Gravitational Radiation from the Electroweak Phase Transition

Arthur Kosowsky, Rutgers

Collaborators

M. Kamionkowski
M.S. Turner } PRD 55, 7368 (1994)

Andrew Mack (Rutgers)
T. Kahniashvili
(Georgian Acad. Sci.)

Gravitational Radiation is a direct probe of the universe at $t < 1$ second

Cosmological Evolution: (Einstein equation)

$$\ddot{h}_{ij} + \underbrace{2\frac{\dot{a}}{a}\dot{h}_{ij}}_{\text{Expansion Drag}} + k^2 h_{ij} = \underbrace{8\pi G a^2 \Pi_{ij}}_{\text{Source}}$$

Π_{ij} is the tensor piece of T_{ij}

- k scales with a (conformal expansion)

Homogeneous solutions:

$h \propto j_0(k\eta)\ ,\ y_0(k\eta)$ Radiation dominated

$h \propto \dfrac{j_1(k\eta)}{k\eta}\ ,\ \dfrac{y_1(k\eta)}{k\eta}$ Matter dominated

- Amplitude scales as a^{-1} $\rho = \langle \dot{h}\dot{h} \rangle$

Green function for radiation era:

$$G(\eta,\eta') = \frac{\sin(k(\eta-\eta'))}{k}\frac{\eta'}{\eta}$$

For $\Pi_{ij} = \Pi_{ij}(k)$,

$$h(\eta,k) = \frac{2\pi G\Pi(k) z_{eq}^2 \eta_{eq}^2}{(3-2\sqrt{2})k\eta}\int_{\eta_*}^{\eta} d\eta' \, \frac{\sin(k\eta - k\eta')}{\eta'} \quad , \; \eta < \eta_{eq}$$

Match to homogeneous solution during matter era:

$$\dot{h}(\eta,k) \simeq 4\pi G\eta_0^2 z_{eq} \ln\left(\frac{z_*}{z_{eq}}\right) k\,\Pi(k)\,\frac{j_2(k\eta)}{k\eta} \quad , \; \eta > \eta_{eq}$$

Phase Transitions

Occur in early universe when symmetry is broken

QCD Quark-hadron transition
$T \approx 1\ \text{GeV} \quad (m_p)$

Electroweak symmetry breaks
$T \simeq 100\ \text{GeV} \quad (m_W)$

GUT ? Grand unification symmetry broken
$T \simeq 10^{16}\ \text{GeV}$?

First-order phase transitions proceed via nucleation, expansion and percolation of bubbles

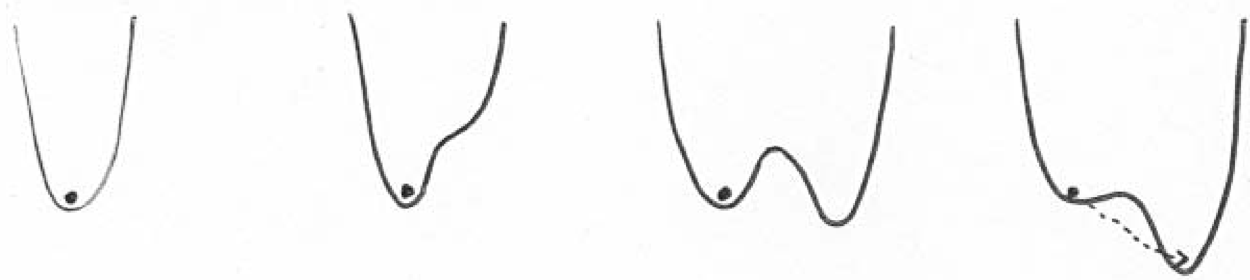
Time scale set by bubble nucleation rate

$$\Gamma = \Gamma_0 e^{\beta t}$$

β is nucleation time scale

Typically $\quad \beta \simeq 4 \ln\left(\frac{m_{Pl}}{T}\right) H \simeq 100\, H$

Effective potential:

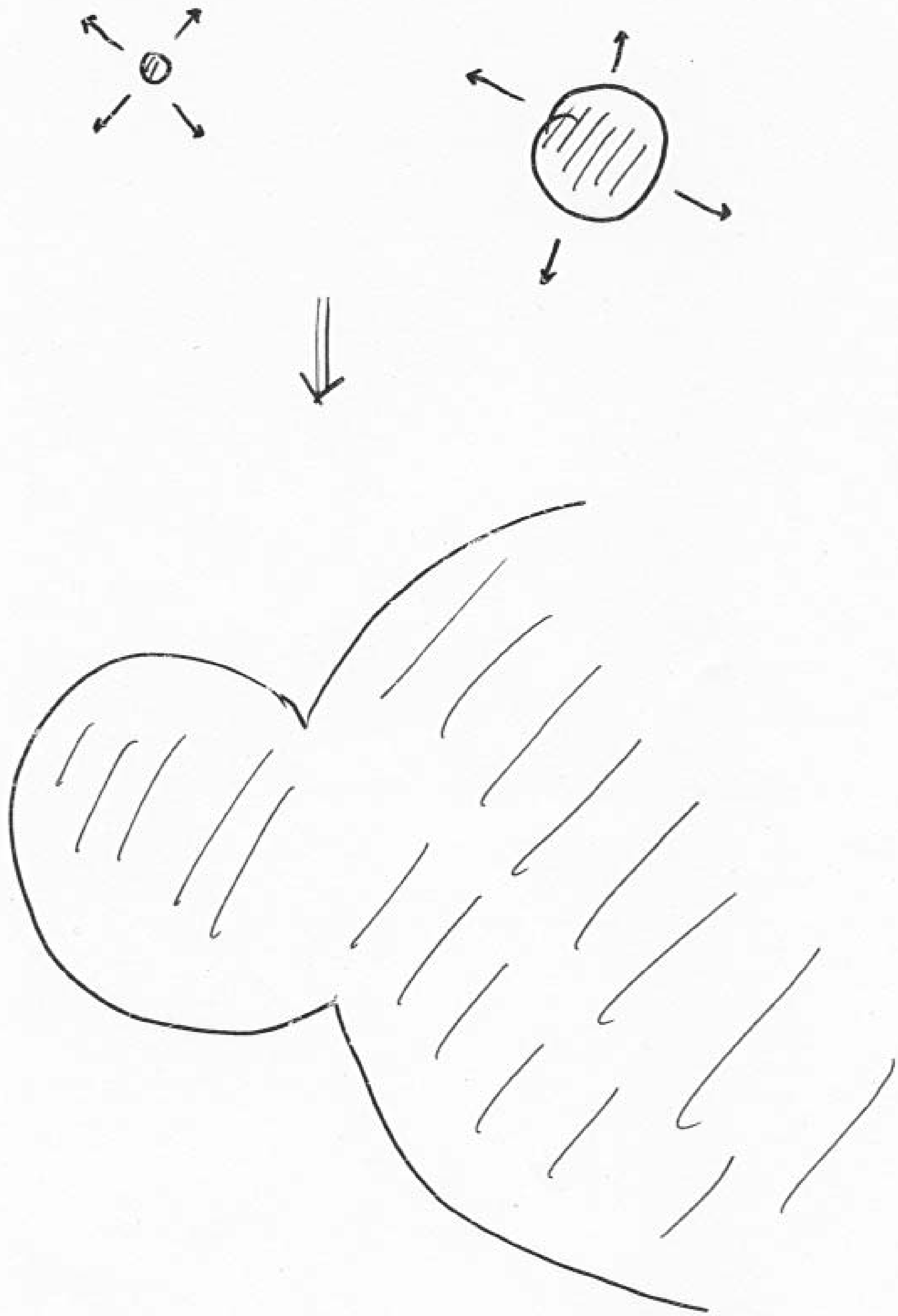


Length scale set by bubble wall velocity v

$$L \simeq v \beta^{-1}$$

v determined by $\dfrac{\text{Latent heat}}{\text{Thermal energy}}$

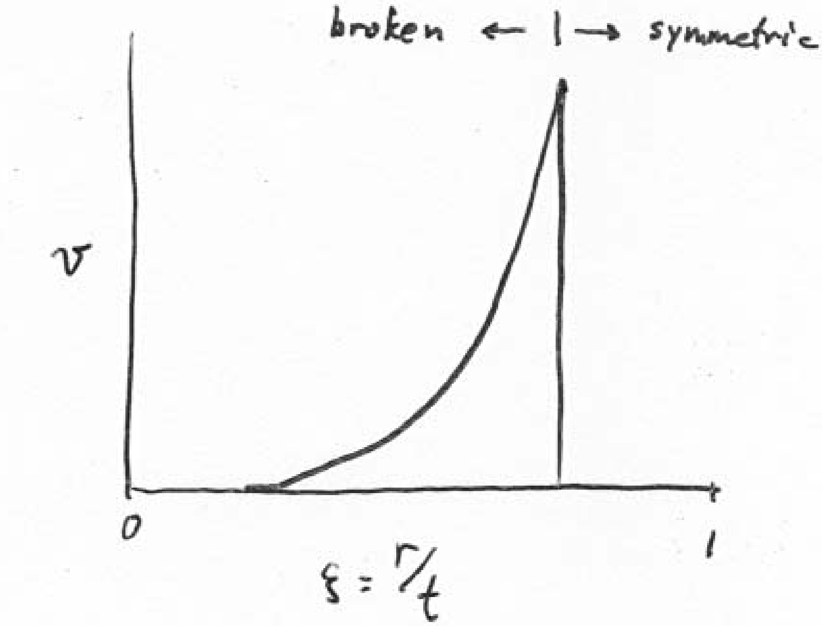
Expansion driven by latent heat
Bubble walls can contain much of latent heat as kinetic energy



Hydrodynamic Bubble Walls

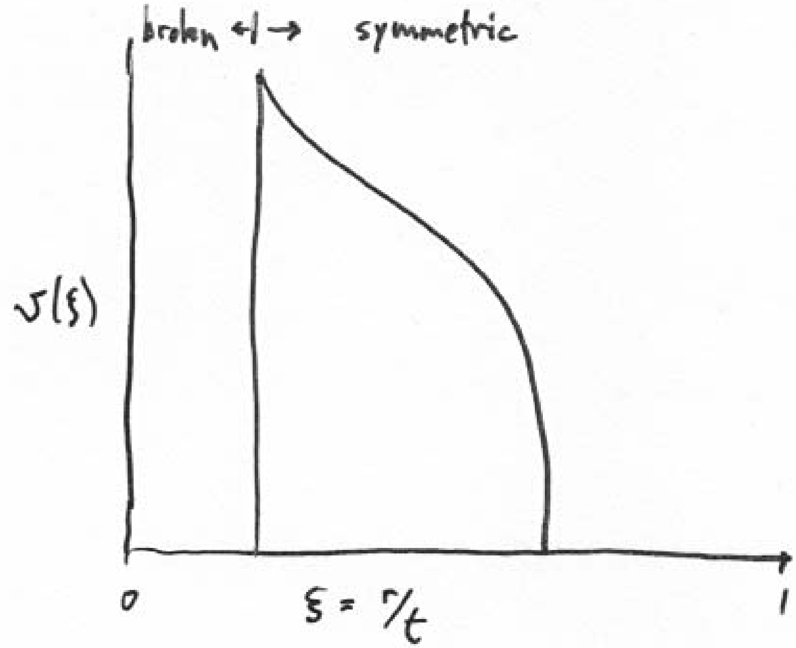
Detonation

$v_0 > c_s$



Deflagration

$v_0 < c_s$



Mode depends on $\dfrac{\text{latent heat}}{\text{thermal energy}}$, hydrodynamic stability

$T(\xi) = w v^2 \gamma^2$ for spherical bubbles

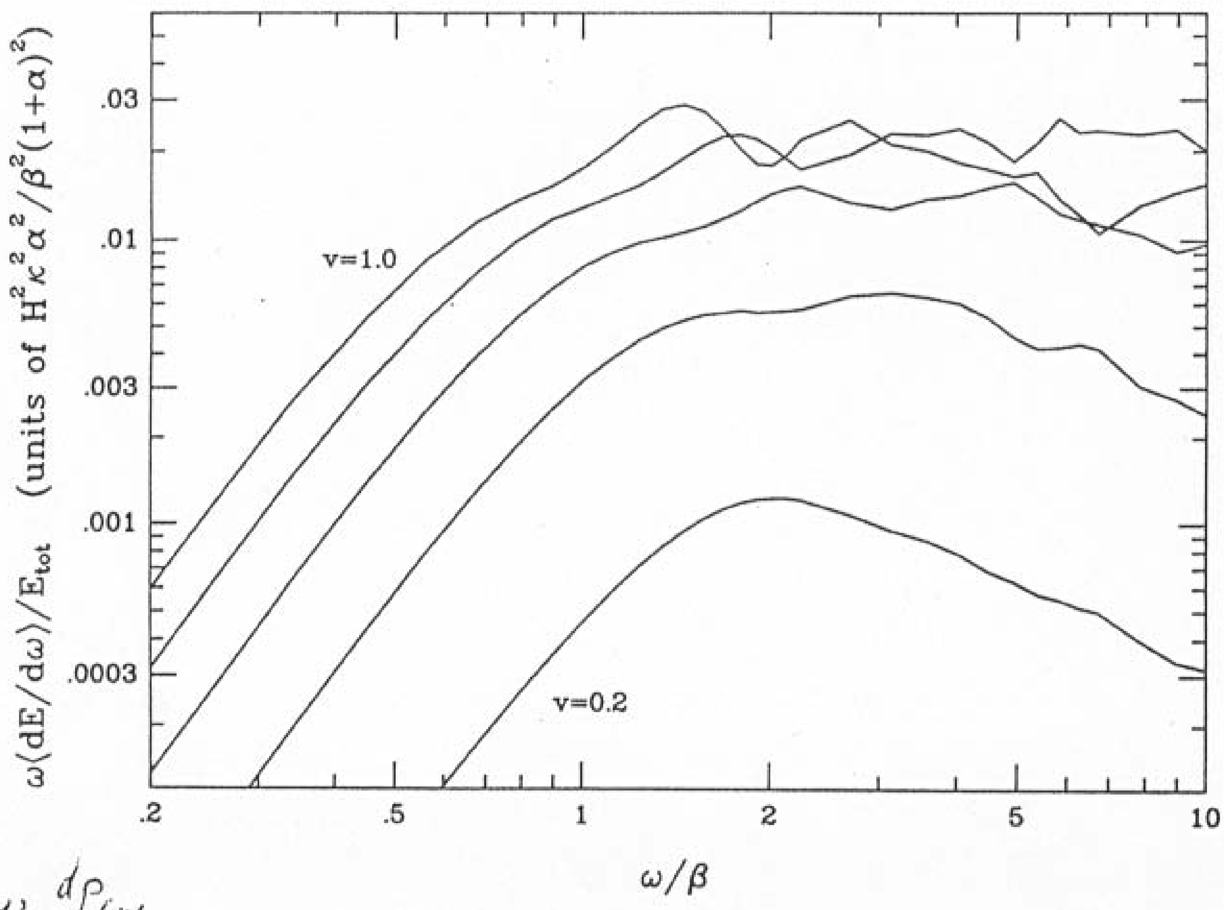


FIG. 7. The energy per octave radiated in gravity waves for a phase transition with spherical bubbles expanding at velocity v, for $v = 0.2$, $v = 0.4$, $v = 0.6$, $v = 0.8$, and $v = 1.0$.

$\omega \dfrac{d\rho_{GW}}{d\omega}$

$$\frac{E_{\rm GW}}{E_{\rm tot}} \approx 0.07\kappa^2 \left(\frac{H}{\beta}\right)^2 \left(\frac{\alpha}{1+\alpha}\right)^2 \left(\frac{v^3}{0.24+v^3}\right) . \quad (30)$$

Note that in the strong-detonation limit $v \to 1$ and $\alpha \to \infty$, this reduces to the vacuum-bubble result of Ref. [3].

The radiation spectra in Fig. 7 depend on the parameters v, κ, β, and $\epsilon = 3w_1\alpha/4$. A particular phase transition is characterized by the temperature at which it occurs and its latent heat, or equivalently by w_1 and α. For

a phase transition (the critical Reynolds number for the onset of turbulence is around 2000), especially if bubble walls are unstable to perturbations and become highly nonspherical.

In the case of fully developed turbulence the distribution of the turbulent kinetic-energy density is expected to take the stationary Kolmogoroff form [19]

$$k\frac{d\rho_{\rm turb}}{dk} \propto k^{-2/3}. \quad (31)$$

Bubble Collisions

$$\Omega_G h^2 \simeq 10^{-6} \kappa^2 \left(\frac{H_*}{\beta}\right)^2 \left(\frac{\alpha}{1+\alpha}\right)^2 \left(\frac{v^3}{0.24+v^3}\right) \left(\frac{100}{g_*}\right)^{1/3}$$

$$f_{max} \simeq 5.2\times 10^{-8}\,\mathrm{Hz} \left(\frac{\beta}{H_*}\right)\left(\frac{T_*}{1\,\mathrm{GeV}}\right)\left(\frac{g_*}{100}\right)^{1/6}$$

$$h_c(f_{max}) \simeq 1.8\times 10^{-14} \kappa \left(\frac{\alpha}{1+\alpha}\right)\left(\frac{H_*}{\beta}\right)^2\left(\frac{1\,\mathrm{GeV}}{T_*}\right)\left(\frac{v^3}{0.24+v^3}\right)^{1/2}\left(\frac{100}{g_*}\right)^{1/3}$$

$\alpha = \dfrac{\text{vacuum energy}}{\text{thermal energy}}$

β^{-1} = nucleation time scale

κ = fraction of vacuum energy into wall kinetic energy

v = wall velocity

$$h_c(f) \equiv 1.3\times 10^{-18} \left[\Omega_G h^2\right]^{1/2} \left(\frac{1\,\mathrm{Hz}}{f}\right)$$

Valid for detonations $v > c_s = 1/\sqrt{3}$

Strongly first-order phase transition

Kolmogoroff Turbulence

- Energy injected on some length scale, produces coherent motions

- Cascade develops, "eddys" on smaller scales
- At some smaller scale, viscosity becomes important: damps bulk motion, energy into heat

- For constant rate of energy flow from larger to smaller scales, Kolmogoroff spectrum of turbulent energy

$$E(k) \equiv \frac{1}{w}\frac{d\rho_{turb}}{dk} = C_k \bar{\varepsilon}^{2/3} k^{-5/3}$$

$C_k \approx 1$ Kolmogoroff constant

$\bar{\varepsilon} = 2\nu \int_{k_s}^{k_D} k^2 E(k)\, dk$ Energy dissipation rate per unit enthalpy

$w = \rho + p$ Enthalpy

ν Kinematic Viscosity

$Re = \left(\frac{k_D}{k_s}\right)^{4/3}$ Reynolds number, $Re \gtrsim 2000$

$T_{ij}(\vec{x}) = w\, u_i(\vec{x})\, u_j(\vec{x})$ Stress-energy tensor (non-diagonal piece)

Model Turbulence

Assume Kolmogoroff spectrum; assume established and dissipated instantaneously.

Characterized by the following set of parameters:

$$\alpha = \frac{\text{vacuum energy}}{\text{thermal energy}} = \frac{4\rho_{vac}}{3w}$$

K = fraction of vacuum energy into turbulent energy

τ = duration of turbulence

L_S = stirring scale

ν = kinematic viscosity

From the Kolmogoroff spectrum, turnover time scale on length scale L

$$\tau_L \simeq \left(\tfrac{2}{3}\right)^{1/2} (2\pi)^{1/3}\, \bar{\varepsilon}^{-1/3} L^{2/3}$$

$\bar{\varepsilon}$ is the energy density dissipation rate per unit enthalpy

$$\bar{\varepsilon} = \frac{27}{8} \nu^3 k_D^4 \quad , \quad k_D \text{ is dissipation scale}$$

$$\simeq \frac{K \rho_{vac}}{w \tau}$$

Turbulence

$$f_{max} \simeq 1.7 \times 10^{-5}\ \mathrm{Hz} \left(\frac{\beta}{H_*}\right) v_b^{-1} \left(\frac{T_*}{100\,\mathrm{GeV}}\right) \left(\frac{g_*}{100}\right)^{1/6}$$

$$h_c(f_{max}) \simeq 5 \times 10^{-17}\, v_b^2 \left(\frac{H_*}{\beta}\right)^2 (\kappa\alpha)^{3/4} \left(\frac{100}{g_*}\right)^{1/3} \left(\frac{100\,\mathrm{GeV}}{T_*}\right) \left(\frac{f}{f_{max}}\right)^{-11/6}$$

$\alpha = \dfrac{\text{vacuum energy}}{\text{thermal energy}}$

$L_S = \beta^{-1} v_b$

$\tau = \max(\beta^{-1}, \tau_s)$

β^{-1} = nucleation time scale

κ = fraction of vacuum energy into turbulence

v_b = bubble wall velocity

For maximum amplitudes,

$$\frac{h_{turb}}{h_{bubbles}} \simeq \frac{1}{4} v_b^{1/2} (\kappa\alpha)^{-1/4}$$

Electroweak Phase Transition

- $SU(2)_L \otimes U(1)_Y \to U(1)_{EM}$
- Effective potentiall for Higgs field
- Particles gain mass
- $T_* \simeq 100\ \text{GeV} \quad (= m_W, m_Z)$

$$H_* \simeq 1.66\, g_*^{1/2} \frac{T_*^2}{m_{Pl}} \quad , \quad H_*^{-1} = 13\ \text{cm}$$

$$\frac{R_*}{R_0} = 8.0 \times 10^{-16} \left(\frac{100}{g_*}\right)^{1/3} \left(\frac{100\ \text{GeV}}{T_*}\right)$$

$$L_0 = L_S \frac{R_0}{R_*} \simeq \frac{R_0}{R_*}\left(10^{-2} v_b H^{-1}\right)$$

$$\simeq 3 \times 10^{12}\ \text{cm} \left(\frac{g_*}{100}\right)^{1/3} \left(\frac{T_*}{100\ \text{GeV}}\right) \left(\frac{v_b}{0.02}\right)$$

$$\nu_0 = \frac{c}{L_0} \simeq 0.01\ \text{Hz}$$

For a given particle physics model, nature of phase transition determined by finite-T effective potential for order parameter

EWPT: Higgs field effective potential

Standard Model: no phase transition for $m_H > m_W$

Minimal Supersymmetric Std. Model:

- Two Higgs doublets H_1, H_2
- $V = m_1^2 H_1^\dagger H_1 + \cdots + \frac{1}{8} g^2 (H_2^\dagger \sigma H_2 + H_1^\dagger \sigma H_1)^2 + \cdots$

  Quadratic, Quartic in H

- Cubic terms only at finite-T

Weak GW signals

Next-to-Minimum SSM

Add a complex gauge singlet N

Superpotential $W = \lambda H_1 H_2 N - \frac{k}{3} N^3$

J. Ellis et al. (1989)

- Now have tree-level cubic term in Higgs potential
- Phase transition can be strong
- Complex parameter space, action for bubble nucleation
- Concrete baryogenesis scenarios

Huber & Schmidt (2001)

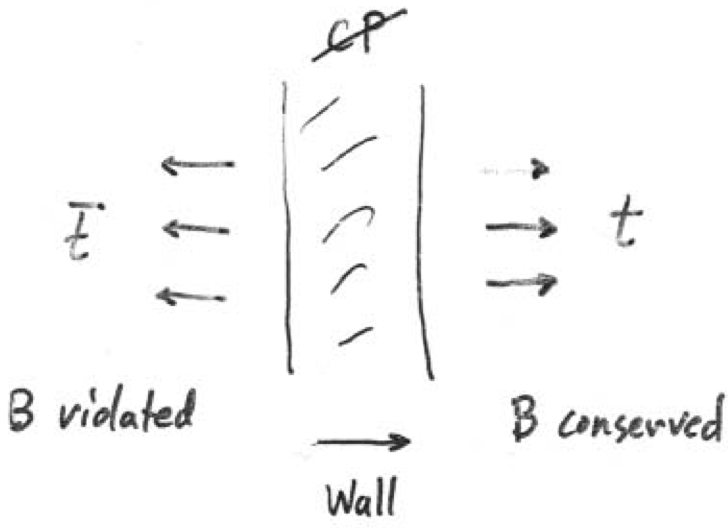
Multiple regions of parameter space giving large signals
Assuming detonation bubbles: (Apreda et al. 2001)

$\frac{\beta}{H_*} = 100$, $\alpha = 1.25$ $\qquad h_{max} \sim 10^{-21}$, $f \sim 2 \times 10^{-3}$ Hz

$\frac{\beta}{H_*} = 20$, $\alpha = 3$ $\qquad h_{max} \sim$ few $\times 10^{-19}$, $f = 4 \times 10^{-4}$ Hz

(Most parameter space gives undetectably small amplitude)

EW Baryogenesis ?



Need both sufficient CP violation (slower wall)
sufficient departure from thermal equilibrium
(faster wall)

Optimal: $v \simeq 0.02$ (?)
$K\alpha \simeq 10^{-4}$ to 10^{-5} (?) (depends on microphysics)

$$h_{max} \simeq 7 \times 10^{-23} (K\alpha)^{5/9} \simeq 2 \times 10^{-25}$$

at $\nu = 0.01$ Hz

Detectable?

LISA design $\simeq 10^{-23}$

LISA Sagnac $\simeq 10^{-24}$ (Hogan & Bender 2001)