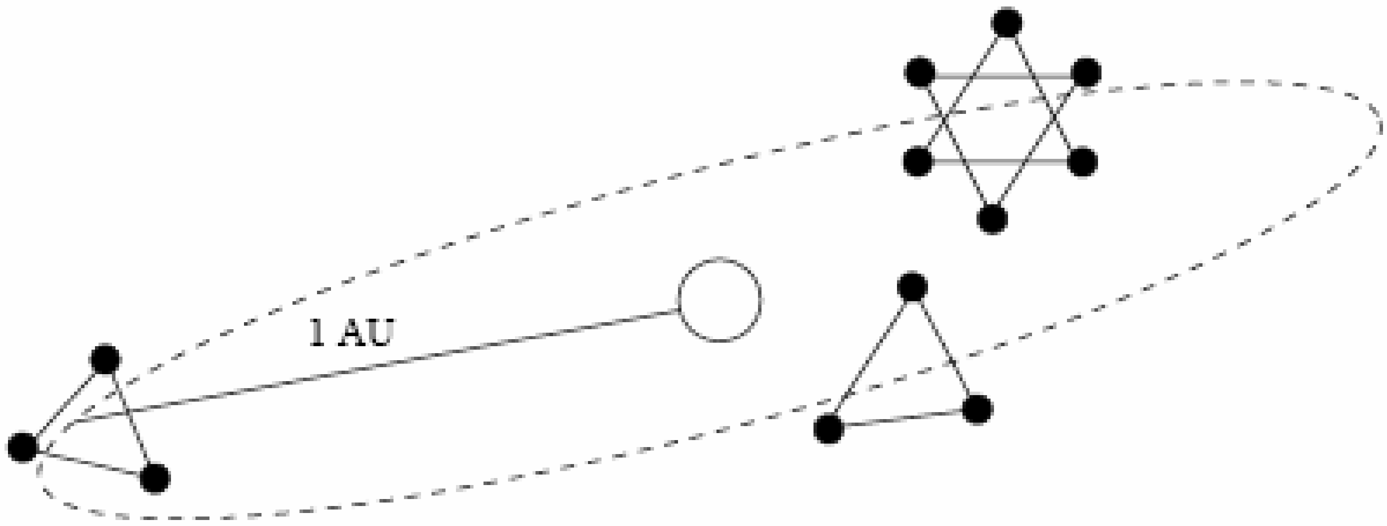
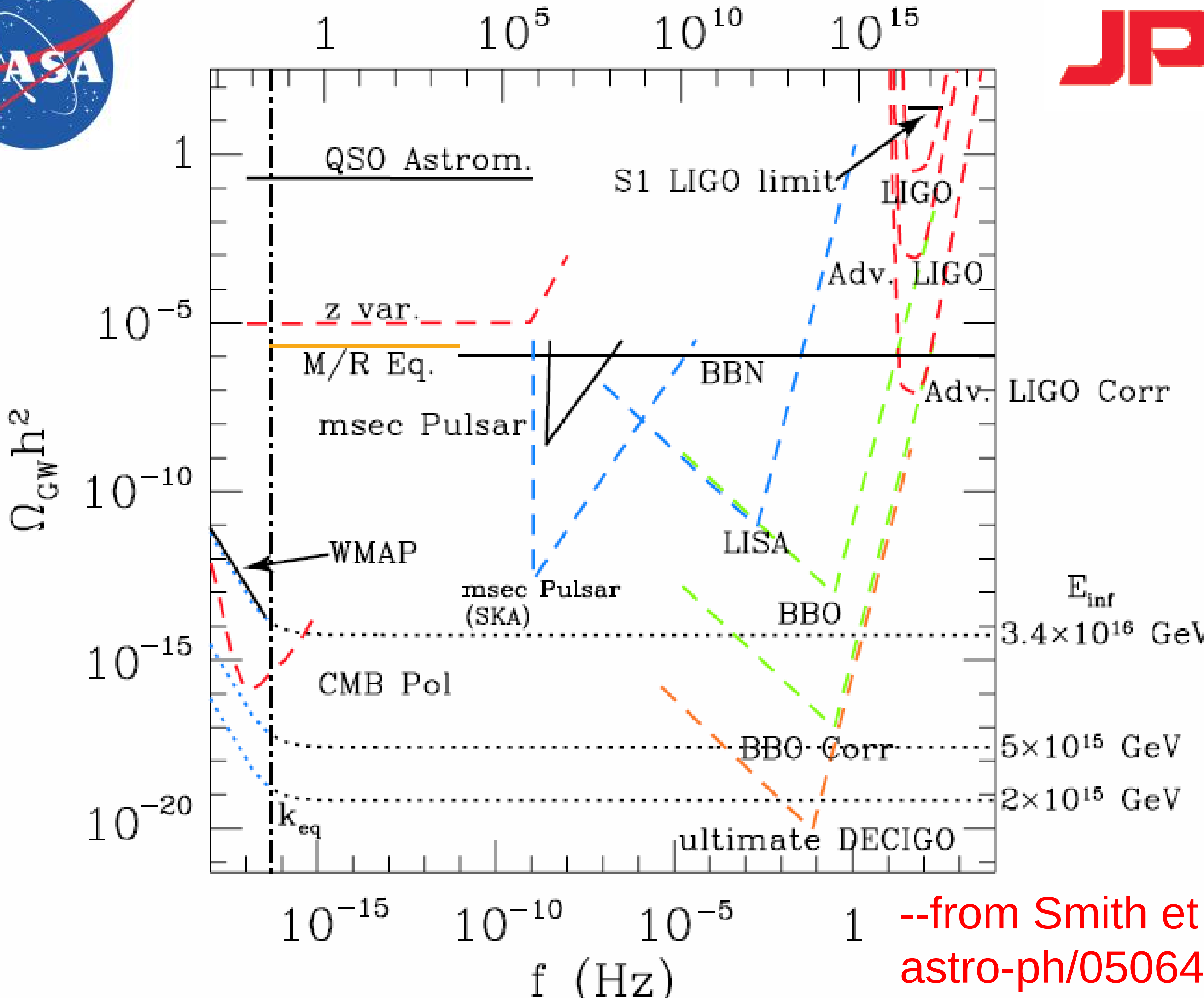
BBO/Decigo and the Neutron-Star-Binary Subtraction Problem

C.Cutler & J.Harms, PRD 73, 04200 (2006)





--from Smith et al., astro-ph/0506422

Why go to shorter arms, or higher frequency ?

Ans: to escape the WD-WD foreground.

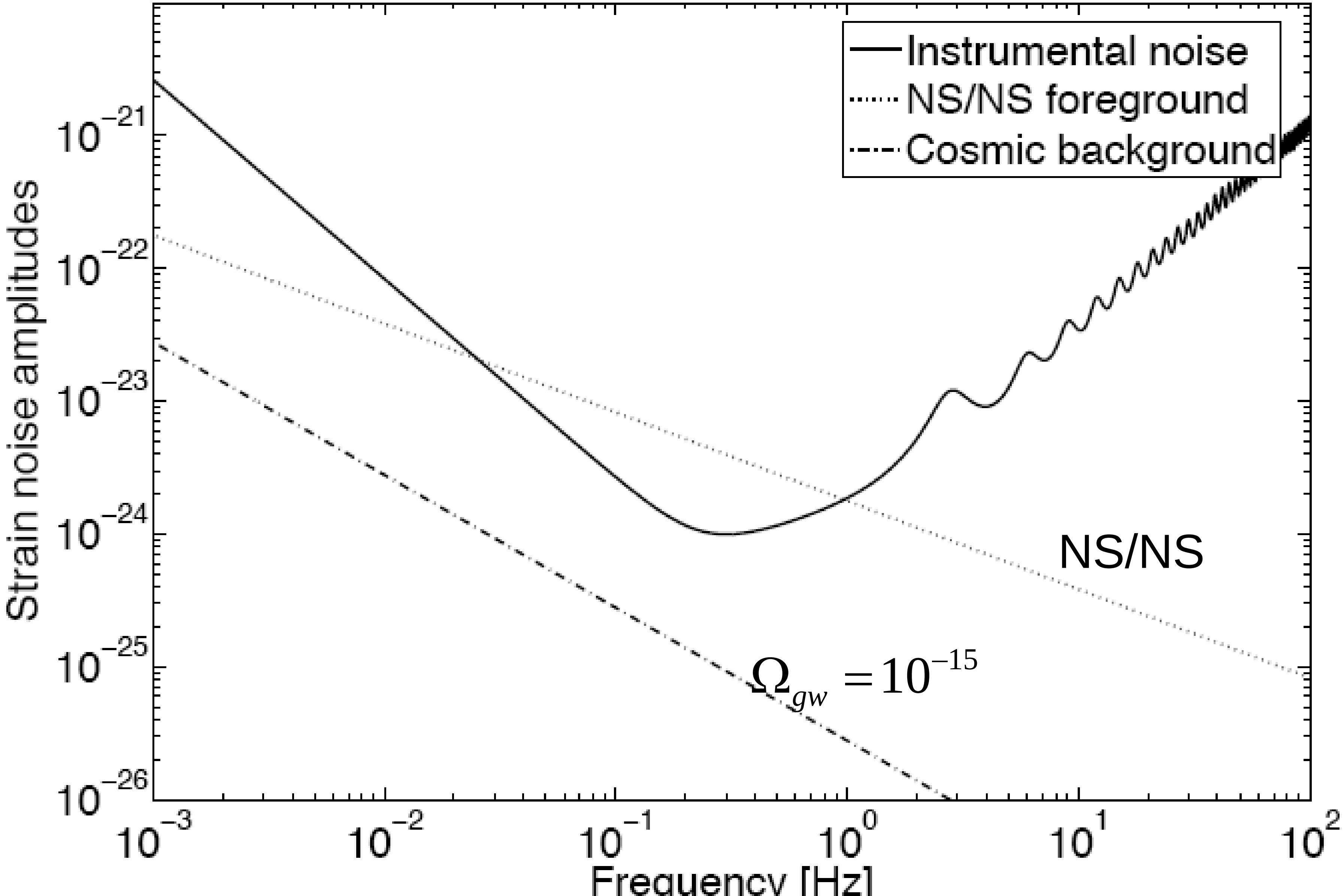
$$\Omega_{GW}^{ex.gal.WD-WD} \approx 3\times10^{-13}(f/1mHz)^{2/3}$$

$$\Omega_{GW}^{gal.WD-WD} \approx 1.5\times10^{-11}(f/1mHz)^{2/3}$$

But the WD-WD foreground disappears above ~0.2 Hz, where even the heaviest WDs merge.

Above ~0.2 Hz, dominant foreground is NS-NS, NS-BH, BH-BH binaries.

BBO Noise Curve



Nominal BBO parameters (used in paper)

	Symbol	Value
Laser power	P	$300\,\mathrm{W}$
Mirror diameter	D	$3.5\,\mathrm{m}$
Optical efficiency	ϵ	0.3
Arm length	L	$5 \cdot 10^7\,\mathrm{m}$
Wavelength of laser light	λ	$0.5\,\mu\mathrm{m}$
Acceleration noise	$\sqrt{S_{\mathrm{acc}}}$	$3 \cdot 10^{-17}\,\mathrm{m}/(\mathrm{s}^2\sqrt{\mathrm{Hz}})$

TABLE I: BBO parameters.

Laser power = 300 x LISA, mirror D =12 x LISA,
arm length = 0.01 x LISA, $S_{acc}^{1/2}$ = 0.01 x LISA

Some Basic Estimates

Typical SNR ~ 140 (the SNR for $z \approx 1.5$, $\mu = 0.5$). That's the TOTAL matched filtering SNR for all 8 synthetic Michelsons, assuming all noise is just instrumental.

$$\frac{M}{r} \approx 5.5 \times 10^{-4} \left(\frac{M[1+z]}{2.8 M_\odot}\right)^{2/3} \left(\frac{f}{0.3\,\mathrm{Hz}}\right)^{2/3}$$

$$t(f) = 4.64 \times 10^5\,\mathrm{s} \left(\frac{\mathcal{M}(1+z)}{1.22 M_\odot}\right)^{-5/3} \left(\frac{f}{1\mathrm{Hz}}\right)^{-8/3}$$

$$\Rightarrow \quad f \approx 0.205\,\mathrm{Hz},\ 0.136\,\mathrm{Hz},\ \text{and}\ 0.112\,\mathrm{Hz}$$

@ t = 1, 3, 5 years before merger,resp. for 2 NSs at low z.

Assumptions re cosmology and merger rate

- We use a spatially flat FLRW mode with

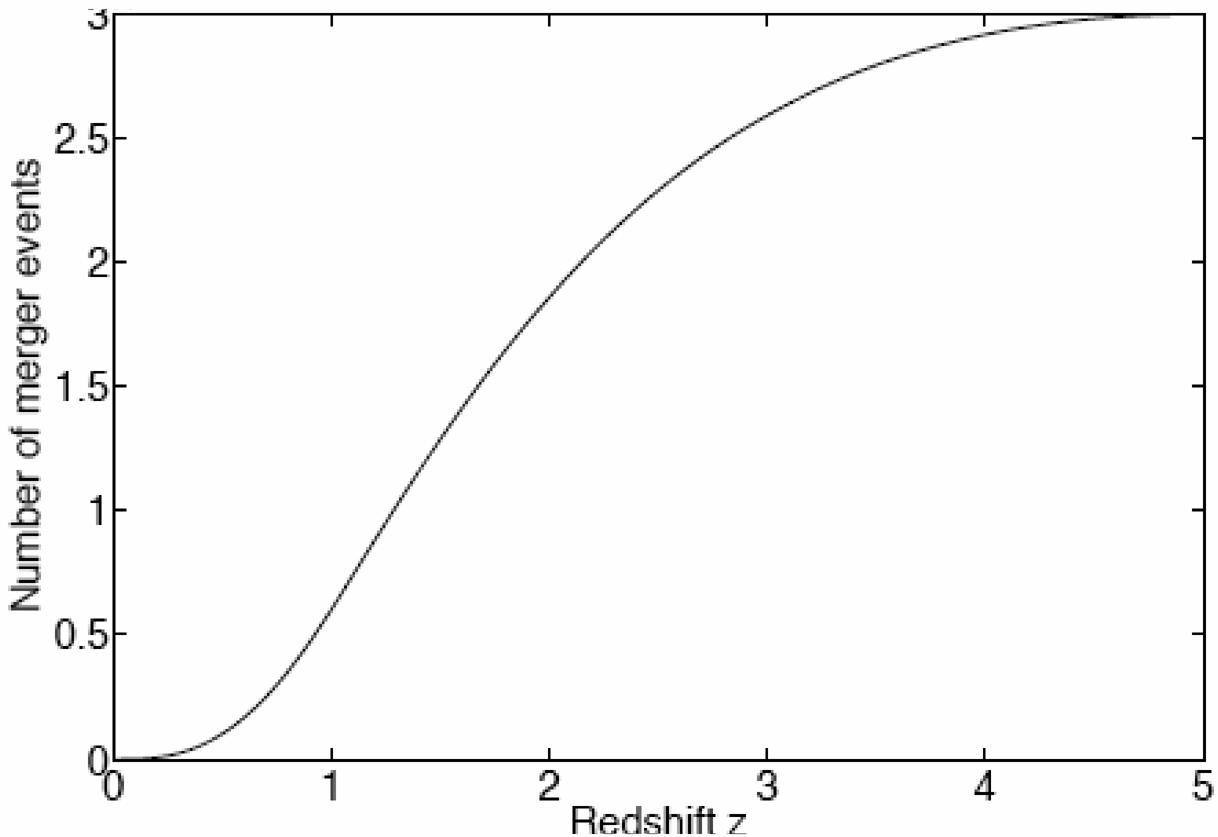
$H_0 = 70\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$, $\Omega_\mathrm{m} = 0.33$ and $\Omega_\Lambda = 0.67$

- Merger rate today: $\dot{n}_0 = 10^{-8}$ -- $10^{-6}\,\mathrm{Mpc^{-3}yr^{-1}}$
- Merger rate in past: $\dot{n}(z) = \dot{n}_0 \cdot r(z)$

where r(z) is following piece-wise linear fit to Schneider et al., (2001)

$$r(z) = \begin{cases} 1 + 2z & z \leq 1 \\ \frac{3}{4}(5 - z) & 1 \leq z \leq 5 \\ 0 & z \geq 5 \end{cases}$$

$$\Delta N_{\rm m} = 3.0 \cdot 10^5 \left(\frac{\Delta\tau_0}{3\,{\rm yr}}\right)\left(\frac{\dot{n}_0}{10^{-7}\,{\rm Mpc}^{-3}{\rm yr}^{-1}}\right)$$

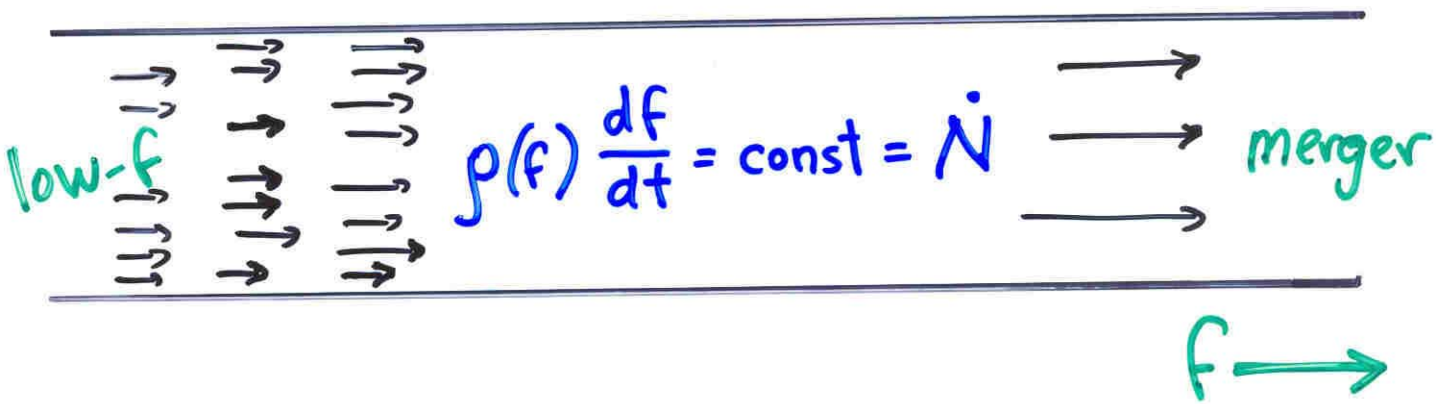


The total number of NS-NS mergers closer than redshift z,

for a 3-yr observation w/ $\dot{n}_0 = 10^{-7}{\rm Mpc}^{-3}{\rm yr}^{-1}$

Density $\rho(f)$ of sources in frequency

Current of sources, flowing in freq. space:



$$\frac{df}{dt} \propto f^{11/3} \Rightarrow \rho \propto f^{-11/3}$$

Unresolved NS binaries represent "confusion noise"

Any individual NS inspiral signal will be buried in the noise, so matched filtering will be required to dig the waveforms out; i.e., for chirping waveform h(t) buried in detector output s(t) , want to do integral

$$\int \frac{\tilde{h}^*(f)\, s(f)\, df}{S_h(f)} \approx \int \frac{h(t)\, s(t)\, dt}{S_h(f(t))}$$

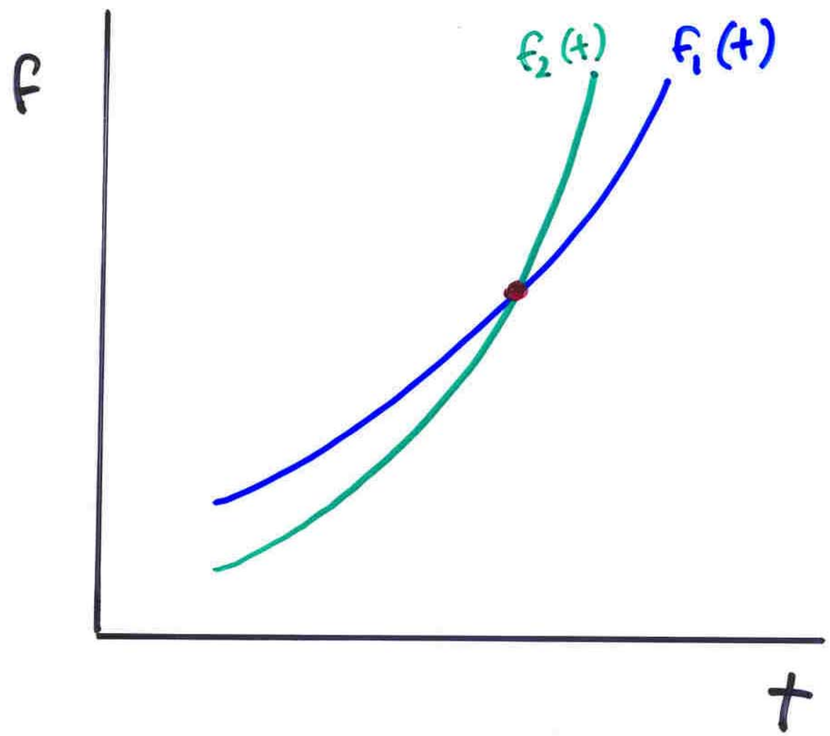
Claim: the resulting integral has the same distribution, whether s(t) is the sum of a very large number of OTHER random chirp waveforms, or s(t) is Gaussian noise with the same spectral density as that sum.

A proof is sketched in paper, but this should also be obvious on general grounds.

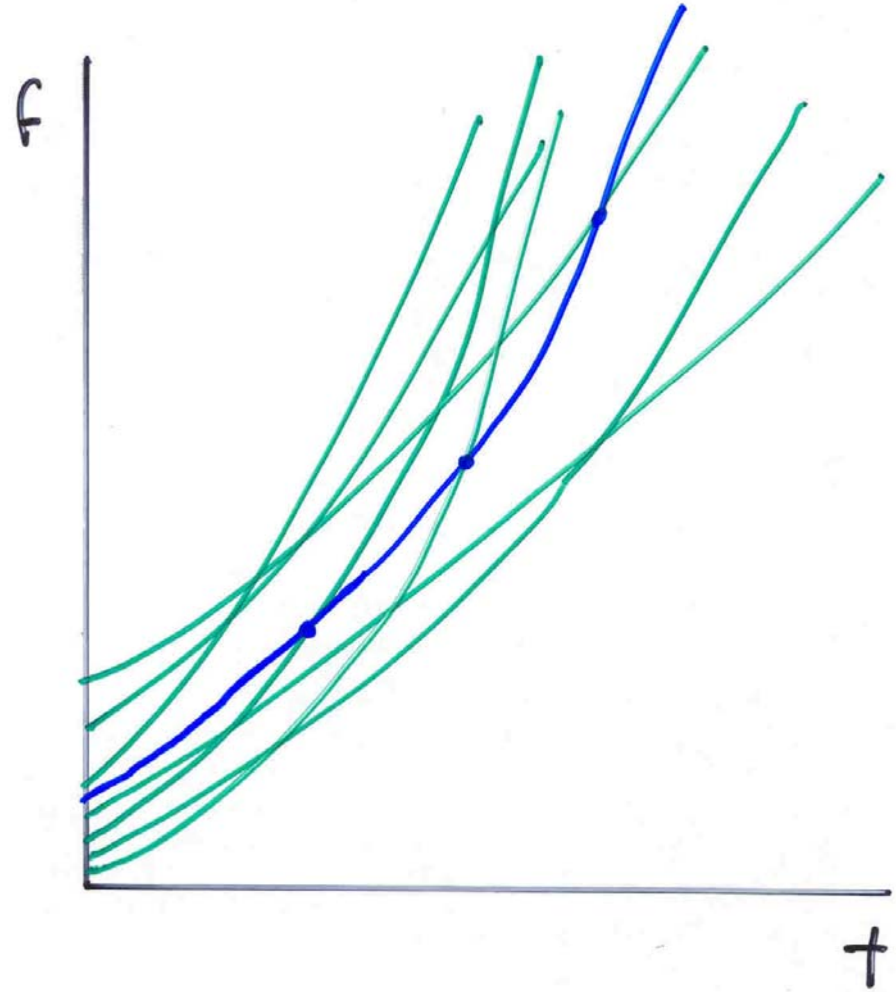
How 2 different chirping waveforms "interfere with" each other:

$\int h_1(t) h_2(t)\, dt$ ---integral dominated by contribution

from short time around crossing of $f_1(t)$ and $f_2(t)$



t-f tracks for 2 merging NS binaries at different z



Rate at which typical track crosses all others is

$$r_c = 0.5(5/3)\rho \dot{f} \frac{\Delta \mathcal{M}_{\rm eff}}{\mathcal{M}_{\rm eff}} \sim \frac{1}{3}\dot{N}$$

--independent of f !

So in its last year, one track crosses ~30,000 others.

Our model of total BBO noise

$$S_h^{tot}(f) = \left[S_h^{inst}(f) + S_h^{NSm,>\bar{z}}(f)\right] \bullet \left[1 - \Lambda(f)\right]^{-1}$$

Confusion noise due to unresolved NS/NS mergers

Effective loss of bandwidth due to subtraction errors

We'll argue that effective loss of bandwidth due to subtraction errors is small enough that we can neglect it for most of the analysis.

Subtraction Errors due to Noise

Quick review of some signal processing:

There's a natural, noise-weighted inner product
$< \quad | \quad >$ on the space of signals, such that

$$SNR^2[h] = < h \,|\, h >$$

and if $h = h(\lambda^\alpha)$ then:

$$\overline{\Delta\lambda^\alpha \Delta\lambda^\beta} = (\Gamma^{-1})^{\alpha\beta} + \mathcal{O}(\mathrm{SNR})^{-1}$$

where $$\Gamma_{\alpha\beta} \equiv \left\langle \frac{\partial \mathbf{h}}{\partial \lambda^\alpha} \,\middle|\, \frac{\partial \mathbf{h}}{\partial \lambda^\beta} \right\rangle$$

Subtraction Errors due to Noise

$$\delta \mathbf{h} \equiv \hat{\mathbf{h}} - \mathbf{h}$$
$$= \partial_\alpha \mathbf{h}\, \delta\lambda^\alpha + \mathcal{O}(\delta\lambda)^2$$

$$\Rightarrow \quad \overline{\langle \delta\mathbf{h} \,|\, \delta\mathbf{h} \rangle} = \langle \partial_\alpha \mathbf{h} \,|\, \partial_\beta \mathbf{h} \rangle\, \overline{\delta\lambda^\alpha \delta\lambda^\beta}$$
$$= \Gamma_{\alpha\beta} \left(\Gamma^{-1}\right)^{\alpha\beta} = N_{\mathrm{p}} \,.$$ (# of params)

$$\Rightarrow \quad \left[\frac{\langle \delta\mathbf{h} \,|\, \delta\mathbf{h} \rangle}{\langle \mathbf{h} \,|\, \mathbf{h} \rangle} \right]^{1/2} = \frac{N_{\mathrm{p}}^{1/2}}{\mathrm{SNR}}$$

~ 0.024 for typical
NS mergers seen by BBO

Subtraction Errors due to Noise

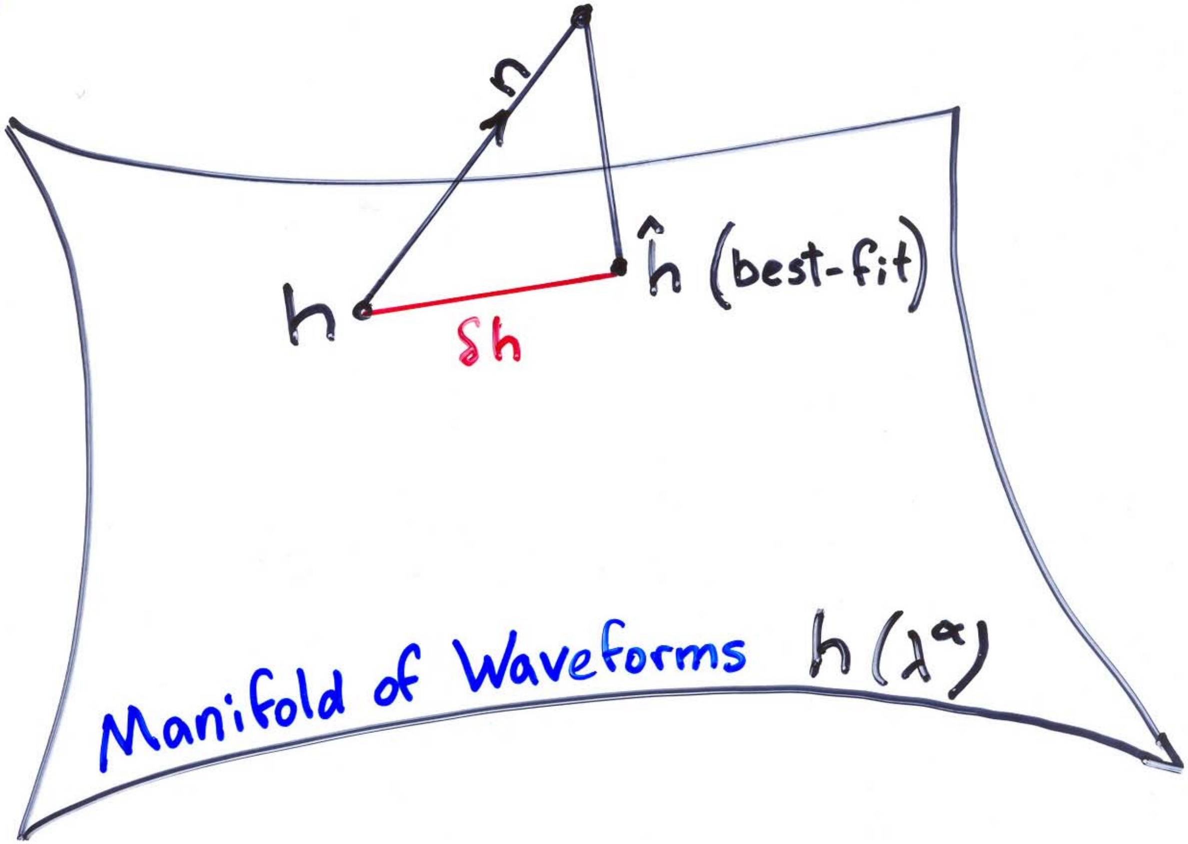
Note same argument applies to entire foreground; i.e., if

$$H(t) = \sum_{i=1}^{N_s} h_i(t)$$

then $\delta H/H \sim \delta h/h \approx 0.024$

since BOTH the total # of params and the total SNR^2 scale linearly with the number of sources.

Recall the foreground lies ~2.5 orders of magnitude above the sought-for inflationary background, so IF this error δH were simply an additive source of noise, it would kill the project. But δH is not a noise source in that way.



δh is (approx) tangent to manifold of waveforms

One idea for mitigating effect of subtraction error

$$\delta H(t) = \partial_\alpha H(t)\delta\lambda^\alpha + \frac{1}{2}\partial_\alpha\partial_\beta H(t)\delta\lambda^\alpha\delta\lambda^\beta + \cdots$$

After subtracting out best fit $\hat{H}$, act on the rest of the data with the projection operator

$$P \equiv I - (\Gamma^{-1})^{\alpha\beta}|\partial_\alpha \mathbf{H}\rangle\langle\partial_\beta \mathbf{H}| \,.$$

This kills the first-order error, at a fractional cost in bandwidth of

~(dim H)/(dim Signal Space) ~ $\dfrac{3\times 10^6}{1.5\times 10^9} \approx 2\times 10^{-3}$

What is detection threshold ρ_{th} ?

(where ρ_{th} is total matched-filter SNR for BBO)

- Significant detection demands

$$N_t \, \mathrm{erfc}(\rho_{th}/\sqrt{2}) \leq 0.01$$

W/ # independent templates $N_t \approx 10^{30} - 10^{36}$

$$\Rightarrow \rho_{th} \geq 12.5 - 13.5$$

What is detection threshold ρ_{th} ?

- But ρ_{th} almost certainly set by computational limitations:

We estimate the BBO NS/NS search is a roughly even match, in difficulty, to a LIGO search for unknown GW pulsars with $f_{\max} = 1kHz$ and $\tau_{\min} = 300\, yr.$

Assuming 10^{17} flops x 1yr of computing power, results from Cutler et al. (2005) for 3-stage hierarchical, stack-slide searches $\Rightarrow \rho_{th} \approx 20$

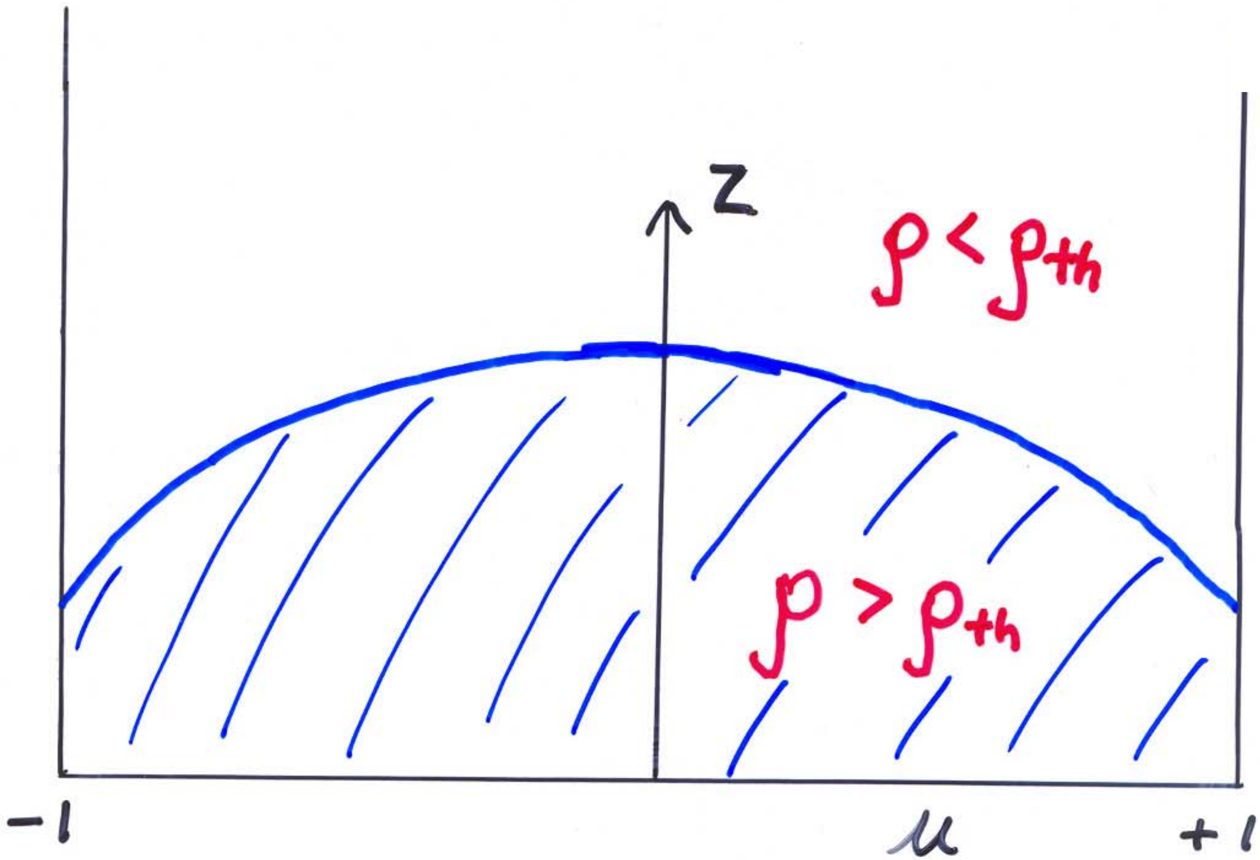
Since this argument is quite rough, we consider 3 cases:

$$\rho_{th} = 20,\ 30,\ 40$$

(Presumed) NS/NS Subtraction Strategy

Start by subtracting out the highest-SNR mergers. This lowers the confusion noise level, so allows you to subtract out some more mergers, which further lowers the noise, allowing even more mergers to be resolved and subtracted out, etc.

Our Goal: to determine self-consistently whether this process stops before (essentially) all binaries are subtracted, and if so, where.



Our Goal: to self-consistently determine the boundary separating detectable and undetectable NS binaries

Our method for finding self-consistent solutions:

(for given, fixed ρ_{th} , $\dot{n}_0$)

1. Let F_G^2 represent a "guess" as to the fraction of the NS merger foreground that is due to unresolved sources. Then the "guessed" total noise is:

$$S_{\mathrm{h}}^{\mathrm{tot}}(F_{\mathrm{G}}, f) = S_{\mathrm{h}}^{\mathrm{inst}}(f) + F_{\mathrm{G}}^2 \cdot S_{\mathrm{h}}^{\mathrm{NSm}}(f)$$

2. Find the boundary in the μ–z plane satisfying

$$\rho_{\mathrm{th}}^2 = 8 \cdot \frac{2f(\mu)}{3\pi^{4/3}} \frac{(\mathcal{M}(1+\bar{z}))^{5/3}}{D_L^2(\bar{z})} \int_0^\infty \mathrm{d}f \, \frac{f^{-7/3}}{S_{\mathrm{h}}^{\mathrm{tot}}(F_{\mathrm{G}}, f)}.$$

where $f(\mu) \equiv \frac{5}{16}(\mu^4 + 6\mu^2 + 1).$

Method of self-consistent solution (cont'd)

3. Given that boundary between detectable and undetectable sources, calculate the fraction F^2 of the NS merger foreground that is due to the undetectable ones.

4. Look for fixed points of $F(F_G)$,

or equivalently, for zeroes of the function

$$F(F_G) - F_G$$

RESULTS

Standard BBO sensitivity and any realistic merger rate:

$$F_{20} = F_{30} = 0,$$

$$F_{40} = 0.0015\,,$$

RESULTS (2)

Standard/2 BBO sensitivity; i.e. $S_{\mathrm{h}}^{\mathrm{inst}} = 4 \cdot S_{\mathrm{h}}^{\mathrm{st.inst}}$

ρ_{th} \ $\dot{n}_0$	$S_{\mathrm{h}}^{\mathrm{inst}} = 4 \cdot S_{\mathrm{h}}^{\mathrm{st.inst}}$		
	10^{-8}	10^{-7}	10^{-6}
20	0.030	0.10	0.30
30	1.4	4.9	22
40	3.0	11	110

Table of ratios $[S_{\mathrm{h}}^{\mathrm{NSm},>\bar{z}}(f)/S_{\mathrm{h}}^{\mathrm{GW}}(f)]^{1/2}$ @ $f = 1\,\mathrm{Hz}$

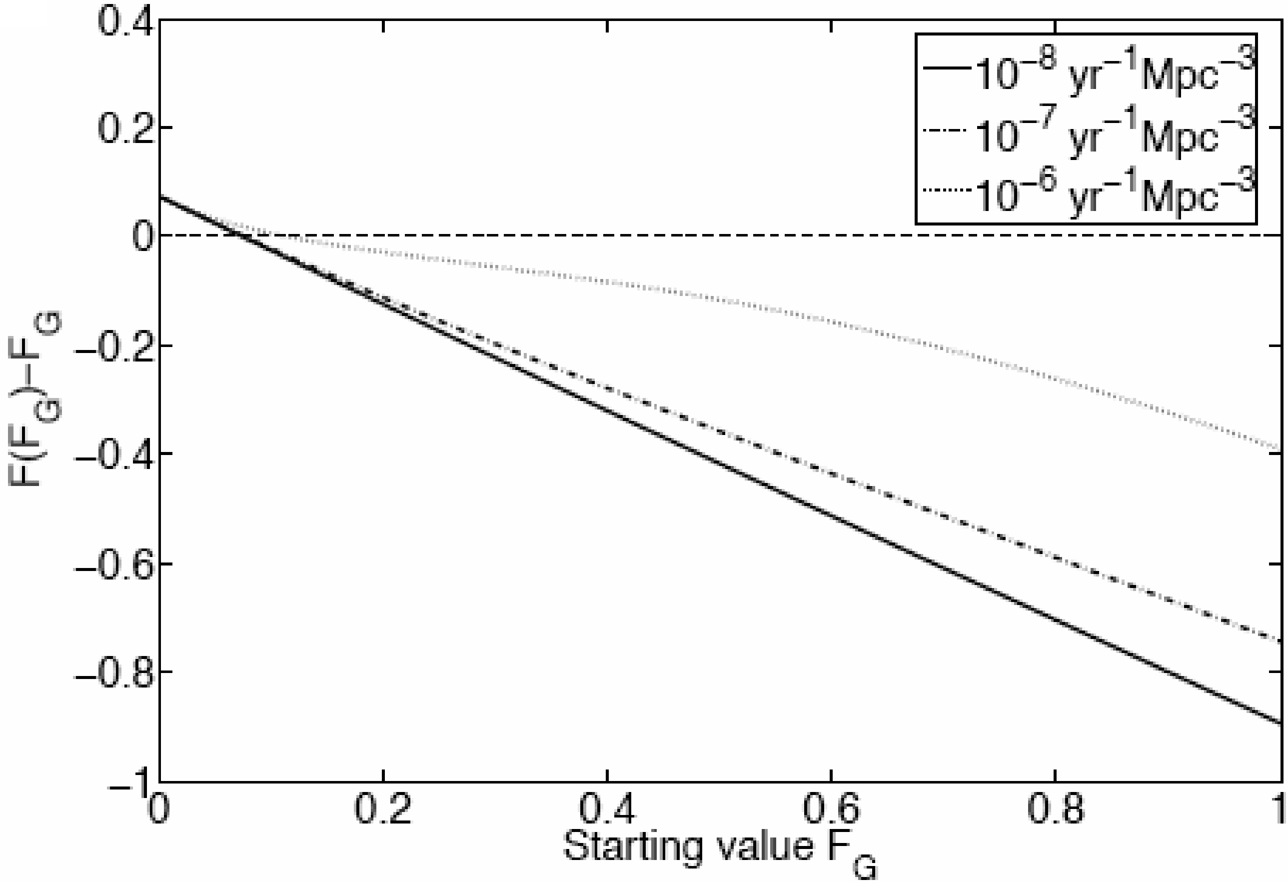
Normalized to $\Omega_{gw}(f) = 10^{-15}$

RESULTS (2')

Standard/2 BBO sensitivity; i.e. $S_{\mathrm{h}}^{\mathrm{inst}} = 4 \cdot S_{\mathrm{h}}^{\mathrm{st.inst}}$

$\dot{n}_0$ / ρ_{th}	$S_{\mathrm{h}}^{\mathrm{inst}} = 4 \cdot S_{\mathrm{h}}^{\mathrm{st.inst}}$		
	10^{-8}	10^{-7}	10^{-6}
20	0.0015	0.0015	0.0015
30	0.071	0.077	0.11
40	0.15	0.17	0.55

Table lists $F' = (S_{\mathrm{h}}^{\mathrm{NSm},>\bar{z}} / S_{\mathrm{h}}^{\mathrm{NSm}})^{1/2}$



Shows the function $F(F_{\rm G}) - F_{\rm G}$ for three merger rates: $\dot{n}_0 = \{10^{-8}, 10^{-7}, 10^{-6}\}\,{\rm yr}^{-1}{\rm Mpc}^{-3}$.

All curves for standard/2 sensitivity and $\rho_{th} = 30$

Understanding the failure for standard/2 sensitivity

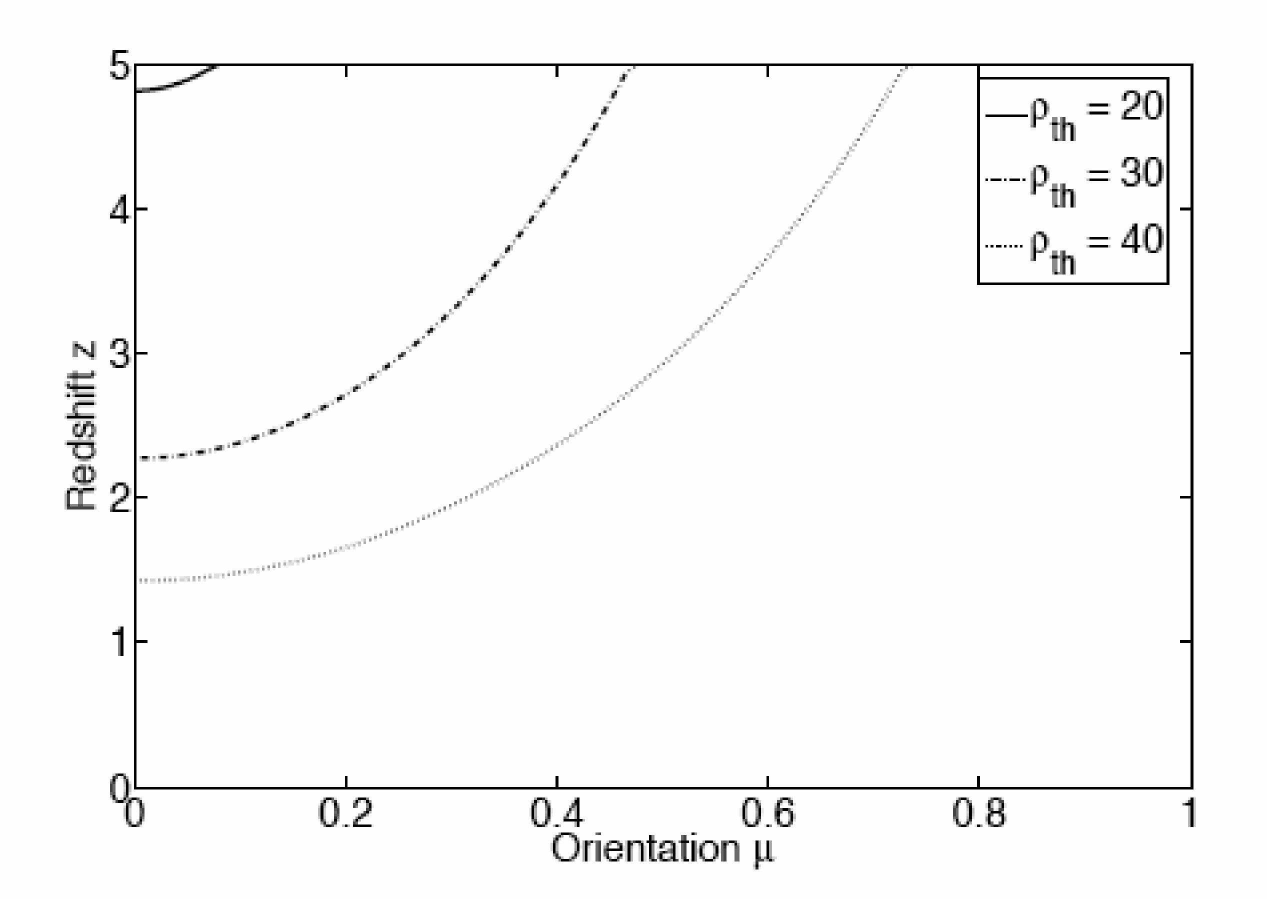


Fig. shows undetectable regions of $\mu - z$ space for $\acute{Y}_0 = 10^{-7}$ and standard/2 sensitivity.

RESULTS (3)

Standard/4 BBO sensitivity; i.e. $S_{\rm h}^{\rm inst}(f) = 16 \cdot S_{\rm h}^{\rm st.inst}(f)$

$$[S_{\rm h}^{\rm NSm,>\bar{z}}(f)/S_{\rm h}^{\rm GW}(f)]^{1/2} > 3$$

even for $\rho_{th} = 20$ and $\dot{n}_0 = 10^{-8}$

That's at $f = 1\,{\rm Hz}$ and normalized to $\Omega_{gw}(f) = 10^{-15}$

Summary

- Showed that standard BBO noise curve indeed allows for essentially full subtraction of NS/NS foreground. BBO/4 sensitivity would clearly be inadequate. Whether BBO/2 sensitivity suffices depends on ρ_{th} & η_o.

- Devised "projection method" for handling subtraction errors due to noise -- at small cost in bandwidth.

➢Paper catalogues relevant physical effects:

- Effects of spin on orbital plane precession and inspiral phase

- Effects of small, non-zero eccentricity on waveform phase, and the contributions of $n \neq 2$ harmonics to foreground

- Effects of P^4N terms on inspiral phase