Tidal Excitation of White Dwarf Oscillations

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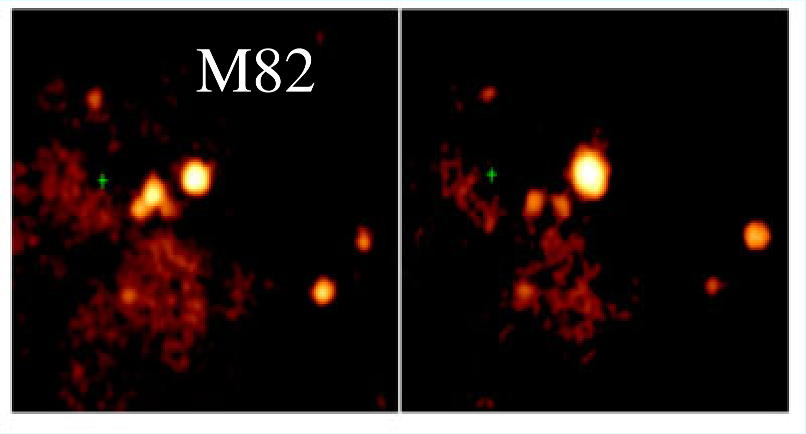
Outline

- Motivation
- White Dwarfs
- Roche and Gravitational Limits
- White Dwarf Capture
- General Relativistic Evolution
- Tidal Resonances in Elliptical Orbits
- Back Reaction and Lagrangian
- Circular Orbits
- Nonlinear and Explosive Evolution
- Summary

Motivation

- White dwarfs
  - billions per galaxy; MACHOs ???
- Massive and Intermediate Holes (MHole, kHole)
  - ~million solar mass in galactic nuclei
  - ULX -kHole?? $\Omega \sim 10^{-6}$?? 10^{5} per galaxy???
  - Rare captures could be interesting, GRB/SNIa*???
- WD-WD
  - P > 5 min (Ramsay et al)
  - LISA signal/noise
- WD-NS
  - ~50, P>3hr (Lorimer)
- WD-BH?

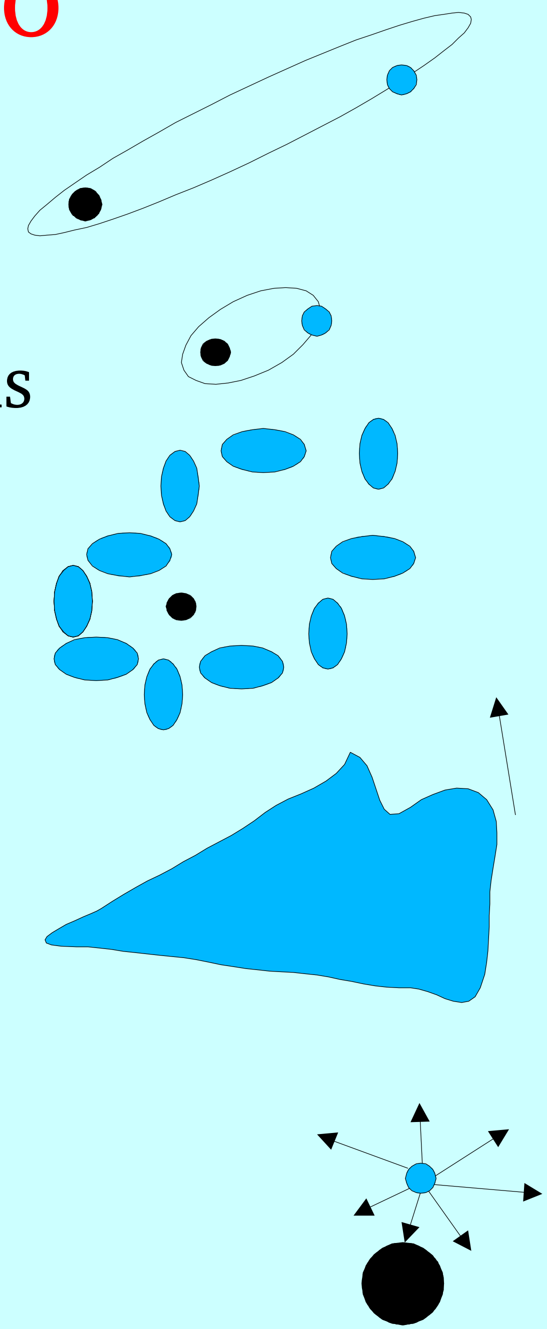


Model for NS-NS/BH mergers?

Ho,Lai

Merger Scenario

- Capture of WD by compact object
- Gravitational radiation inspiral
- Resonant excitation of WD oscillations
- Damping / breaking of waves
- Heating of stellar interior
- Nuclear burning/explosion
- Trapped debris => SNIa*???
  - Relativistic torus
  - EM bomb powered by hole spin??
- Gravitational radiation source



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White Dwarfs

- Common – mostly low mass
  - 0.6 M_{sun}; higher mass higher frequencies
  - R ~ 8600 km,
  - E_{bind} ~1.6 x 10^{43} J = 150 keV/n
    - cf E_{ign} ~ 10-50 keV/n; E_{burn} ~ 700keV/n
  - $E_{orb} \sim 5.6 \times 10^{43} M_3^{2/3} f_{-3}^{2/3}$ J
- "Simple" fluids
  - Electron degeneracy pressure support – cold
- Oscillation modes~$Y_{lm}(\theta,\phi)\, u_n(r)$
  - p-modes – too high frequency for resonance
  - g-modes - superficial when cold
  - f (Helmholtz) – mode ν_{22} ~ 89mHz

Roche/Gravitational Limits

- WD on circular orbit overflows its Roche lobe when $f_R \sim 0.05(GM/R^3)^{1/2} \sim 18$ mHz
  - Roche limit: $a_R \sim 2 \times 10^5 M_3^{1/3}$ km
    - ~1000 R_{NS} for NS companions
  - For elliptical orbit, $f_R \sim 18(1\text{-}e)^{3/2}$ mHz
- For modestly spinning hole, $r_{capt} \sim 6\ m \sim 10^4 M_3$ km
  - $r_{capt} > r_R$ for $M > 10^5 M_{sun}$.
  - $3 \times 10^5 M_{sun}$ for extreme prograde Kerr holes

Tidal Capture

- Globular cluster X-ray sources (Fabian et al 1977)
- Capture cross sections (Press & Teukolsky 1978)
- Tidal capture when maximum angular frequency at peribothon exceeds Roche angular frequency

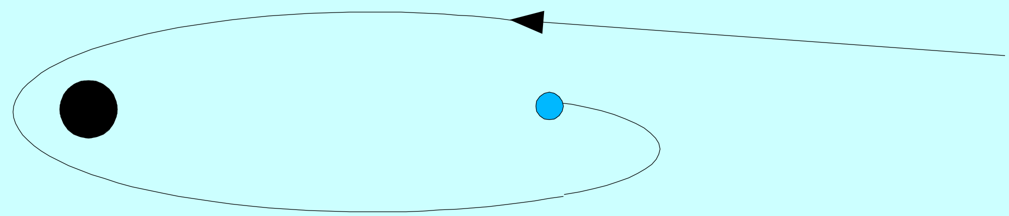
$$f_{capt} \sim f_R \sim 18(1\text{-}e)^{3/2}\ \mathrm{mHz}$$

- Immediate post capture evolution complex
  - Stellar heating in subsequent passages (Ray et al 1986)
  - If only loosely bound may not be retained (Sigurdsson & Rees)
  - Extracts more energy than angular momentum from the orbit.

Gravitational Bremsstrahlung

$$\Delta E = \frac{85\, G^{7/2} M_{wd}^{2} M^{5/2}}{12\, 2^{1/2} c^{5} r_{min}^{7/2}}$$

$$\Delta L = \frac{144\, \Delta E}{85\, n_{max}}$$



Bremsstrahlung cross section dominates for M > 10^5 M_{sun}.

Smaller cross section for capturing into tightly bound elliptical orbit

Unanticipated, small eccentric orbits are a feature of Sgr A*, ESP.

Three -body processes are also possible

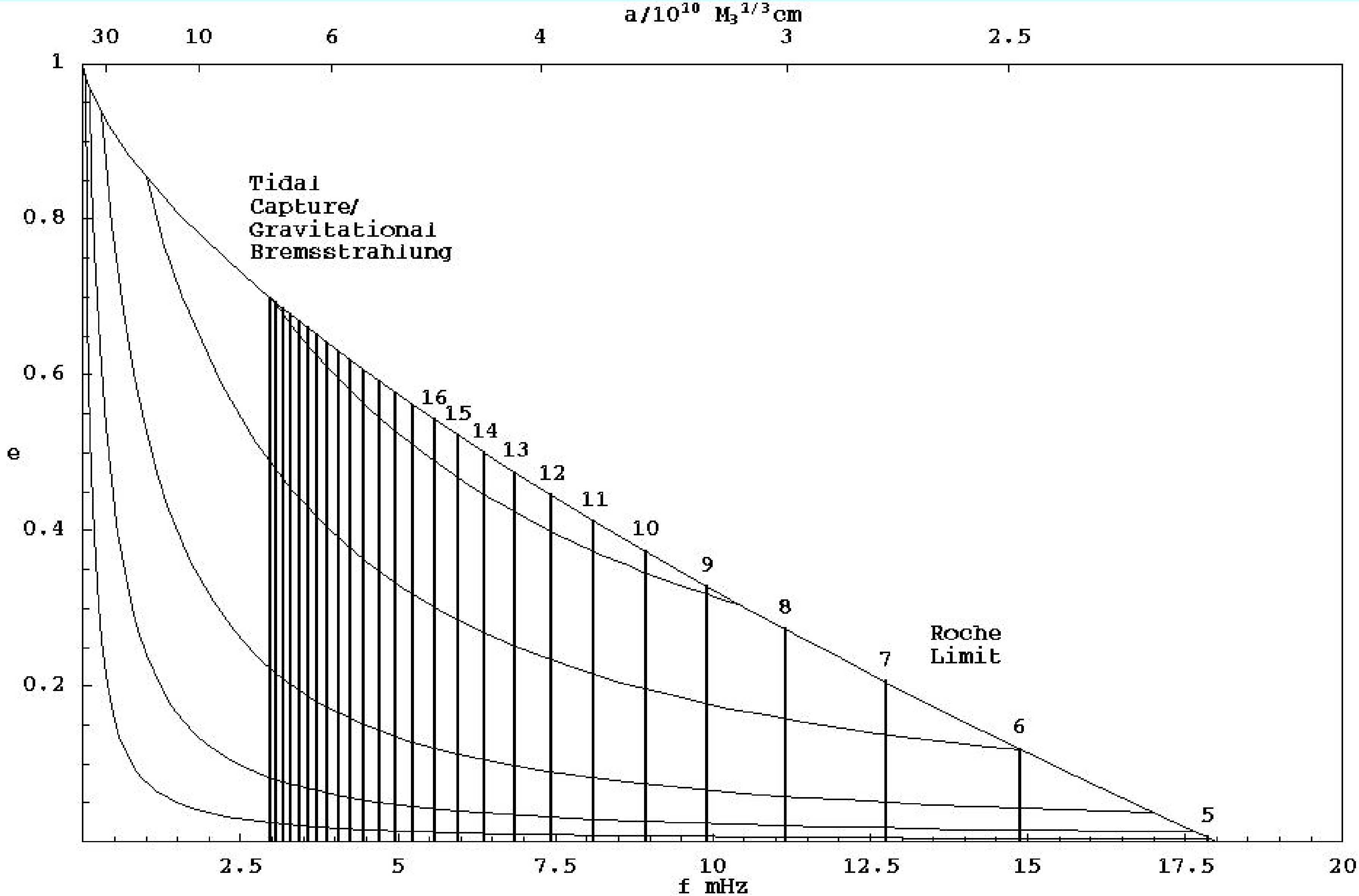
Gravitational Inspiral

$$f(e) = f(0.54)(1-e^2)^{3/2} e^{-18/19} (1+121e^2/304)^{-1305/2299}$$

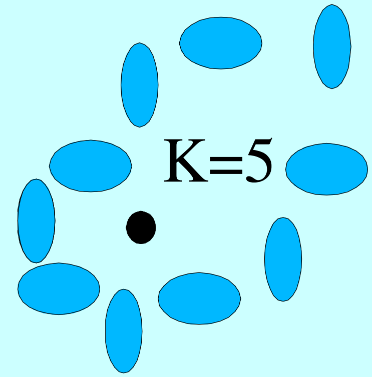
$$\frac{dt}{df} = \frac{5}{96}(2\pi)^{-8/3} f^{-11/3} M^{-2/3} \mu^{-1} \frac{(1-e^2)^{7/2}}{(1+73e^2/24+37e^4/96)}$$

Peters & Mathews

Orbit evolves from capture through resonances
to tidal destruction or horizon crossing


a/10^10 M3^1/3 cm
30
10
6
4
3
2.5
1
0.8
0.6
0.4
0.2
e
Tidal
Capture/
Gravitational
Bremsstrahlung
16
15
14
13
12
11
10
9
8
7
6
5
Roche
Limit
2.5
5
7.5
10
12.5
15
17.5
20
f mHz

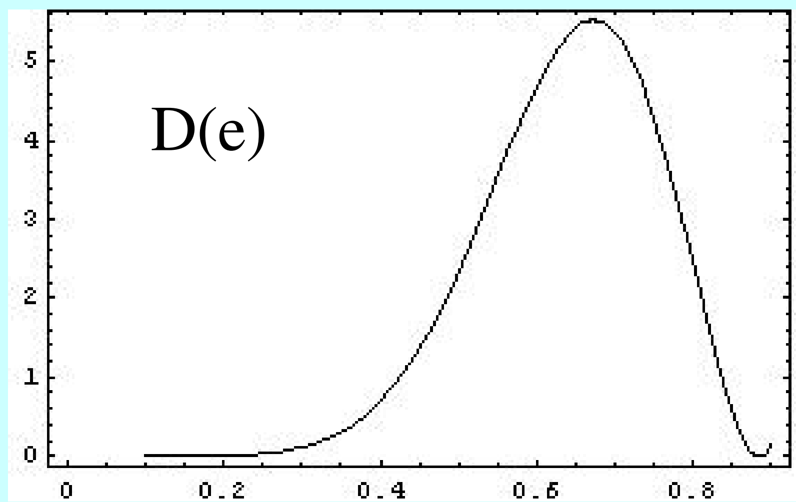
Tidal Resonances



- Expand gravitational field about WD in Y_{lm}.
- Most important resonance is l=m=2; f=ν/k.
  - Other resonances can be important
- Compute change in mode amplitude for unperturbed orbit as star passes through resonance. Use amplitude formalism

$$\vec{\xi} = \sum \frac{\vec{\xi}_j}{\omega_j} \int dm\, \vec{\xi}_j \cdot \int dt\, e^{i\omega_j (t-t')} \nabla \Phi$$

- Compute energy and angular momentum change
  - $\Delta E = \left(\frac{GM^2}{R}\right) D(e) \left(\frac{M_{eff}}{M}\right)\left(\frac{f^2}{df/dt}\right)$
  - Nb $\Delta L = m\, \Delta E / 2\pi\nu$



Simple Harmonic Oscillator

$$\xi'' + \Omega_{\sim}^{2}\xi = A(t)$$

$$E = \frac{1}{2}[A(\Omega_{0,}t)^{2}]$$

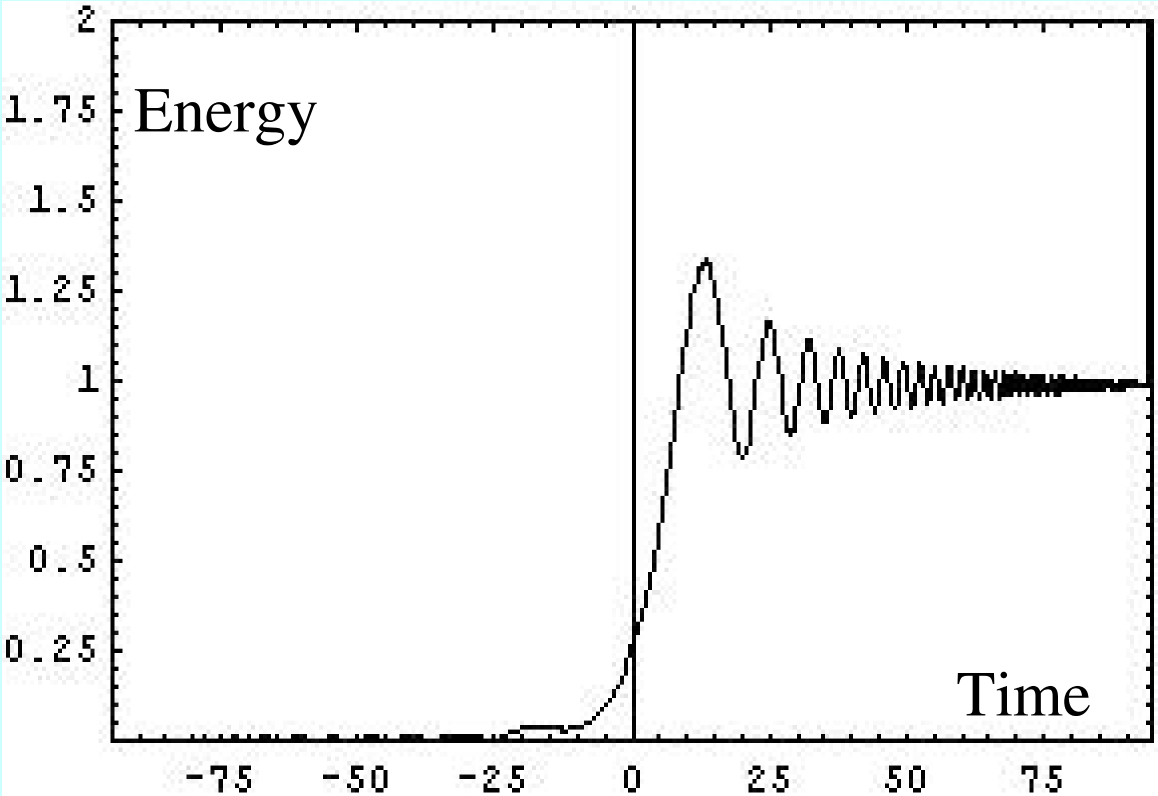
$$A(t) = A_0 \cos(\Omega_0 t + \frac{1}{2}\Omega' t^2)$$

At resonance, force and velocity in phase for

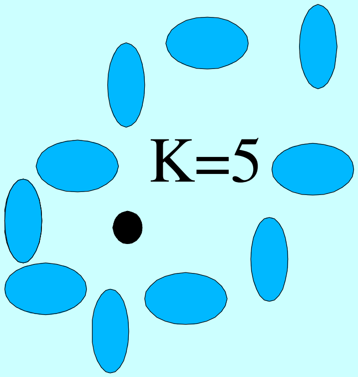
$$\Delta t \approx \Omega'^{-1/2}$$

$$E \approx \frac{A_0^2}{\Omega'}$$

Energy transfer independent of damping



Tidal Resonances



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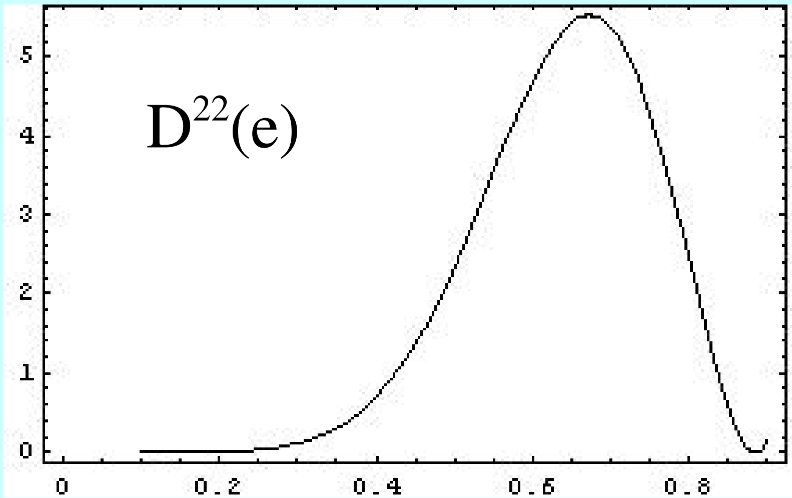
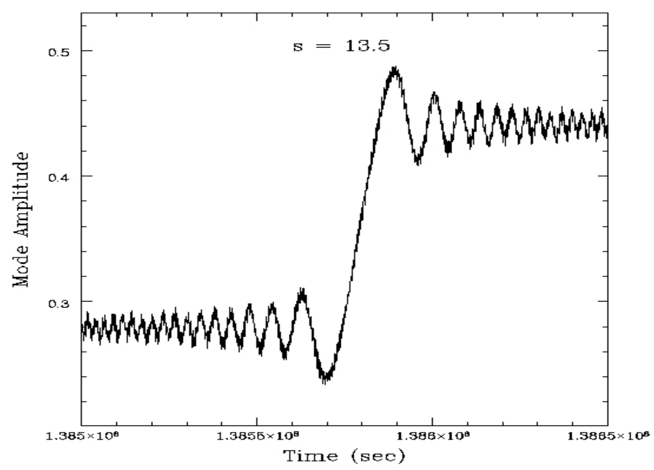
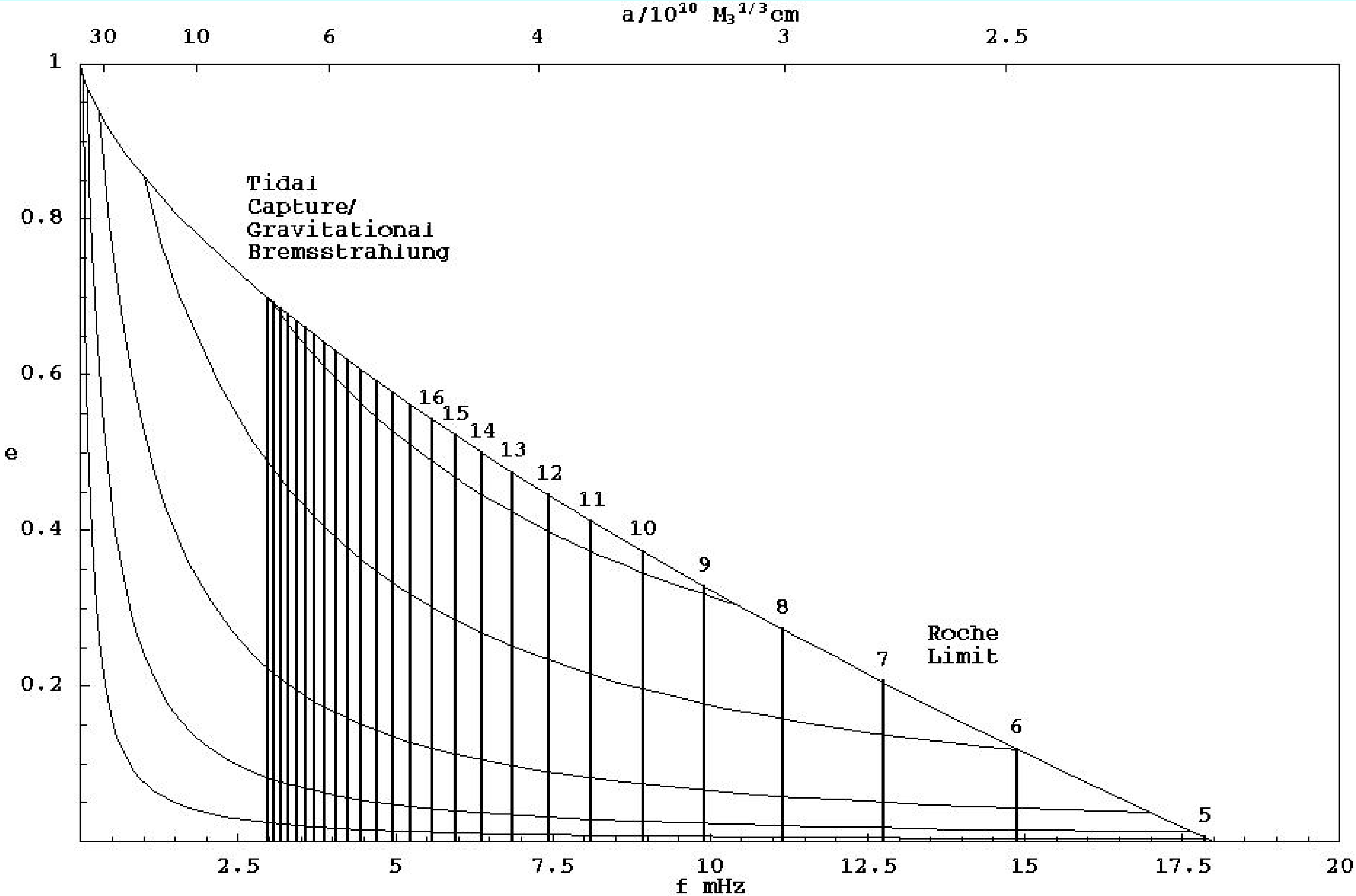
  - Nb $\Delta L = m\, \Delta E / 2\pi\nu$
- $\Delta E > Mc^2$!



Back Reaction

- Joint evolution of oscillation and orbit
- Lagrangian approach [Salmon 1988, Rathore, Broderick & Blandford 2002]
- $L = L_{eq} + L_{int} + L_{pert}$.
- Vary wrt ϕ, Φ, ρ
- Include relativistic corrections to 7/2 order (Iyer & Will)
- Compute stress tensor and conserve energy and angular momentum
- Resonances shifted and energy transfer diminished





$(\delta r/r) = 0.15\ E_{41}^{\ 0.5}$ $E_{osc}=\Sigma\Delta E_k$

Circular Orbits

- Only excite in fundamental k=m => g-modes
- If interior is hot, $T \sim 10^8$K, then modes have large effective mass and significant energy transfer can occur
- We need to heat star to have mode to heat the star!
- For WD binaries at $f \sim 3$mHz,
  - $E_{orb} \sim 10^{42}$ J,
  - $E_{nl} \sim 0.1 E_{bind} \sim 10^{42}$J
- Effects on the waveform could be large
  - Sidebands broaden line

Mode Damping

- Conventional damping negligible
- Nonlinear wave-wave coupling
  - Parametric excitation of p-modes
- Turbulence near density jumps if present (Zimmerman)

Summary

- White dwarf – compact object captures could happen at an interesting rate
- Tidal capture at low mass, gravitational bremsstrahlung at high mass
- Resonant tidal interaction important and can lead to nonlinear oscillation for elliptical orbits
- Problem involves back reaction
- Lessons for NS-NS/BH mergers
- Wave breaking/damping might detonate the star leading to a peculiar SNIa/GRB