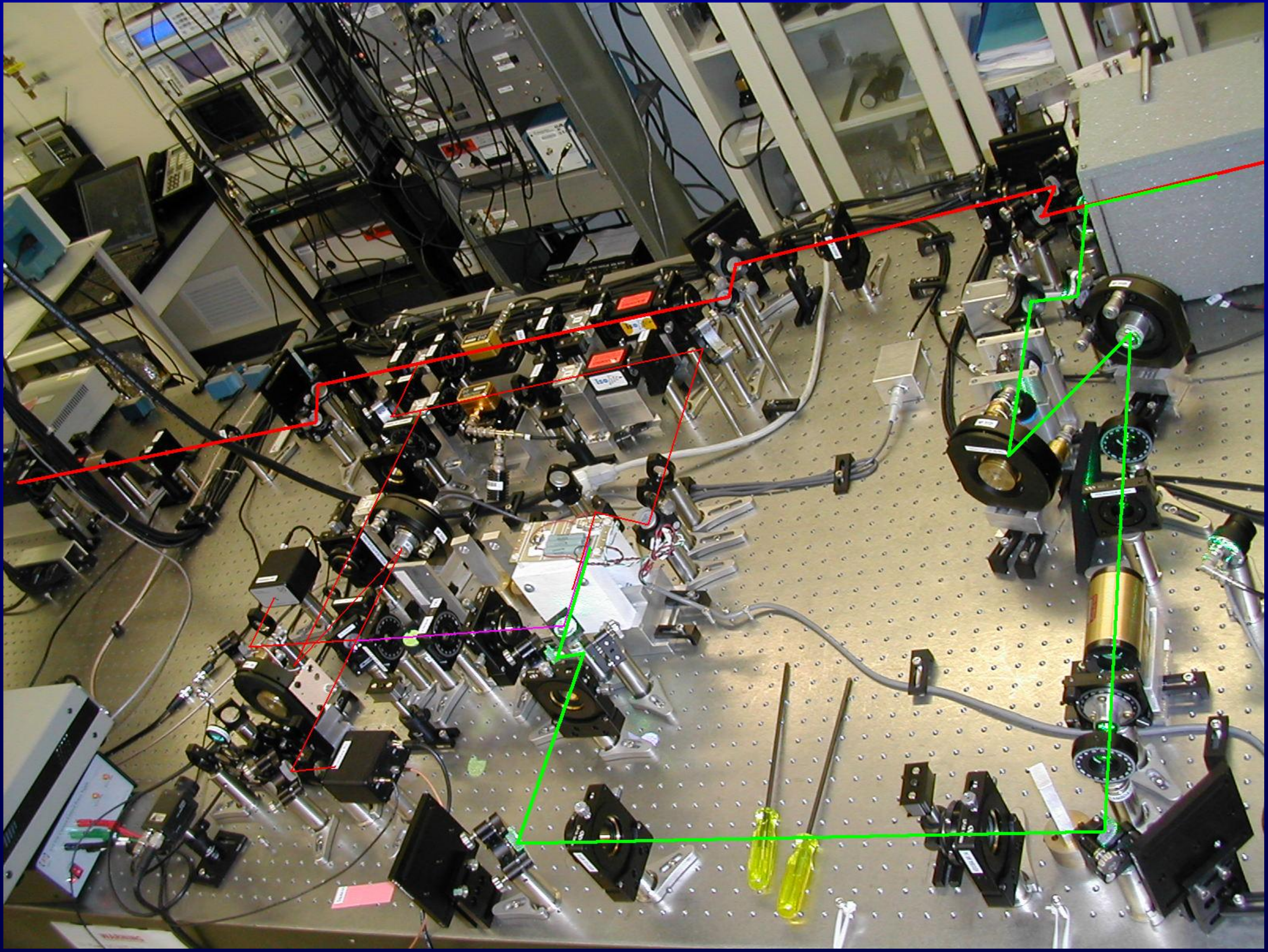
Parametric amplification

The output photon quadratures are correlated

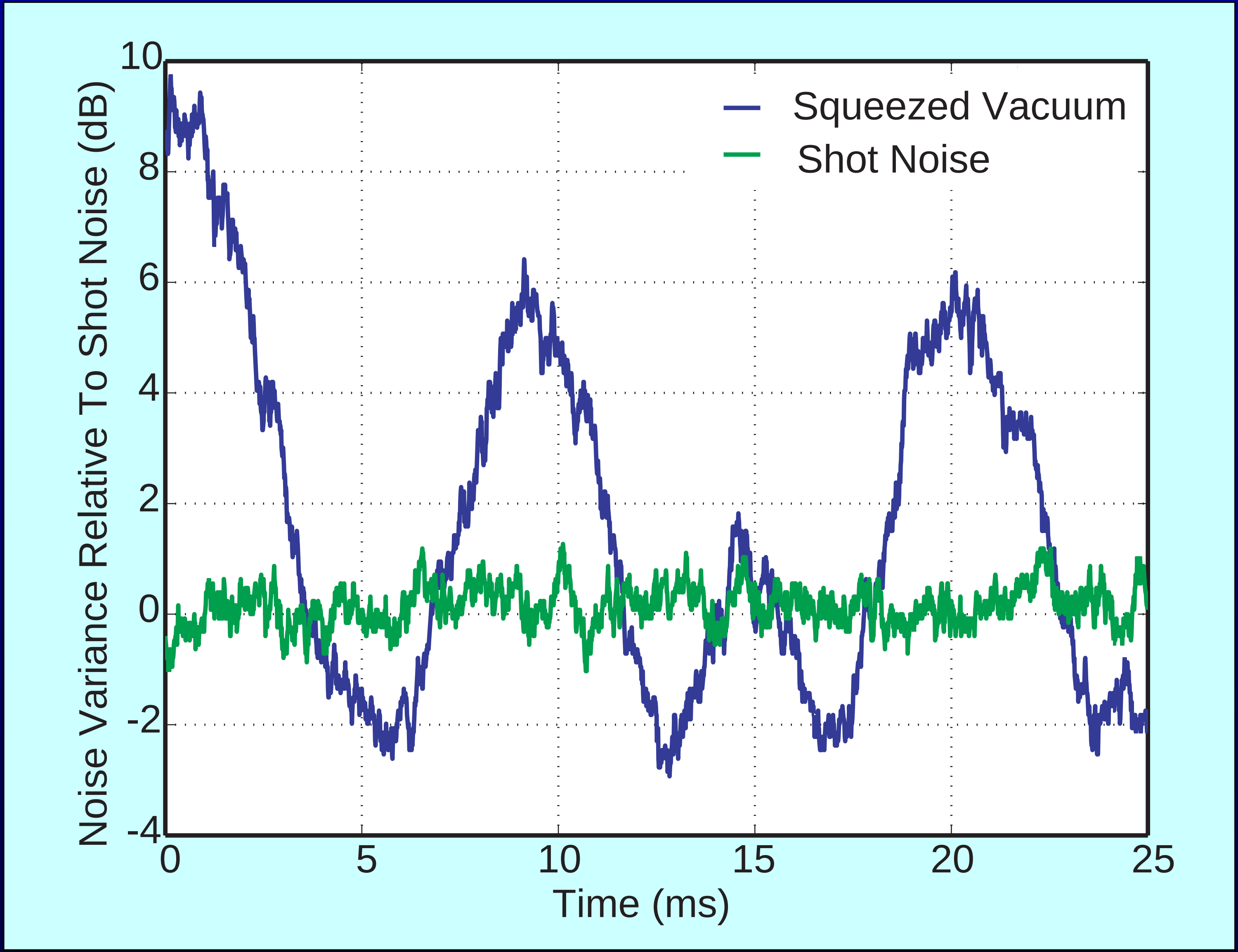
Optical Parametric Oscillator



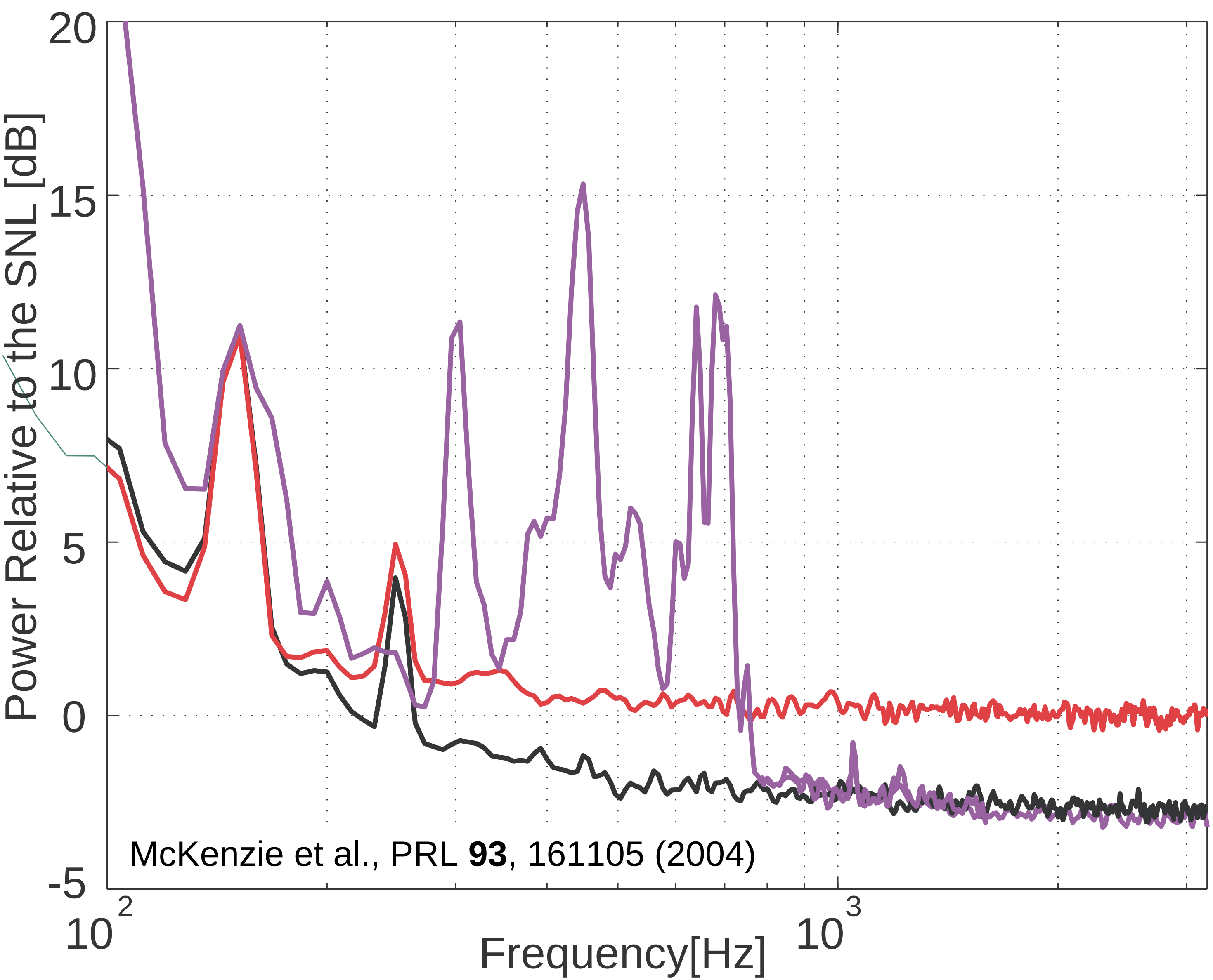
Typical Experimental Setup



Squeezed Vacuum

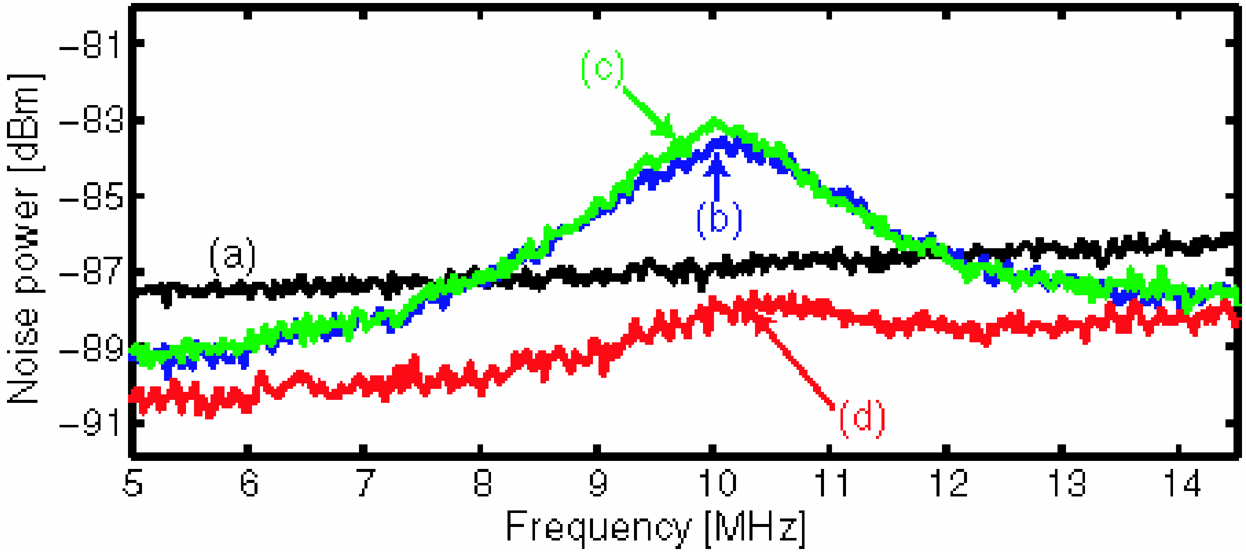


Low frequency squeezing at ANU



McKenzie et al., PRL **93**, 161105 (2004)

Injection in a power recycled Michelson interferometer



Vahlbruch *et al.* Phys. Rev. Lett., **95** 211102 (2005)

K.McKenzie *et al.* Phys. Rev. Lett., **88** 231102 (2002)

Crystal squeezing experiments
Progress in last 5 years

- Squeezing at audio frequencies
- Improved crystals in use
- Filter cavities for frequency-dependent squeezing
- Detailed calculations of noise couplings to establish fundamental limits
- Table top configurations
- Coherently controlled squeezing
- 3 to 4 dB at frequencies > 100 Hz
- Table top interferometers and 40 m (soon)

Light reading tonight…

- Squeezed state injection in power-recycled Michelson
  McKenzie, et al., Phys. Rev. Lett. **88** (2002)
- Audio frequency squeezing
  McKenzie, et al., Phys. Rev. Lett. **93** (2004)
- Squeeze angle rotation
  Chelkowsky, et al., Phys. Rev. A **71** (2004)
- Squeeze amplitude attenuation
  Corbitt, et al., Phys. Rev A **72** (2004)
- Squeezed state injection in dual-recycled Michelson ifo
  Vahlbruch, et al., Phys. Rev. Lett. **95** (2005)
- Far from fundamental limits
  Goda, et al., Phys. Rev. A **72** (2005)
- Incoherent control ('noise locking')
  McKenzie, et al., J. Opt. B **7** (2005)
- Avoid kilometer-scale optical cavities with EIT filters, e.g.
  Mikhailov, et al., PRA in press (2006)

Squeezing using radiation pressure coupling

The principle

- Use radiation pressure as the squeezing mechanism
  - Consider an optical cavity with high stored power and a phase sensitive readout
  - Intensity fluctuations (radiation pressure) drive the motion of the cavity mirrors
  - Mirror motion is then imprinted onto the phase of the light
- Analogy with nonlinear optical media
  - Intensity-dependent refractive index changes couple amplitude and phase

$$\boxed{\phi \sim \frac{2\pi}{\lambda_0} n(I)\, z}$$

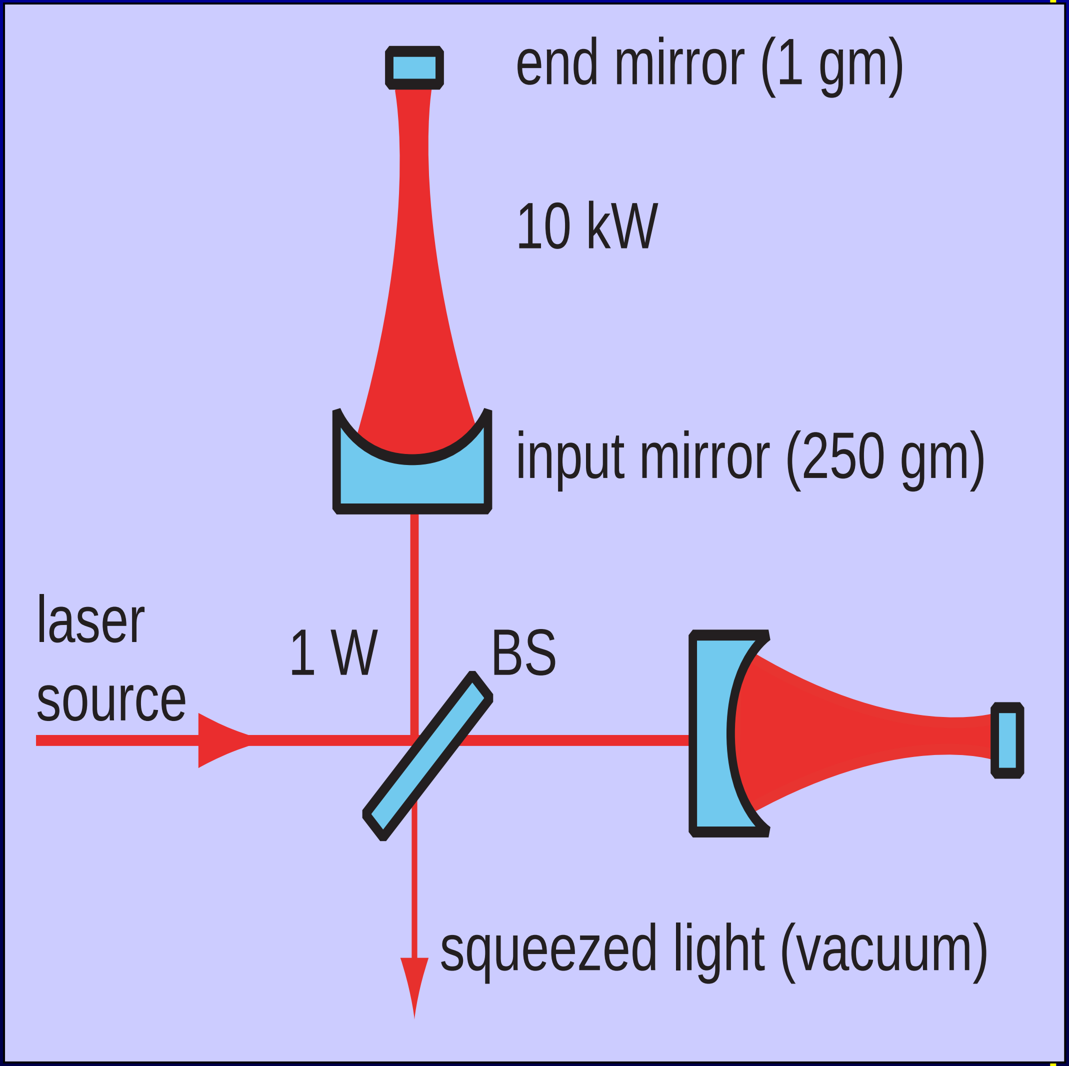
The experimental concept

- A “tabletop” interferometer to generate squeezed light as an alternative to nonlinear optical media
- Use radiation pressure as the squeezing mechanism
- Relies on intrinsic quantum physics of optical field–mechanical oscillator correlations
- Squeezing produced even when the sensitivity is far worse than the SQL
    - Due to noise suppression a la optical springs

The “ponderomotive” interferometer

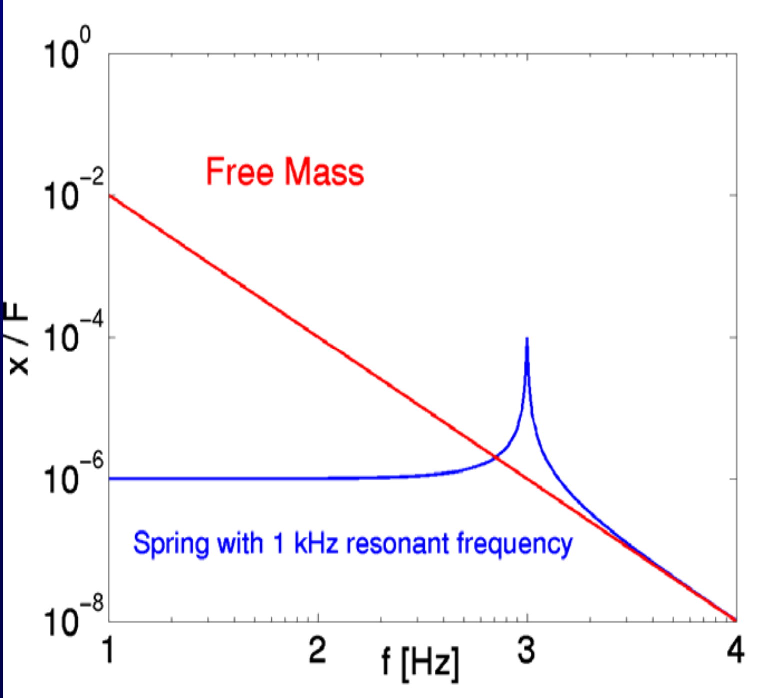
Key ingredients

- Low mass, low noise mechanical oscillator mirror – 1 gm with 1 Hz resonant frequency
- High circulating power – 10 kW
- High finesse cavities 15000
- Differential measurement – common-mode rejection to cancel classical noise
- Optical spring – noise suppression and frequency independent squeezing



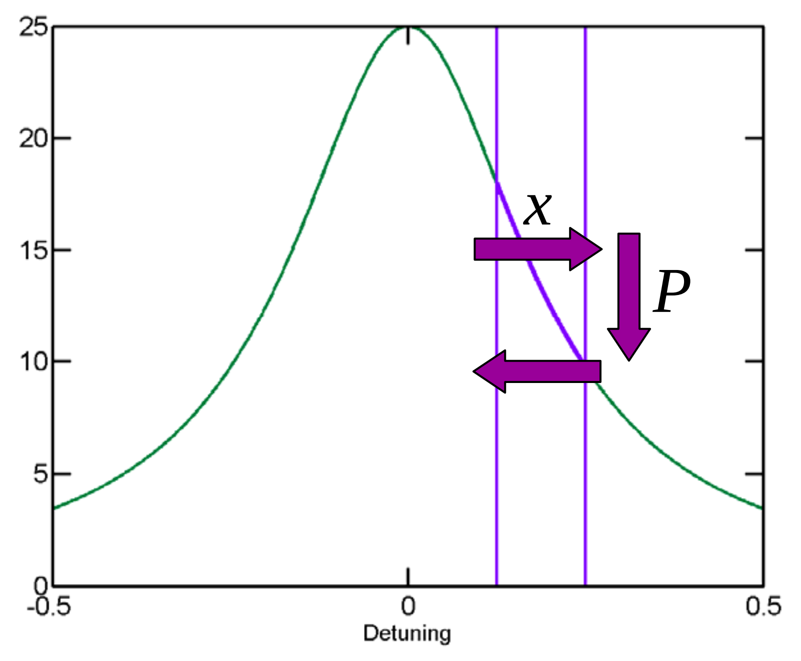
Optical Springs

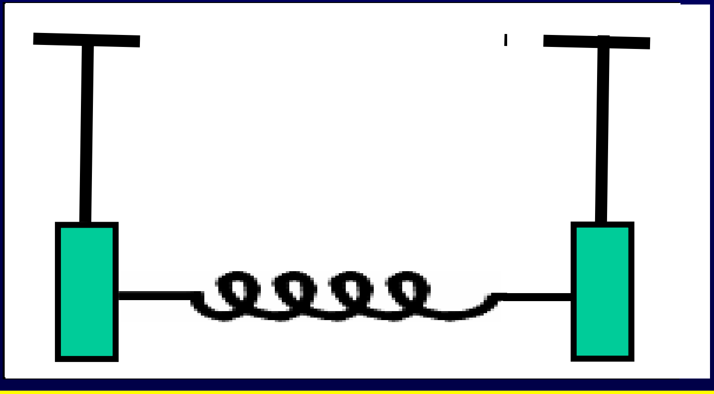
- Modify test mass dynamics
- Suppress displacement noise (compared to free mass case)
- Why not use a mechanical spring?
  - Displacements due to thermal noise introduced by the high frequency (mechanical) spring will wash out the effects of squeezing
- Connect low-frequency mechanical oscillator to (nearly) noiseless optical spring
- An optical spring with a high resonant frequency will not change the thermal force spectrum of the mechanical pendulum
  - Use a low resonant frequency mechanical pendulum to minimize thermal noise
  - Use an optical spring to produce a flat response out to higher frequencies



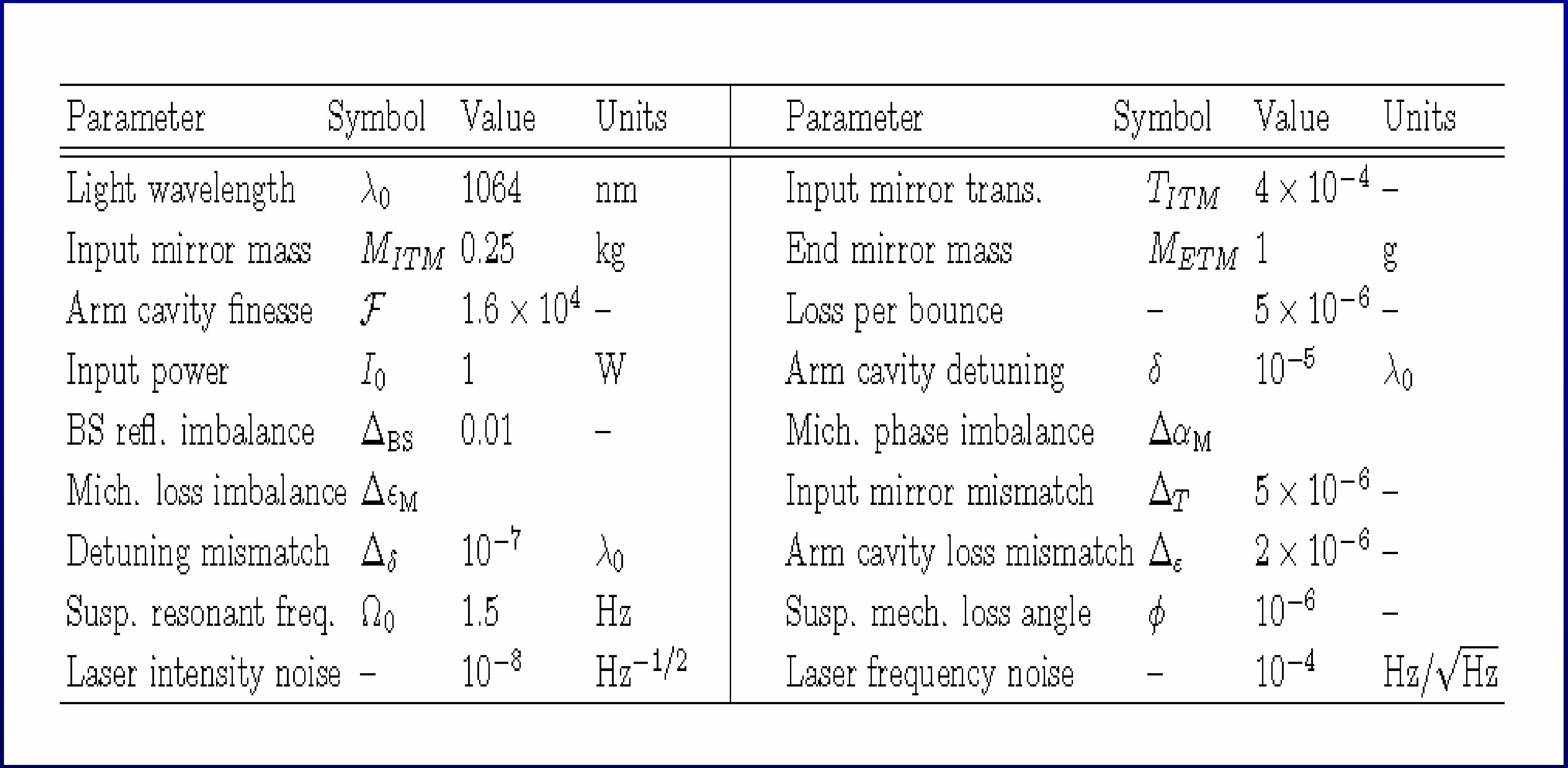
How to make an optical spring?

- Detune a resonant cavity to higher frequency (blueshift)
  - Detuning increases
  - Cavity becomes longer
  - Power in cavity decreases
  - Radiation-pressure force decreases
  - Mirror ‘restored’ to original position
  - Cavity becomes shorter
  - Power in cavity increases
  - Mirror still ‘restored’ to original position



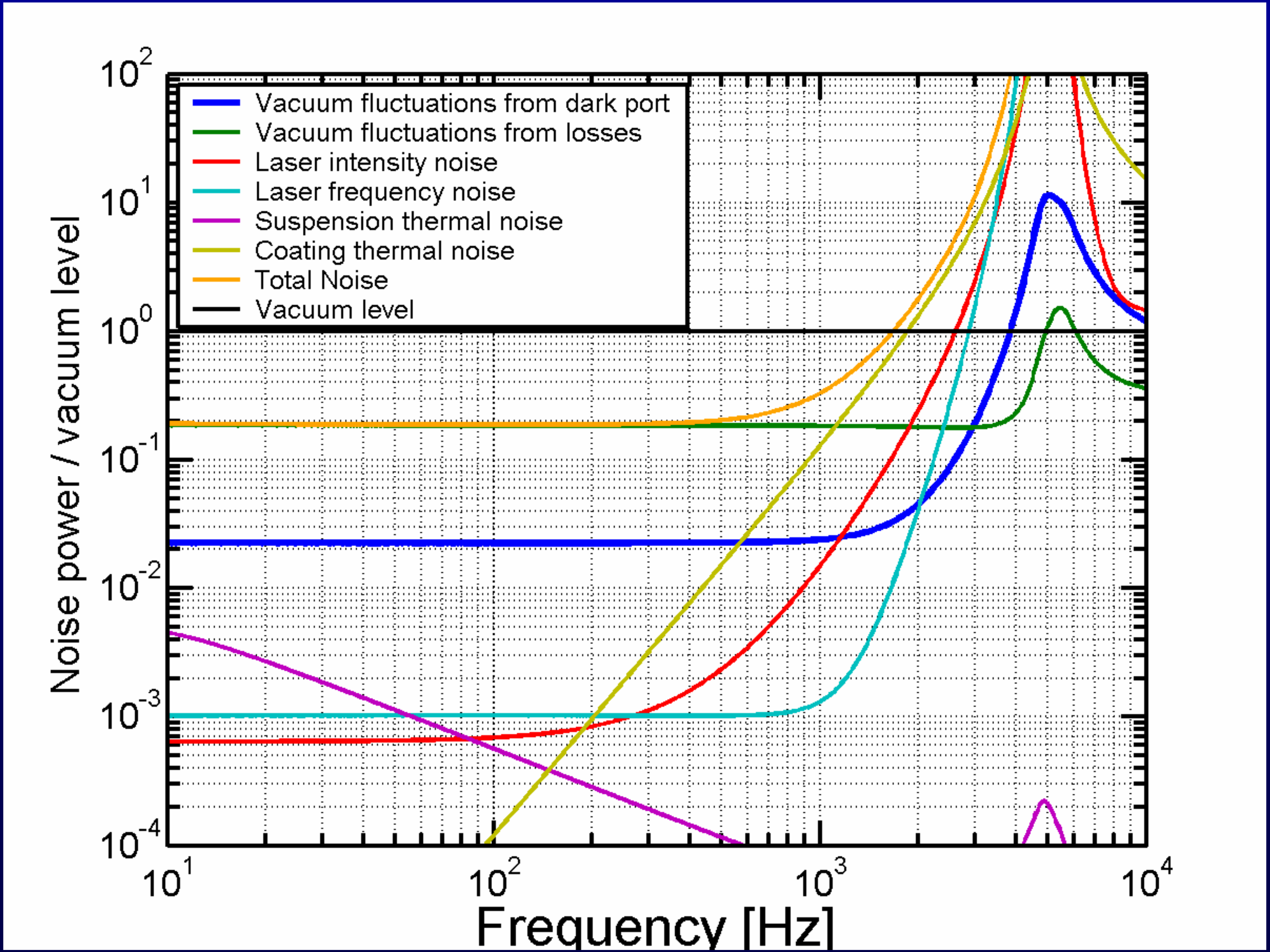


Assumed experimental parameters



Parameter	Symbol	Value	Units	Parameter	Symbol	Value	Units
Light wavelength	λ_0	1064	nm	Input mirror trans.	T_{ITM}	4×10^{-4}	–
Input mirror mass	M_{ITM}	0.25	kg	End mirror mass	M_{ETM}	1	g
Arm cavity finesse	$\mathcal{F}$	1.6×10^4	–	Loss per bounce	–	5×10^{-6}	–
Input power	I_0	1	W	Arm cavity detuning	δ	10^{-5}	λ_0
BS refl. imbalance	Δ_{BS}	0.01	–	Mich. phase imbalance	$\Delta\alpha_{\mathrm{M}}$		
Mich. loss imbalance	$\Delta\epsilon_{\mathrm{M}}$			Input mirror mismatch	Δ_T	5×10^{-6}	–
Detuning mismatch	Δ_δ	10^{-7}	λ_0	Arm cavity loss mismatch	Δ_ε	2×10^{-6}	–
Susp. resonant freq.	Ω_0	1.5	Hz	Susp. mech. loss angle	ϕ	10^{-6}	–
Laser intensity noise	–	10^{-8}	$\mathrm{Hz}^{-1/2}$	Laser frequency noise	–	10^{-4}	$\mathrm{Hz}/\sqrt{\mathrm{Hz}}$

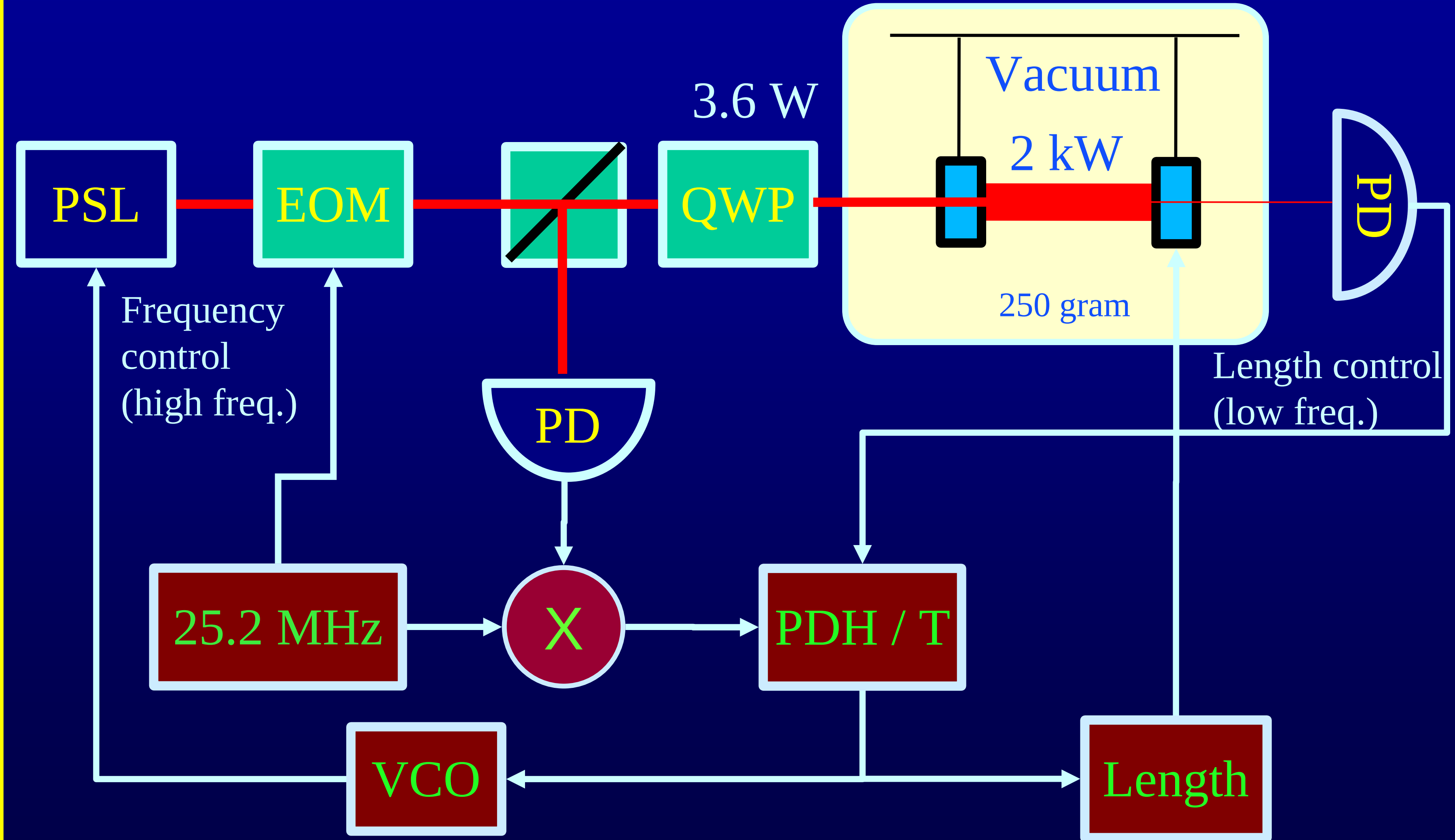
Noise budget



Experimental progress

- Experiment carried out in three phases
  - Phase I → linear cavity with two 250 g suspended mirrors, finesse of 1000, ~4 W of input power
  - Phase II → cavity with one 250 g and one 1 g suspended mirror, finesse of 8000, ~1 W of input power
  - Phase III → two identical cavities and Michelson interferometer
- Ultimate goal – quantum-limited radiation pressure for ponderomotive squeezing interferometer

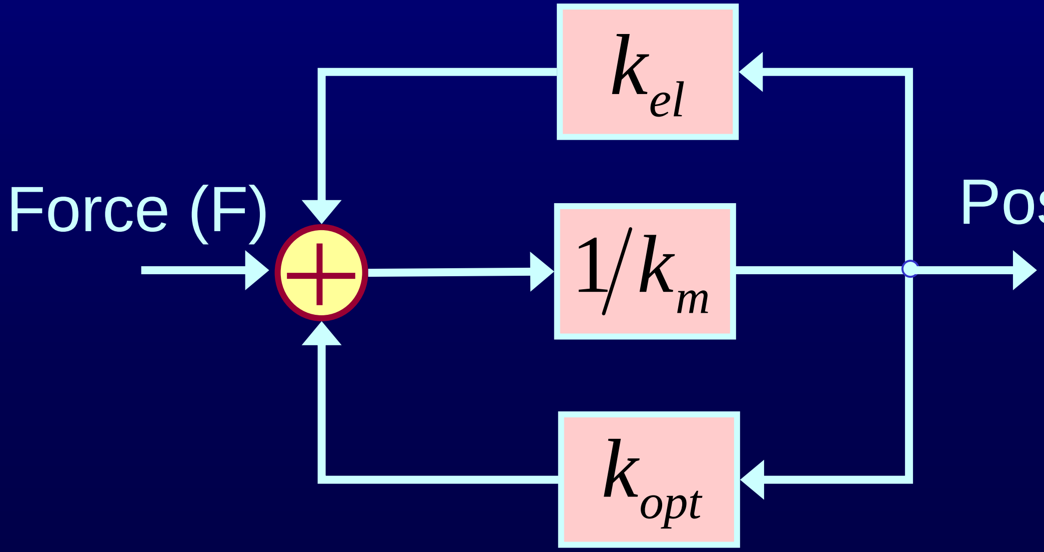
Phase I Experiment



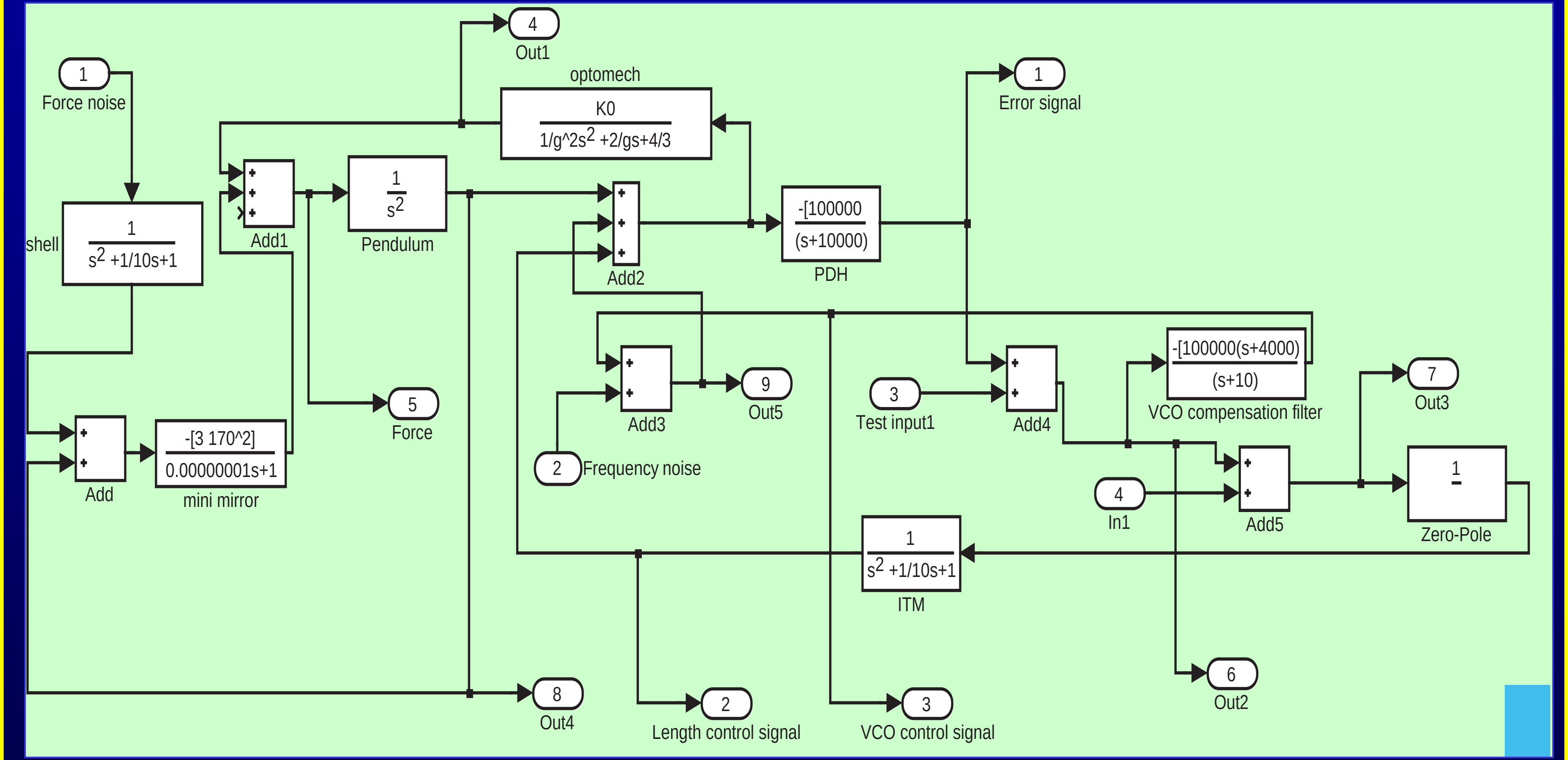
Optical Feedback Model

- Mechanical oscillator
- Optical feedback
  - Optical spring
  - Parametric Instability
- Electronic servo

$$\frac{x}{F} = \frac{1/k_m}{1+\left(k_{opt}+k_{el}\right)/k_m}$$



A Simulink Model



Properties of optical springs

- Optical rigidity

$$K(\Omega) = \frac{K_0(I_0, \delta, \gamma, F)}{(1+i\Omega/\gamma)^2 + (\delta/\gamma)^2}$$

unstable → F(v)

- Modified dynamics

$$-\Omega^2 x = -\left(\Omega_0^2 + \underbrace{\frac{K(\Omega)}{M_{eff}}}_{\Omega_{OR}^2}\right) + i\frac{\Omega_0}{Q_m}\Omega + \frac{F_{ext}}{M_{eff}}$$

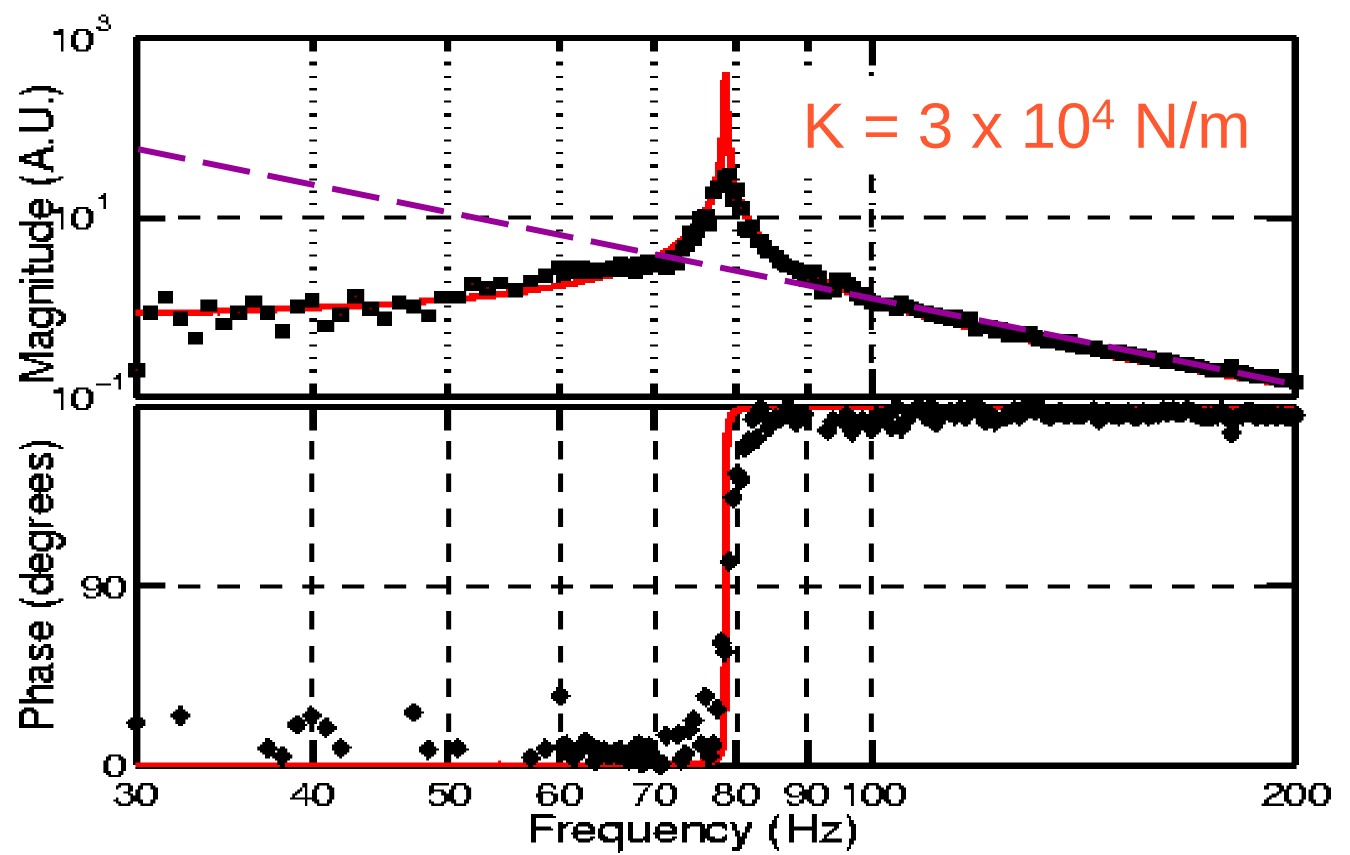
- Noise suppression

$$\frac{x}{F_{ext}} = \frac{\Omega^2}{K/M_{eff} - \Omega^2}\frac{x^{(0)}}{F_{ext}}$$

$$x \to \frac{\Omega^2}{\Omega_{OR}^2}x^{(0)} \quad \text{when } \Omega \ll \Omega_{OR}$$

Optical Spring Measured

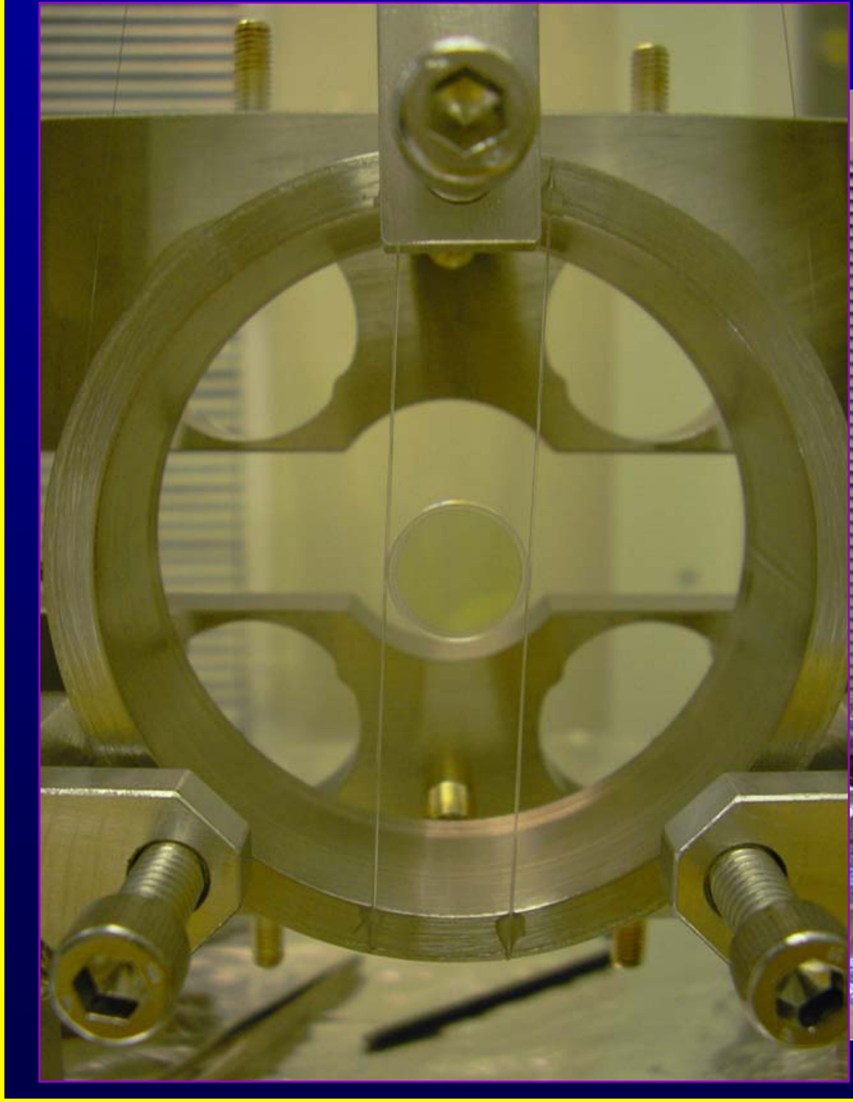
- Phase increases by 180°, so resonance is unstable!
- But there is lots of gain at this frequency, so it doesn't destabilize the system

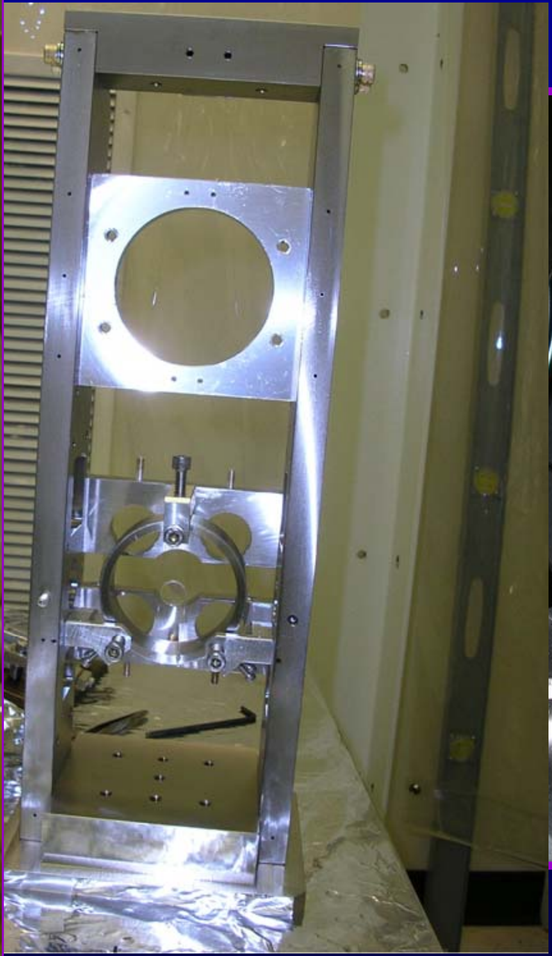


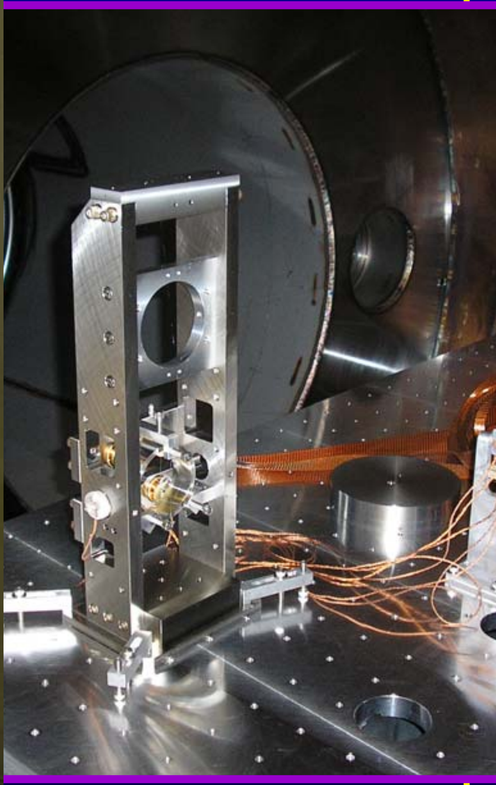
Phase II Cavity

- Use 250 g input and 1 g end mirror in a suspended 1 m long cavity with goal of
    - R < 50 at full power
    - <1 MW/cm^2 power density
    - Optical spring resonance at > 1 kHz
- Final (low thermal noise) suspension for 1 gm mirror not ready yet, so
    - Double suspension
- Goals for this stage
    - See noise reduction effects
    - Get optical spring out of the servo bandwidth
    - See instability directly and damp it

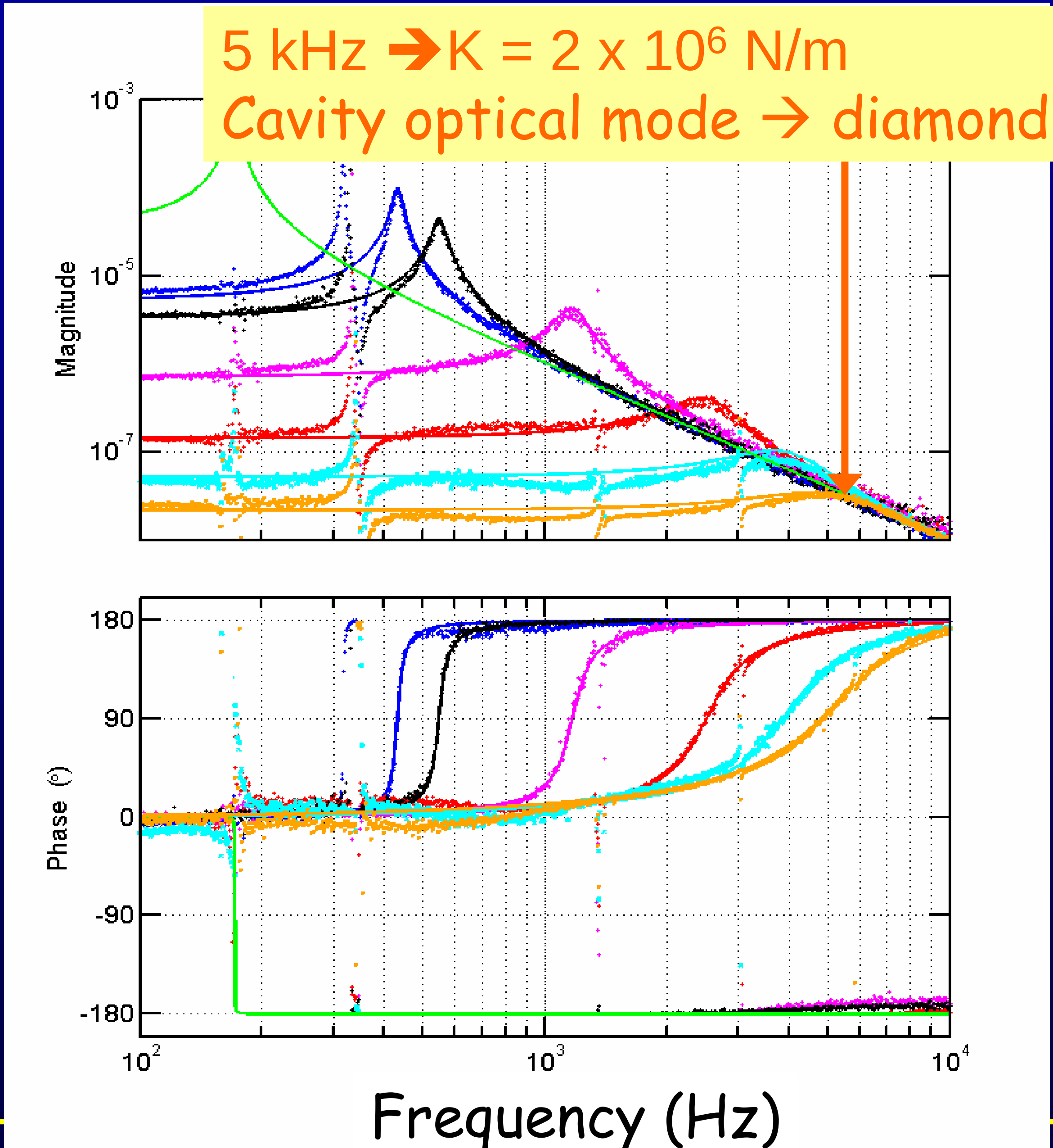
Double suspension for mini mirror (the "MOS")







Noise suppression



5 kHz ➔K = 2 x 10^6 N/m

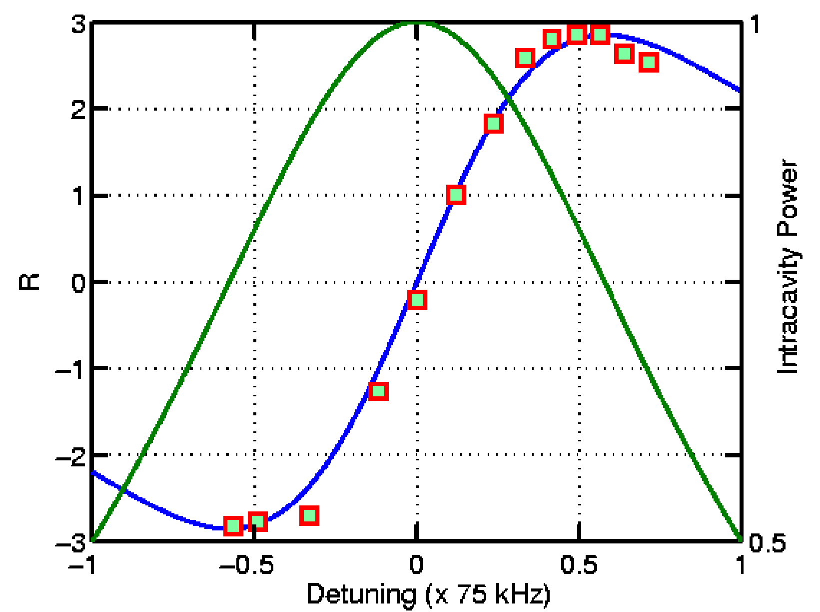
Cavity optical mode → diamond rod

Why is this interesting/important?

- Optical systems that are radiation pressure dominated
    - Study modified mirror oscillator dynamics
    - Manipulate optical field quadratures
- Test of low noise optical spring
    - Suppression of thermal noise
- First ever demonstration of ponderomotive squeezing
- Probes quantum mechanics of optical field-mechanical oscillator coupling at 1 g mass scales (MACROscopic quantum measurement)
- Role of feedback control in these quantum systems

Spin-offs from high power radiation pressure experiments

- Simulations and techniques useful for AdLIGO and other GW interferometers
  - Developed first fully quantum optical simulation code
  - Michelson detuning
- Parametric instability observed and stabilized
- Diamond rods?



- Mini-mirror suspension uses many AdLIGO suspension innovations
- Tests of stabilizing optical spring resonance out of band

Light reading tonight…

- Beyond quantum noise in GW interferometers
  Kimble, et al., Phys. Rev. D **65** (2001)
- Quantum noise in signal recycled interferometers
  Buonanno and Chen, Phys. Rev. D **64** (2001)
- Optical spring effect in signal recycled interferometers
  Buonanno and Chen, Phys. Rev. D **65** (2002)
- Quantum formalism including radiation pressure
  Corbitt, et al., Phys. Rev. A **72** (2005)
- Ponderomotive squeezing experiment description
  Corbitt, et al., Phys. Rev. A **73** (2006)
- Optical spring and parametric instability measured
  Corbitt, et al., gr-qc/0511022 (2005)
- Noise suppression
  Corbitt, et al., in preparation
- Optical spring effect in signal recycled interferometers (40m)
  Miyakawa, et al., in preparation

Conclusions

- Advanced LIGO is expected to reach the quantum noise limit in most of the band
- QND techniques needed to do better
- Squeezed states of the EM field appears to be the most promising approach
  - Crystal squeezing mature
  - Ponderomotive squeezing getting closer
- Factors of 2 to 5 improvements foreseeable in the next decade
  - Not fundamental but technical
- Need to push on this to be ready for future instruments
- Radiation pressure tests on these mass scales – path to macroscopic quantum state measurement