Small-Signal Distortion in Feedback Amplifiers for Audio[1]

James Boyk[2] and Gerald Jay Sussman[3]

April 22, 2003

[2]Pianist in Residence, Lecturer in Music in Electrical Engineering, and Director of the Music Lab, California Institute of Technology

[3]Matsushita Professor of Electrical Engineering, Department of Electrical Engineering and Computer Science, Massachusetts Institute of Technology

Abstract

We examine how intermodulation distortion of small two-tone signals is affected by adding degenerative feedback to three types of elementary amplifier circuits (single-ended, push-pull pair, and differential pair), each implemented with three types of active device (FET, BJT and vacuum triode). Although high precision numerical methods are employed in our analysis, the active devices are modeled with rather simple models. We have not investigated the consequences of more elaborate models.

Though negative feedback usually improves the distortion characteristics of an amplifier, we find that in some cases it makes the distortion "messier." For instance, a common-source FET amplifier without feedback has a distortion spectrum displaying exactly four spurious spectral lines; adding feedback introduces tier upon tier of high-order intermodulation products spanning the full bandwidth of the amplifier (as suggested by Crowhurst in 1957). In a class-B complementary-pair FET amplifier, feedback mysteriously boosts specific high-order distortion products.

The distortions we are dealing with are small, but we speculate that they may be psychoacoustically significant.

This work also casts light on the relative virtues of the three types of active devices and the three circuit types. For instance, a FET pair run in class-A produces zero distortion even without feedback.

Some experienced listeners report favorably on the sound quality of non-feedback amplifiers. This is surprising, because such amplifiers have much more nonlinear distortion than amplifiers that use negative feedback. Indeed, the appropriate use of negative feedback improves almost all of the theoretical and measurable parameters of an amplifier. Of course, the listeners may be mistaken. Alternatively, some subtle consequent of negative feedback may be responsible for the difference in perception. Here we investigate one such possibility.

Of the various proposals attempting to explain how these perceptual differences might arise, most have suggested bad design errors in the application of feedback. For example, "transient intermodulation distortion" occurs in amplifiers with inadequate slew rate;[1] but real-world transients have rise times that are easy to accommodate with modern circuits.[2]

Another suggestion is that feedback amplifiers are more sensitive to radio-frequency interference. The idea is that the output impedance of a feedback amplifier may be very low at audio frequencies, but rises as the frequency increases. This is because feedback amplifiers must be compensated to ensure stability, and the most common compensation scheme introduces a principal pole at low frequencies that lowers the loop gain as the frequency increases, so that the output impedance rises. If the impedance is high enough, strong radio-frequency fields, as occur in our environment, can come in through the output and wreak havoc by rectification, shifting the bias conditions of the amplifier. This is certainly possible, but a well-shielded amplifier with appropriate filters need not have this problem.

Yet another idea is that the clipping behavior of feedback amplifiers is different from that of non-feedback amplifiers: clipping is sharper and recovery from clipping may be problematic. This also is true, but it does not explain the perceptual differences that may remain even in the case that the amplifiers are not driven to clipping: listeners report differences in the low-level details and the sound of the "room," the recording venue. One of us (JB) has

[1] Transient intermodulation distortion occurs when the amplifier cannot slew fast enough to follow the transient. During such a transient, the amplifier is pinned to the slew trajectory and cannot follow variations in the input. Apparently this effect was known as early as Roddam in 1952 [8], but it only became widely known in audio circles with the work of Otala [7].

[2] Boyk [2] has made a survey of wideband spectra from real musical sources. Although some of these show significant energy above 20 kHz they put limits on the rate of change of the sound pressure level in most ordinary music waveforms.

observed that the introduction of feedback into one particular (microphone pre-) amplifier seems to "separate" the very high frequencies from the rest of the range, as though a badly-integrated super-tweeter had been added to the monitor system. This yields an unnatural sound that seems correlated with but disconnected from the program material.

We investigate the possibility that the difference in sound quality is not an accident of the particular design but is an inherent characteristic of negative feedback. The idea is not new with us. In 1957, Norman Crowhurst [4] observed that since the intrinsic nonlinearity of an amplifier must produce harmonic and intermodulation products from the components of the program material, feedback will combine these products with the program to produce further distortion products. Since many of the products in each "generation" are higher or lower in frequency than the signals that produce them, the effect will be to create products extending over the full bandwidth of the amplifier. Although the total amount of this distortion is very small—much smaller than the lower-order distortion produced by the same amplifier without feedback—Crowhurst observed, "The logical result of this process would be a sort of program-modulated, high-frequency 'noise' component, giving the reproduction a 'roughness'." We speculate that this "noise," constantly changing as it is (because it is correlated with the program material), may interfere with the listeners' perceptions of low-level detail. Such speculation is not new with us either. As far back as 1950, Shorter [9] was worried about the perceptual effect of high-order distortion products; and the idea has been periodically revisited by many authors, including, most recently, Daniel H. Cheever [3], who developed a new measurement strategy which attempts to quantify the effect of this kind of distortion on perception.

In what follows, we examine the responses of nine elementary circuits to two-tone inputs, comparing in each case the behavior without feedback to that with feedback (in some cases more than one amount of feedback). Three of the nine basic circuits are simple stages using a single FET, BJT or vacuum triode; another three use pairs of these devices in complementary (FET, BJT) or push-pull (triode) configurations; and the final three are differential-input circuits. Each circuit is studied at a signal level which best reveals the behavior of interest.

As usual in such work, the real subject of our study is the behavior of certain mathematical equations and relations. When we ascribe the behavior to circuits, we are assuming that the active devices are modeled perfectly by

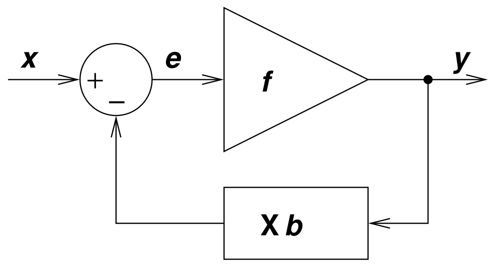

Figure 1: A feedback amplifier

the stated "laws" (square for FETs, three-halves for triodes, exponential for BJTs); that complementary pairs are perfectly symmetrical; that the tubes used in push-pull are identical; and that transformers are perfect. Though none of these is true, the results may yet be useful.

Our analysis is stateless; that is, we assume no frequency-dependent elements. Real amplifiers have such elements, but we can see the essential behavior without considering them.

The core of this paper is the numerically-derived spectra that we obtain for a variety of amplifiers, each being considered both with and without feedback. We begin, however, with a formal derivation of analytic estimates for the lowest order spectral lines, which we can use to check the validity of the numerical work; and a bit of circuit analysis. The reader may wish to skip this preliminary analysis and proceed to the discussion of the spectra that begins on page 10 under "Our spectra," touching down at figure 2 on the way.

Feedback in a nonlinear system

In figure 1 the amplifier is modeled by a function f that is in general nonlinear. The feedback path is assumed to be a linear path that multiplies by a constant b. Thus, the equation relating the output y to the input x is

$$y = f(x - by). \tag{1}$$

If f were linear, say $f(e) = Ae$ we could solve for y to get the familiar Black's formula,

$$y = \frac{Ax}{1 + Ab}. \tag{2}$$

If A is very large then $y \approx x/b$, allowing us to reliably make amplifiers with gain $1/b$ using amplifiers with large, but uncontrolled gain. However, the distortion we are interested in is due to the *nonlinearity* of f.

Assume that f may be expressed as a power series

$$y = A_1 e + A_2 e^2 + A_3 e^3 + \cdots \tag{3}$$

with no offset term, so if $e = 0$ then $y = 0$. To account for the feedback we can substitute $(x - by)$ for e to obtain

$$y = A_1(x - by) + A_2(x - by)^2 + A_3(x - by)^3 + \cdots. \tag{4}$$

In general, we can solve for y, producing a power series that represents the entire transfer function of the feedback amplifier. This series

$$y = a_1 x + a_2 x^2 + a_3 x^3 + \cdots \tag{5}$$

can be obtained by taking derivatives of equation (4):

$$\begin{aligned}
a_1 &= \left.\frac{dy}{dx}\right|_{x=0} = \frac{A_1}{1 + A_1 b} && (6)\\
a_2 &= \left.\frac{1}{2}\frac{d^2y}{dx^2}\right|_{x=0} = \frac{A_2}{(1 + A_1 b)^2} && (7)\\
a_3 &= \left.\frac{1}{6}\frac{d^3y}{dx^3}\right|_{x=0} = \frac{(A_3 A_1 - 2A_2^2)b}{(1 + A_1 b)^5} && (8)\\
&\cdots
\end{aligned}$$

We see that a_1 is the gain we would expect if the amplifier were linear, and the higher-order terms are the distortion. If we make the input a sinusoid $x(t) = C\cos\omega t$, expand powers using the multiple angle formulas,[3] and collect like terms, we get a Fourier series showing the harmonic components. Considering only the first three terms of equation (5) we get:

$$y = \left(Ca_1 + \frac{3}{4}C^3 a_3\right)\cos\omega t + \frac{1}{2}C^2 a_2 \cos 2\omega t + \frac{1}{4}C^3 a_3 \cos 3\omega t. \tag{9}$$

Thus, for small signals (C small) the relative size of the second harmonic and third harmonic distortion terms are:

$$\begin{aligned}
\mathrm{HD}_2 &\approx \frac{1}{2}C\frac{a_2}{a_1} = \frac{1}{2}\frac{A_2}{A_1(1 + A_1 b)} && (10)\\
\mathrm{HD}_3 &\approx \frac{1}{4}C^2\frac{a_3}{a_1} = \frac{1}{4}\frac{(A_3 A_1 - 2A_2^2)b}{A_1(1 + A_1 b)^4} && (11)
\end{aligned}$$

[3] For example, $(\cos\alpha)^2 = \frac{1}{2} + \frac{1}{2}\cos 2\alpha$ and $(\cos\alpha)^3 = \frac{3}{4}\cos\alpha + \frac{1}{4}\cos 3\alpha$.

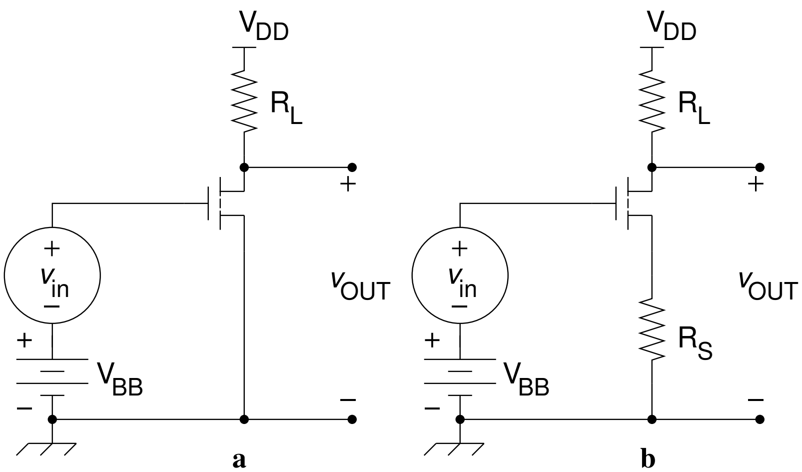

Figure 2: Amplifier **a** has no feedback; amplifier **b** has source feedback.

If we make the input the sum of two sinusoids of different frequencies, we can compute the intermodulation products as well. However this kind of algebra is not usually easy to understand past the first few terms. Since we are interested in the high-order terms, we will not pursue the algebraic approach, but will use it only to check the lowest terms of the numerical results, and to help us understand how the distortion terms are generated.

A tale of two FET amplifiers

The two FET circuits in figure 2 are identical except that amplifier **a** has no feedback while amplifier **b** has source feedback created by non-zero R_S. In the analysis below, we find that the amplifier without feedback introduces only second harmonics and first-order sums and differences of signal components; and that while adding feedback lowers these distortion products a bit, it also produces all orders of harmonic and intermodulation products in tiers stepping down in level but extending to the full bandwidth of the amplifier.

We will operate the Field-Effect Transistor (FET) in the saturated region, where it exhibits simple square-law nonlinearity: a good approximation for its drain current is

$$i_D = \frac{k}{2}(v_{GS} - V_T)^2, \tag{12}$$

where v_{GS} is the gate-to-source voltage and where k and V_T are parameters of the FET. The FET is in saturation so long as $V_T < v_{GS} < V_T + v_{DS}$, where v_{DS} is the drain-to-source voltage.

Analysis: the no-feedback case

To simulate the circuit we need the output voltage as a function of the input voltage. The output voltage is

$$v_{OUT} = V_{DD} - R_L i_D, \tag{13}$$

so we need the drain current i_D. For amplifier **a**, $v_{GS} = v_{in} + V_{BB}$, so substituting this into equation (12) and then plugging the resulting current into equation (13) we obtain an expression for the output voltage in terms of the input voltage:

$$v_{OUT} = V_{DD} - \frac{k}{2}(v_{in} + V_{BB} - V_T)^2 R_L. \tag{14}$$

Equation (14) is all we really need to obtain a numerical spectrum for any given input signal, as described on page10. But first, to see what we should expect, we do some analysis along the lines of equations (1–11), but now specific to these FET amplifiers.

We can rewrite equation (14) as

$$v_{out} = v_{OUT} - V_{OUT} = a_1 v_{in} + a_2 v_{in}^2, \tag{15}$$

a simple quadratic, where

$$V_{OUT} = V_{DD} - \frac{k}{2}(V_{BB} - V_T)^2 R_L \tag{16}$$

$$a_1 = -k(V_{BB} - V_T) R_L \tag{17}$$

$$a_2 = -\frac{1}{2} R_L k. \tag{18}$$

The incremental gain of amplifier **a** is thus

$$\left.\frac{\partial v_{OUT}}{\partial v_{in}}\right|_{v_{in}=0} = -k(V_{BB} - V_T) R_L = a_1 \tag{19}$$

This helps us choose circuit values to obtain a given gain.

Because this amplifier exhibits a simple quadratic law equation (15), its distortion products can only be second harmonics and sums and differences of the Fourier components.[4] In this simple case it is easy to work out the spectrum symbolically. (We will use this result on page 11 to check our numerical simulation.)

We define the excess gate bias $V_B = V_{BB} - V_T$, and the corresponding drain bias current $I_D = \frac{1}{2}kV_B^2$. Then the total drain current can be rewritten as

$$i_D = I_D\left(1 + 2\frac{v_{in}}{V_B} + \left(\frac{v_{in}}{V_B}\right)^2\right). \tag{20}$$

If we drive this amplifier with a sinusoid

$$v_{in} = A\cos\omega t \tag{21}$$

we obtain

$$i_D = I_D(b_0 + b_1\cos\omega t + b_2\cos 2\omega t) \tag{22}$$

where

$$b_0 = \frac{1}{2}\left(1 + \left(\frac{A}{V_B}\right)^2\right) \tag{23}$$

$$b_1 = 2\frac{A}{V_B} \tag{24}$$

$$b_2 = \frac{1}{2}\left(\frac{A}{V_B}\right)^2. \tag{25}$$

This Fourier series has only three terms. Comparing the magnitude of the second harmonic component to the magnitude of the fundamental we obtain

$$\mathrm{HD}_2 = \frac{b_2}{b_1} = \frac{A}{4V_B}. \tag{26}$$

We could work out the sizes of the Fourier components for the two-tone stimulus, getting terms for the sum and difference as well as the two second harmonics, but for more complicated circuits this would be much harder.

[4] Remember that squares of weighted sums of sinusoids can be expressed as weighted sums of sinusoids with angles that are sums and differences of the given angles.

The feedback case

Amplifier **b** is a bit more complicated. As before, we need an expression for v_{OUT} in terms of v_{in}. The output voltage is

$$v_{OUT} = V_{DD} - R_L i_D, \tag{27}$$

so we need the drain current i_D. But to compute the drain current we must solve a quadratic equation:

$$\frac{k}{2} R_S^2 i_D^2 - (k R_S v_G + 1)\, i_D + \frac{k}{2} v_G^2 = 0, \tag{28}$$

where $v_G = v_{in} + V_{BB} - V_T$. We pick the correct root so that as $R_S \to 0$ the circuit approximates the behavior of amplifier **a**. This gives

$$i_D = \frac{1}{R_S}\left(v_G + \frac{1}{R_S k}\right) - \frac{\sqrt{2}}{R_S\sqrt{R_S k}}\sqrt{v_G + \frac{1}{2R_S k}}. \tag{29}$$

Substituting into equation (27) we obtain

$$v_{OUT} = V_{DD} - \frac{R_L}{2kR_S^2}\left[\sqrt{1 + 2kv_G R_S} - 1\right]^2, \tag{30}$$

which is what we need for the simulation.

The incremental gain is

$$\left.\frac{\partial v_{OUT}}{\partial v_{in}}\right|_{v_{in}=0} = -\frac{R_L}{R_S}\left(1 - \frac{1}{\sqrt{1 + 2R_S\,(V_{BB} - V_T)\,k}}\right), \tag{31}$$

which goes to $-R_L k (V_{BB} - V_T)$ as $R_S \to 0$, as required by the condition that circuit **a** is a special case of circuit **b** ($R_S = 0$).

If we expanded equation (30) as a power series we would see that the square-root term expands into all powers of the incremental input voltage v_{in}. So by contrast with the simple amplifier **a** the feedback amplifier **b** produces not just second harmonics but all orders of harmonics; not just simple sums and differences, but all orders of intermodulation products. This illustrates the idea behind the claim that the feedback amplifier produces a more complex spectrum than the simple amplifier. However, to learn more we have to be quantitative.

Our spectra

Using numerical simulation, we compared the behavior of these amplifier topologies when stimulated by a two-tone signal. As for all examples in this paper, the two tones were at frequencies 3 and 5. Because the simulations are done with no frequency-dependent elements the units of the frequencies do not matter; all that matters is their ratio. The frequencies were chosen to be relatively prime (they are both actually prime) so as to show the maximum number of independent components. Perhaps it would be better to use incommensurate frequencies, such as 3 and 3ϕ, where $\phi = (1 + \sqrt{5})/2$, the golden ratio; but we chose integer frequencies and an integer timespan so that the spectra would come out as clean lines, without spectral leakage or skirts due to the window function. The small errors introduced by choosing integers rather than incommensurate numbers are not significant in our results.

All of the spectra in this paper were developed using numerical-analysis procedures written by one of the authors (GJS). For each circuit, a numerical procedure was written to determine the output voltage in terms of the input voltage, for each moment of time. This was easy when the relationship was given by a single equation, such as equation (14) for the no-feedback single-ended FET stage. Other cases, however, required solving simple nonlinear systems of equations, such as equations (28–30) for the same FET stage with feedback.

In general, equation solutions are accurate to one part in 10^{14} (-280 dB). Other errors sometimes contribute to bring the noise floor 15 dB higher. The two-tone input was generated in a time span of 16 and sampled with 4096 points. The output voltage was computed for each of these input points and transformed with a 4096-point transform to obtain the frequency spectrum, with a maximum representable frequency of 128. However, our spectral plots only show frequencies up to 32. In fact there are no distortion components above the noise floor in our data above a frequency of 127, so our graphs are not contaminated by aliases.

Two-tone spectra of FET amplifiers

The amplifiers were designed to have an incremental gain of -10 (that is, 20 dB, inverting), using formulas (19) and (31). The FETs were assumed to have $k = 0.002$ A V^{-2} and $V_T = 1.0$ V.[5] The other device parameters and operating conditions are given in the table below. We see that both amplifiers are comfortably biased into the saturation region, and that the source resistor in circuit **b** produces only a small amount of feedback (about 1.8 dB). The bias current in amplifier **a** is 1.0 mA; and in **b**, about 0.814 mA. The table shows parameters for these two amplifiers, and for amplifier **c**, with the same topology as **b** but more feedback (about 9.5 dB). We include **c** to demonstrate the robustness of the conclusions.

	a	**b**	**c**	
V_{BB}	2.0	2.0	3.0	V
R_L	5000	6800	15000	Ω
R_S	0	120	1000	Ω
I_D	1.0	0.814	1.0	mA

In these experiments the two tones of the stimulus are of equal amplitude (.05 peak volts), at frequencies 3 and 5. As mentioned above, we use the same stimulus frequencies throughout this paper, though the amplitudes may vary; and all single-ended amplifiers studied have incremental gains of -10 (that is, 20 dB, inverting).

In the spectrum of amplifier **a** (figure 3) the fundamental components (frequencies 3 and 5) have been normalized to 0 dB. The second harmonics (frequencies 6 and 10) are at -38 dB; and the sum and difference intermodulation products (frequencies 2 and 8) are at about -32 dB.

Here we can check the simulations against the theory: The simulations show that the second harmonics are down by -38 dB. If we evaluate the ratio of the second harmonic to the fundamental using equation (26), plugging in 0.05 for A and 1.0 for $V_B = V_{BB} - V_T$, we find that the ratio is 0.0125 or -38.06 dB.

The spectrum of the feedback amplifier **b** (figure 4) is more complicated, as expected. The second harmonics are at about -40.5 dB, and the sum and difference frequencies are at about -34.5 dB. This is a small improvement—about 2.5 dB in each line—compared to the amplifier without feedback. However, there is a new tier of components with peaks at about -80 dB; and even more components down around -120.

[5] These paramters are typical for an N-channel enhancement-mode MOSFET when used for small-signal amplification.

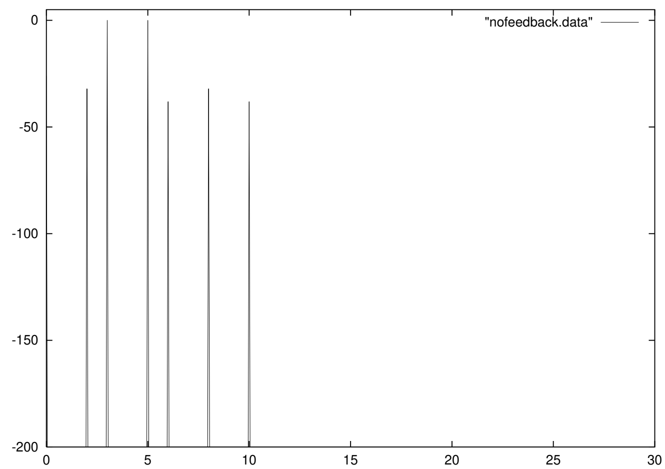

Figure 3: The spectrum of amplifier **a**: a single-ended FET without feedback; the two-tone input has 0.05 peak volts in each component.

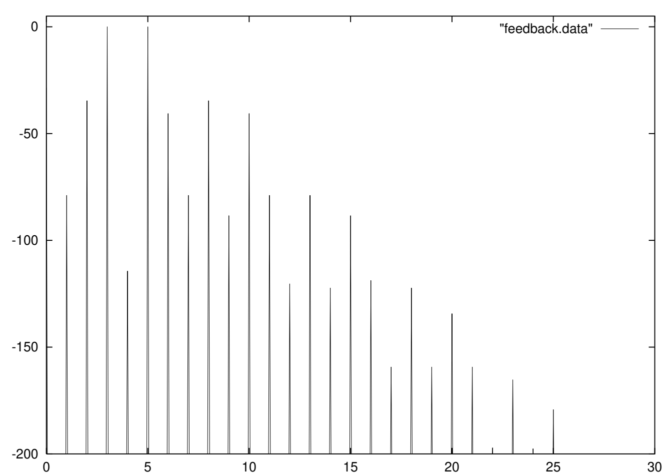

Figure 4: The spectrum of amplifier **b**: a single-ended FET with source feedback; the two-tone input has 0.05 peak volts in each component.

The spectrum of amplifier **c** is not shown. Its additional feedback compared to **b** makes for a distortion spectrum which is similar except that all of the products are pushed down in level.

Returning to feedback amplifier **b**, in figure 5 we expand the vertical scale of the spectrum to see the structure more clearly. We see many tiers of distortion products, each produced by an additional circulation around the feedback loop and 40 dB below the previous tier. The noise just above -300 dB is due to numerical error in the equation solver.

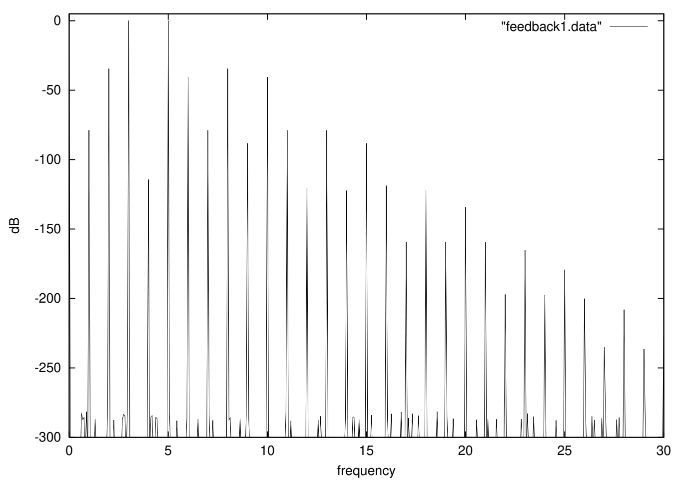

Figure 5: Expanded spectrum of amplifier **b**: a single-ended FET with source feedback; the two-tone input has 0.05 peak volts in each component.

Thus we observe that adding negative feedback to a FET amplifier, while decreasing the overall amount of distortion, significantly changes the distribution of the distortion products. In two-tone tests, feedback introduces new tiers of products, most very weak, but not necessarily insignificant perceptually, as they produce a noise floor *correlated with* the program material. And not only does the amplitude of this noise floor rise and fall with the amplitude of the program material, but its character changes as it rises and falls, higher-order products being more volatile than lower-order ones. If we increase the drive of the FET amplifier by 12 dB, from 0.05 peak volts to 0.2 peak volts, the distortion at frequency 2 rises by 24 dB (12 dB relative to the input signals), which we may take simply as due to higher signal levels. But the components at frequencies 17 and 19 increase by 60 dB (48 dB relative to the input signals). As the level rises, the distortion is thus weighted toward the higher-order terms, and mostly toward higher frequencies.[6] Further experiments (not shown) with the FET amplifier demonstrates this point.

If we change the fundamental signal amplitudes, the amplitudes of the distortion products change so that the spectrum has essentially the same shape in that it is made up of tiers of distortion products, but the amplitude difference between the tiers changes. As the fundamental signal amplitudes decrease the spacing increases, so small signals are *relatively* less distorted than large ones.

BJTs are different

Consider the two Bipolar Junction Transistor (BJT) circuits shown in figure 6. BJTs are quite different from FETs in that, while FETs have a square-law relationship between the controlled current and the control voltage, the relationship for a BJT is exponential. For a correctly biased BJT a good approximation of the collector current i_C in terms of the base-emitter voltage v_{BE} is

$$i_C = I_0 \left(e^{\frac{q}{kT} v_{BE}} - 1 \right), \tag{32}$$

where I_0 is a very small number, such as 10^{-12}, that depends on the geometry of the transistor; and $\frac{q}{kT}$ is about 38.68 for a temperature of 300 K.

Circuit **d** is a simple BJT common-emitter amplifier with no feedback, while circuit **e** is the same but with emitter feedback created by the inclusion of non-zero R_E. We consider two values of R_E, 100 and 1000 ohms, and refer to the two resulting amplifiers, having respectively about 14 and 32 dB of

[6] This strong dependence of the amplitude of a distortion product on its order is not accidental, but it is inherent in the mathematics of the situation: each distortion product arises from terms of the series expansion that have powers of the input equal to the sum of the absolute values of the integer weights of the fundamental components. So frequency $17 = 4 \cdot 3 + 1 \cdot 5 = -1 \cdot 3 + 4 \cdot 5$ has integer weights 4 and 1 which sum to 5. Thus the exponent associated with it is 5 and so a 12 dB change in the input produces a $5 \times 12 = 60$ dB change in the output at this frequency. Of course, $17 = 7 \cdot 5 - 6 \cdot 3$ as well, with an exponent of 13. So this term grows even faster than the fifth-order ones already described, but it starts out so much smaller that under reasonable assumptions it never exceeds the fifth-order terms. The lowest-order combinations of input frequencies that give any particular component are usually dominant. The conditions under which this series converges, and under which the low-order terms are dominant are mathematically very interesting, but they are beyond the scope of this paper.

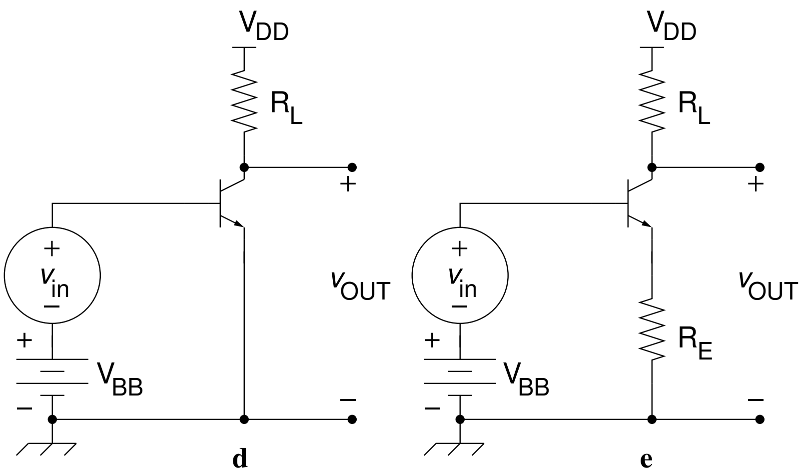

Figure 6: Amplifier **d** has no feedback; amplifier **e** has emitter feedback.

feedback, as **e** and **f**. We bias all three amplifiers to have a gain of -10 with a collector current of about 1 mA. The analysis of these BJT amplifiers is similar to the analysis of the FET amplifiers. The operating conditions are summarized in the table below. The numbers are not round because we wanted to bias circuit **d** to 1 mA.

	d	**e**	**f**	
V_{BB}	0.5453	0.6453	1.5453	V
R_L	263.2	1264	10270	Ω
R_E	0	100	1000	Ω
I_C	1.0	1.0	1.0	mA

The nonlinearity of a BJT is extremely sharp, so we drive the input with a peak voltage of only 0.0004 Volts in each frequency. If we used 0.05 Volts, as with the FETs, the distortion of the no-feedback BJT amplifier would be very bad and we would not be able to understand the effect of the feedback. To avoid this distortion in real-world circuits that use BJTs, degenerative feedback is employed to ensure that the signal appearing across the base-emitter junction of the transistor is very small.

The spectrum of **d**, the BJT amplifier without feedback (figure 7), is complicated, and remarkably similar to the spectrum of the FET amplifier *with* feedback, showing several tiers of distortion products. We can understand

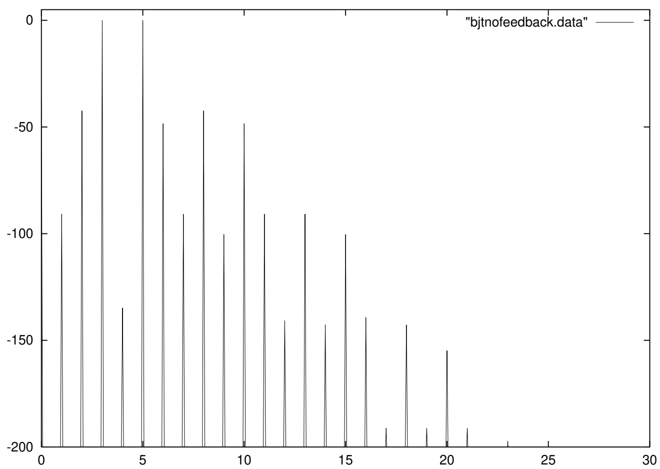

Figure 7: The spectrum of amplifier **d**: a single-ended BJT without feedback; the two-tone input has 0.0004 peak volts in each component.

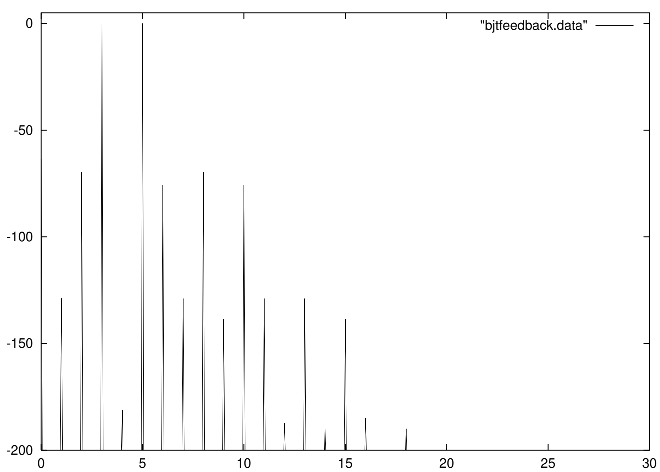

Figure 8: The spectrum of amplifier **e**: a single-ended BJT with emitter feedback; the two-tone input has 0.0004 peak volts in each component.

this by realizing that the power-series expansion of the BJT's exponential characteristic contains terms of all order in the input voltage. In the FET amplifier the high-order terms are constructed by the feedback process; in the BJT, they are inherent.

The spectrum of **e**, the BJT amplifier with feedback (figure 8) is like the spectrum without feedback except that all of the distortion products are substantially attenuated. When we increase the feedback, as in circuit **f**, we see (figure 9) that the performance of the amplifier again improves dramatically, so that the principal distortion lines are suppressed below -100 dB and the next tier is suppressed below -180 dB. Thus, in the case of the BJT, feedback improves the nonlinear distortion of the amplifier.

Although the no-feedback BJT amplifier started out much worse than the no-feedback FET amplifier it is vastly improved by feedback; and because the transconductance of the BJT is much higher than the transconductance of the FET, we can use much more feedback and still obtain the same overall gain.

The extra gain also allows enough feedback so that a BJT amplifier can accomodate a greater range of input voltages without exceeding a given distortion level, even though the intrinsic nonlinearity of the BJT is much sharper than the nonlinearity of the FET, seriously restricting the input range of the bare BJT relative to the bare FET. For example, if we try to drive BJT circuit **f** (with 32 dB of feedback) with the same size input that we used with the FET amplifiers (peak voltage 0.05 in each component), the distortion in the BJT amplifier (figure 10) is 13 dB lower than the distortion in the FET amplifier **c** with only 9.5 dB of feedback.

Comparison of figure 9 and figure 10 illustrates another interesting point: For the BJT stage with or without feedback, as for the FET stage wth feedback, higher-order distortion products are emphasized as the amplitude of the input signals is increased.[7] This is generally true of any nonlinear system that shows tiers of intermodulation products.

Vacuum triodes

To analyze a triode we start with the behavior of a vacuum diode. This can be approximately characterized by the Child-Langmuir law, which gives the

[7]See the footnote 6

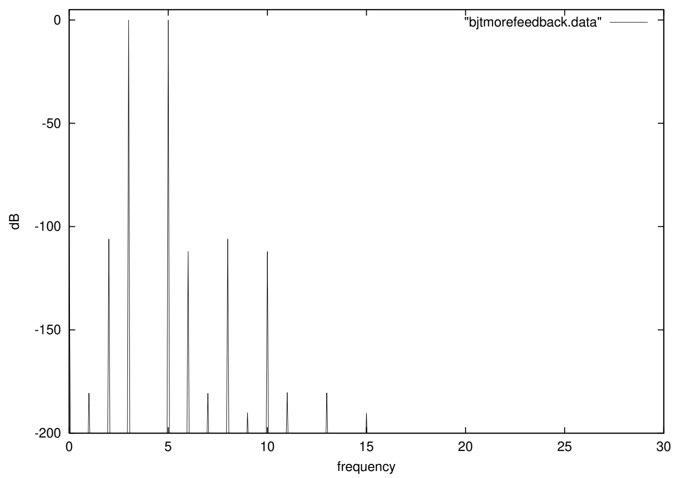

Figure 9: The spectrum of amplifier **f**: a single-ended BJT with more emitter feedback; the two-tone input has 0.0004 peak volts in each component.

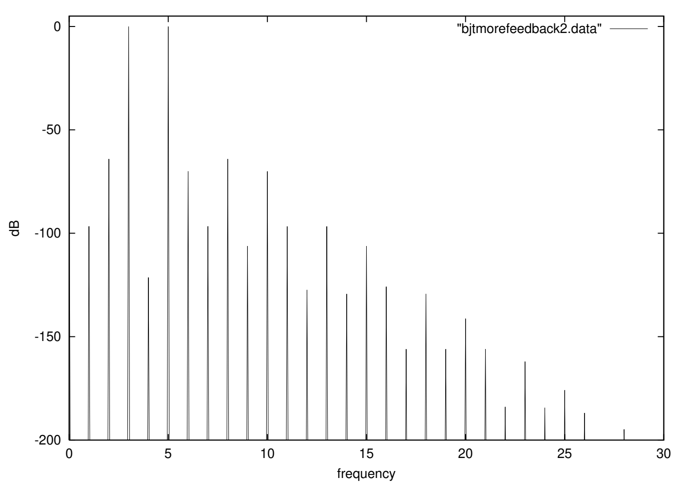

Figure 10: The spectrum of amplifier **f**: a single-ended BJT with more emitter feedback; the two-tone input has 0.05 peak volts in each component.

anode current i_P as a function of the voltage from anode to cathode v_{PK}

$$i_P = \frac{\sqrt{2}}{9\pi d^2}\sqrt{\frac{e}{m}}v_{PK}^{3/2}, \tag{33}$$

where d is the distance from the anode to the cathode, and e/m is the charge-to-mass ratio of the electron. This forms a theoretical basis for a model for the behavior of a vacuum triode by modifying equation (33) to include the influence of the control grid [6]. The influence of the control grid is parameterized by the amplification factor μ, which is defined to be

$$\mu = \frac{\partial v_{PK}}{\partial v_{GK}}, \tag{34}$$

where v_{GK} is the grid-to-cathode voltage. With this, the model is

$$i_P = K(v_{PK} + \mu\, v_{GK})^{3/2}, \tag{35}$$

where the perveance K and the amplification factor are determined by the geometry of the device. In published specifications, the amplification factor and the transconductance

$$g_m = \frac{\partial i_P}{\partial v_{GK}} \tag{36}$$

are often provided for a given operating point, and we have to deduce the perveance.[8]

For the FET amplifiers we had single equations (14, 30) that gave v_{OUT} in terms of v_{in}. For the BJT, the situation was similar, though we did not give the equations. In the FET case, equation (30) was an explicit solution, but in the BJT case a numerical solution was necessary. But for the tube circuit **h** (amplifier **g** is the same, but with $R_K = 0$; see figure 11), we have instead a fairly complicated set of equations:

$$\begin{aligned} v_{PK} &= V_{BB} - (R_L + R_K)i_P & (37)\\ v_{GK} &= v_{in} - V_{CC} - R_K i_P & (38)\\ i_P &= K(v_{PK} + \mu\, v_{GK})^{3/2} & (39)\\ v_{OUT} &= V_{BB} - R_L i_P. & (40) \end{aligned}$$

[8] For example, for a section of a 6DJ8 [1] double triode, the data sheet gives $g_m \approx 0.0125$ amps/volt and $\mu \approx 33$ when operating at $V_{PK} = 90$ volts and $V_{GK} = -1.3$ volts, so we have $K \approx 3.7 \times 10^{-5}$.

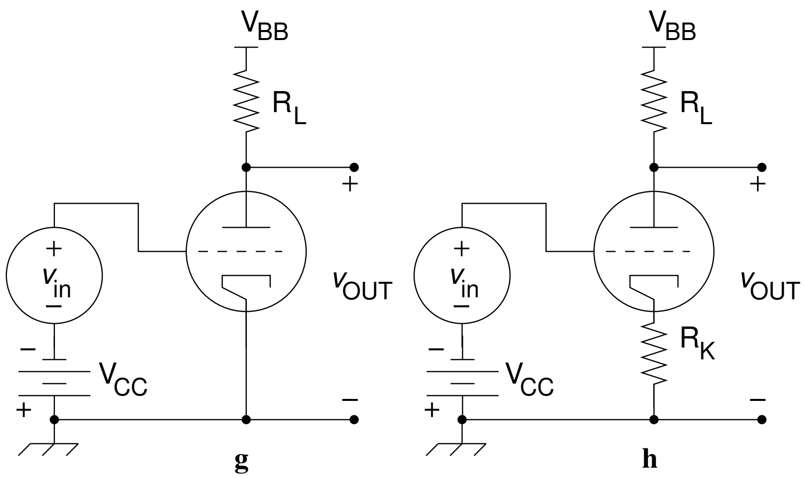

Figure 11: Amplifier **g** has no feedback; amplifier **h** has cathode feedback.

It is not illuminating to attack these equations analytically. However, the 3/2 power law in equation (39) can be expressed as a power series that has terms of all order, and this will be part of the answer even when $R_K = 0$. Thus, we can expect the triode results to be something like the BJT results obtained above, in that distortion products of all orders should appear in the spectrum of the simple amplifier **g**. The question is: Does the addition of feedback make things better or worse?

The numerical results are not surprising. We simulated the amplifier with a 6DJ8 triode[9] under the following operating conditions:

	g	**h**	
V_{BB}	100	140	V
V_{CC}	-1.2	0	V
R_L	1200	3500	Ω
R_K	0	150	Ω
I_P	11.7	11.1	mA

Both of these amplifiers have a gain of approximately −10. The spectrum of the no-feedback tube amplifier **g** has all orders of distortion products, as predicted. But the distortion products are generally smaller than the

[9]See footnote 8

corresponding products in either the no-feedback FET or BJT amplifiers (**a** and **d**), because the triode is fundamentally more linear than either the BJT or the FET.

The spectrum of amplifier **h**, with cathode feedback (figure 13), is cleaner than that of **g** (figure 12). All of the products are pushed down considerably, making it comparable to the spectrum of the BJT with emitter feedback (figure 8). However, here the amplitude is 0.05 volts for each component; the comparable BJT spectrum is for a drive of .0004 peak volts. If we compare with the spectrum of a BJT with "more" emitter feedback driven with 0.05 peak volts (figure 10) we find that the tube amplifier with small cathode feedback is still somewhat cleaner.

Additional experiments (not shown) show that smaller signals produce smaller distortion components, for triode amplifiers with and without cathode feedback.

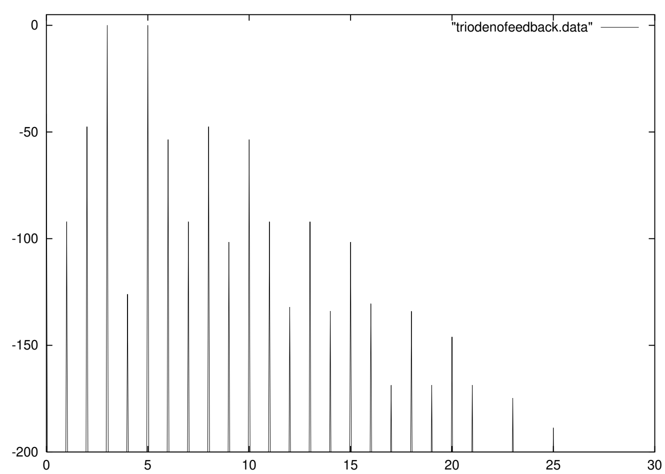

Figure 12: The spectrum of amplifier **g**: a single-ended vacuum triode without feedback; the two-tone input has 0.05 peak volts in each component.

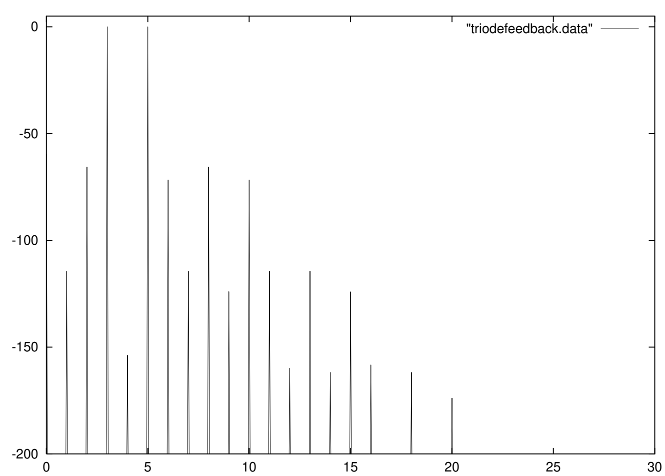

Figure 13: The spectrum of amplifier **h**: a single-ended vacuum triode with cathode feedback; the two-tone input has 0.05 peak volts in each component.

Details of single-ended amplifiers

In the following table we summarize the relative amplitudes of the principal spectral lines that occur in the output of each of the elementary amplifiers that we have considered:

	FET			BJT			Triode	
ckt	**a**	**b**	**c**	**d**	**e**	**f**	**g**	**h**
drive	0.05	0.05	0.05	0.0004	0.0004	0.0004	0.05	0.05
fb dB	0	1.8	9.5	0	14	32	0	4.2
1		-78.9	-92.7	-90.7	-128.9	-182.1	-92.1	-114.5
2	-32.0	-34.5	-51.1	-42.4	-69.6	-106.0	-47.5	-65.6
3	0	0	0	0	0	0	0	0
4		-114.3	-125.4	-134.8	-181.3	-193.7	-126.0	-153.8
5	0	0	0	0	0	0	0	0
6	-38.1	-34.5	-57.1	-48.4	-75.7	-112.0	-53.5	-71.6
7		-78.9	-92.7	-90.8	-128.9	-177.9	-92.1	-114.5
8	-32.0	-34.5	-51.1	-42.4	-69.6	-106.0	-47.5	-65.6
9		-88.4	-102.2	-100.3	-138.4	-187.0	-101.6	-124.0
10	-38.1	-40.5	-57.1	-48.4	-75.7	-112.0	-53.5	-71.6
11		-78.9	-92.7	-90.8	-128.9	-180.7	-92.1	-114.5
12		-120.3	-131.4	-140.8	-187.2	-197.8	-132.1	-159.8
13		-78.9	-92.7	-90.8	-128.9	-182.6	-92.1	-114.5
14		-122.3	-133.4	-142.7	-190.2	-200.5	-134.0	-161.8
15		-88.4	-102.2	-100.3	-138.4	-184.8	-101.6	-124.0
16		-118.7	-129.8	-139.2	-185.0	-188.8	-130.5	-158.2
17		-159.2	-167.6	-191.1		-201.4	-168.7	-202.4
18		-122.3	-133.4	-142.7	-189.1	-209.7	-134.0	-161.8
19		-159.2	-167.6	-191.1			-168.7	-202.4
20		-134.3	-145.4	-154.8	-199.0		-146.0	-173.8

Summary of single-ended amplifiers

For all cases, input signals were at frequencies 3 and 5 (unscaled). Input voltage levels were always equal for the two components and were as given below. Unless otherwise stated, (a) Higher input level raises the relative distortion and (b) emphasizes higher-order distortion products. (c) Adding more feedback lowers all distortion products.

1. FET: 0.05 peak volts in each input component

   Without feedback, distortion consists of only 2nd harmonics and first-order sums and differences of the two input components; that is, only the four frequencies 2, 6, 8, 10. Feedback creates complex new distortion products extending over the full bandwidth of the amplifier and thus constituting a kind of "noise floor."

2. BJT: 0.0004 peak volts in each input component

   Distortion without feedback is complex, resembling that of the FET amplifier *with* feedback.

3. Triode: 0.05 peak volts in each input components

   Distortion without feedback is complex, resembling that of the FET amplifier with feedback, but distortion products are distinctly lower in level that in the no-feedback FET amplifier at the same input level, or even the no-feedback BJT amp at its much lower level.

Output stages using two devices

The push-pull output stage is ubiquitous in audio amplifiers. It provides symmetrical low-impedance drive, and it introduces no even-harmonic distortion.

BJT complementary-pair output stage

Consider the idealized BJT stage in figure 14. We assume that the PNP and NPN transistors have identical behavior, except for signs, and that they are symmetrically biased and operated in class A (that is, neither transistor is cut off for any input signal). One way to understand this stage is as a push-pull emitter follower; it has no voltage gain, but plenty of power gain.

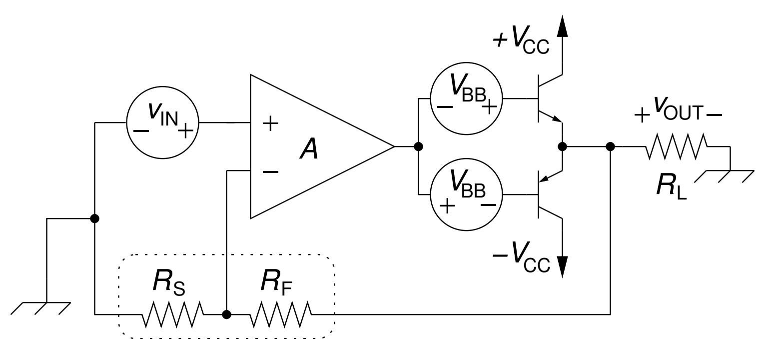

Figure 14: An idealized complementary-pair stage constructed from BJTs, driven by an ideal linear differential amplifier of voltage gain A.

The analysis is then quite simple. For the two transistors we have

$$i_{CN} = +I_0\left(e^{+\frac{q}{kT}v_{BEN}} - 1\right) \quad (41)$$

$$i_{CP} = -I_0\left(e^{-\frac{q}{kT}v_{BEP}} - 1\right), \quad (42)$$

where

$$v_{BEN} = A\Delta v - v_{OUT} + V_{BB} \quad (43)$$

$$v_{BEP} = A\Delta v - v_{OUT} - V_{BB} \quad (44)$$

and

$$i_{OUT} = i_{CN} + i_{CP} \tag{45}$$

$$\Delta v = v_{IN} - \frac{R_S}{R_F + R_S} v_{OUT}. \tag{46}$$

From these we can deduce that

$$v_{OUT} = 2R_L I_0 e^{\frac{q}{kT} V_{BB}} \sinh\left(\frac{q}{kT}\left(Av_{IN} - \left(\frac{AR_S}{R_F + R_S} + 1\right) v_{OUT}\right)\right). \tag{47}$$

For each v_{IN} we must numerically solve for v_{OUT}.

By adjusting V_{BB} this complementary-pair stage could be biased for any amount of crossover distortion. The stage has approximately unity gain, but by adjusting the prescalar gain A and the resistive divider (R_S and R_F) we can see how the stage would behave in an amplifier with any gain and feedback we please. We tested the BJT pair stage only in class A, however. As in the single-ended cases, we set things up so that the overall gain is always 10. In our experiments we set $v_{BB} = 0.6$ volts. So, with $I_0 = 10^{-12}$ amperes we have a resting bias current of about 8 mA through the transistors.

In the no-feedback case, we prescale the input by 10 (to give the desired overall gain), so the actual signal driving the bases of the complementary pair had a peak voltage of 0.05 volts for each component. (The variation in the base-emitter voltage remains small here because this is a follower circuit.) With this we got the spectrum of the output that appears in figure 15.

We introduce feedback by boosting the prescalar gain to 100, and feeding back 0.089 of the output signal. With this we get the spectrum of the output that appears in figure 16. As we would expect, the feedback improves the result by suppressing all of the distortion products. Whether or not we apply feedback in this class-A amplifier the relative distortion decreases with the signal amplitude.

FET complementary-pair output stage

We can replace the complementary-pair of BJTs in figure 14 with FETs. The most common way to do this is to use the P-channel FET as the pullup and the N-channel FET as the pulldown. This yields a symmetrized inverting amplifier stage with voltage gain, as in a CMOS inverter stage. An unusual alternative, which is more analogous to the complementary BJT stage analyzed above, is to make a symmetrical source follower, as in figure 17.

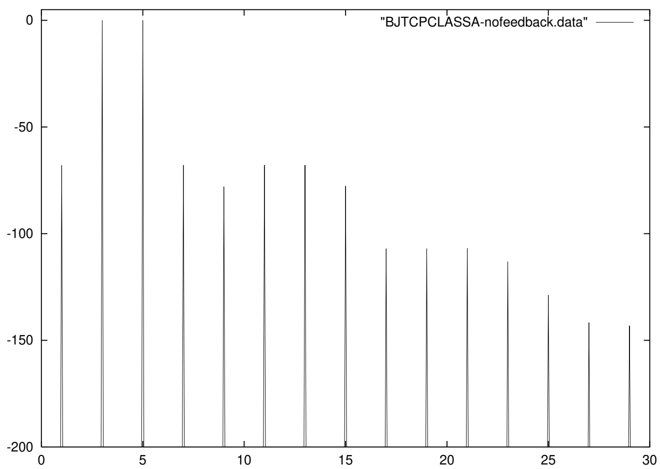

Figure 15: The spectrum of the complementary-pair BJT amplifier without feedback; the two-tone input has 0.005 peak volts in each component.

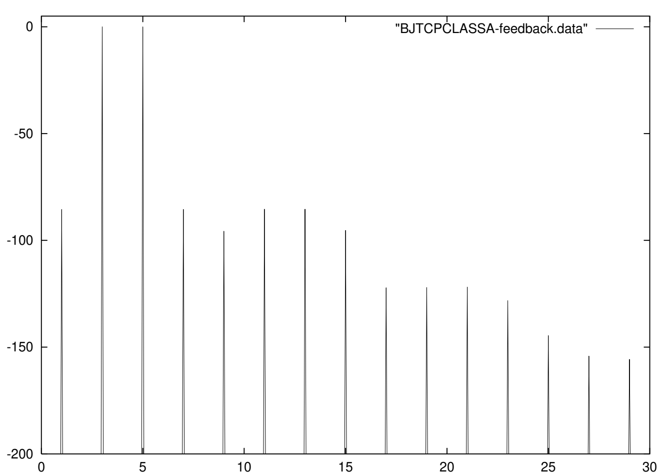

Figure 16: The spectrum of the complementary-pair BJT amplifier with 19 dB of feedback; the two-tone input has 0.005 peak volts in each component.

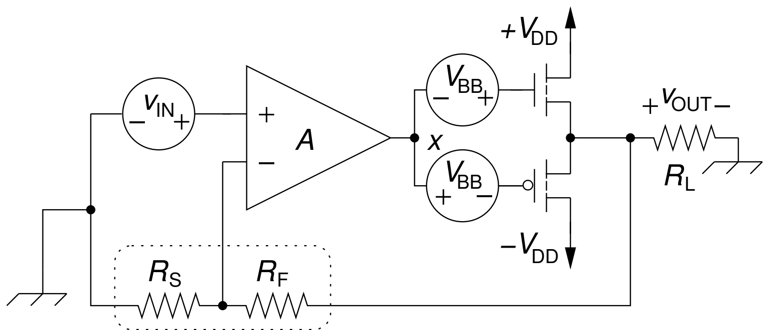

Figure 17: An idealized complementary-pair stage constructed from FETs, driven by an ideal linear differential amplifier of voltage gain A. The node potential at the output of the differential amplifier is labeled x.

If we let the excess bias $V_B = V_{BB} - V_T$ and we let $v = x - v_{out}$, where x is the node potential at the output of the differential amplifier, then

$$i_{DN} = \begin{cases} 0 & \text{if } v + V_B < 0 \\ +\frac{k}{2}(v + V_B)^2 & \text{otherwise} \end{cases} \tag{48}$$

$$i_{DP} = \begin{cases} 0 & \text{if } v - V_B > 0 \\ -\frac{k}{2}(v - V_B)^2 & \text{otherwise} \end{cases} . \tag{49}$$

By adjusting V_{BB} we can choose to make this stage operate class A or class B (for any input, one transistor is always cut off and the other is active) or any combination.

A remarkable feature of this complementary pair is that if we set $V_{BB} >> V_T$, so the stage is operated class A, then the distortions exactly cancel and the result is linear! In this case

$$v_{OUT} = \frac{2R_L V_B k}{1 + 2R_L V_B k} x; \tag{50}$$

feedback has no effect except to control the gain.

If we set the bias $V_{BB} = V_T$ then $V_B = 0$, the amplifier operates in class B, and there is distortion. Where the BJT emitter follower had voltage gain close to unity, the FET source follower has substantially less gain. So to attain an overall gain of 10 with a load of 1000 Ω and no feedback, the prescalar must have a gain of about 20.

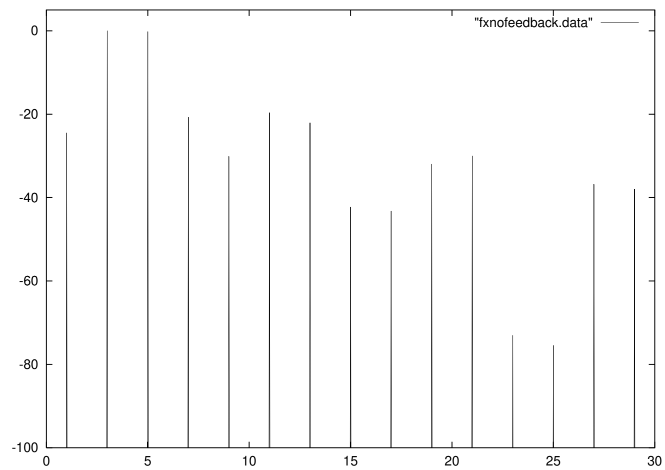

Figure 18: The spectrum of the class-B complementary-pair FET amplifier without feedback; the two-tone input has 0.06 peak volts in each component. Note that the vertical range here is 100 dB rather than the usual 200 dB.

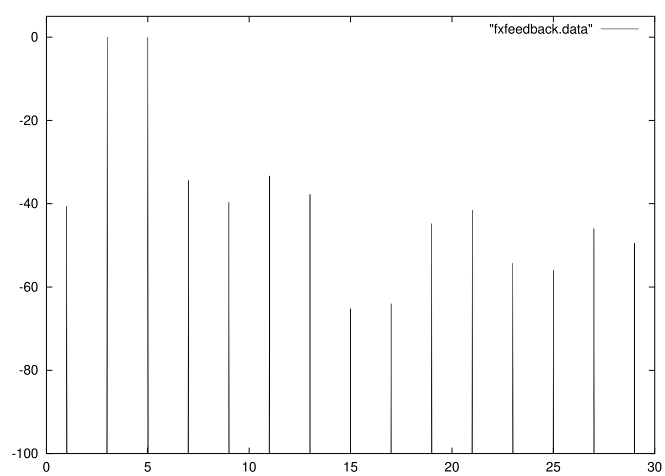

Figure 19: The spectrum of the class-B complementary-pair FET amplifier with 18 dB of feedback; the two-tone input has 0.06 peak volts in each component. Note that the vertical range here is 100 dB rather than the usual 200 dB.

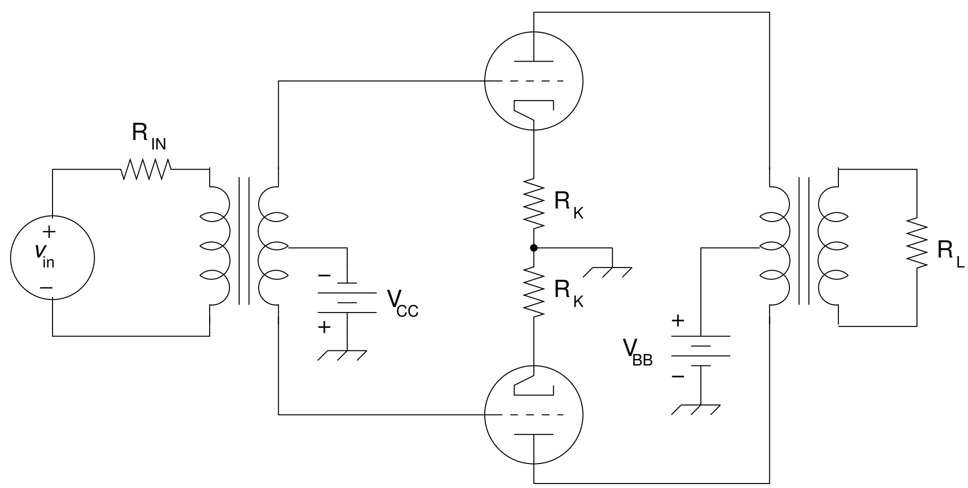

Figure 20: An idealized push-pull stage constructed from vacuum triodes.

We can maintain an overall gain of 10 while introducing 18 dB of feedback by increasing the prescalar gain to 100.

The spectra are interesting. Figure 18 shows the spectrum of the amplifier without feedback; figure 19, that of the amplifier with feedback. While feedback generally improves the signal, the distortion is still pretty bad; and the products at frequencies 23 and 25 are much worse with feedback than without feedback. Indeed, the component at frequency 25 is almost 20 dB worse with feedback!

This effect disappears with larger excitation signals, where the crossover region is a smaller portion of the waveform. As the size of the excitation increases the distortion generally decreases and the feedback becomes uniformly effective. For smaller excitations the relative distortions worsen both with and without feedback.

Vacuum triode push-pull output stage

Because vacuum triodes do not come in complementary pairs a push-pull amplifier built from triodes is configured somewhat differently from one built with semiconductor devices. The traditional method is to use center-tapped transformers to provide a phase inversion at input and the output. Unfortunately, transformers that provide good performance over a wide range of frequencies and amplitudes are expensive and hard to manufacture, especially if they must be used with significant bias current; thus most modern audio amplifiers do not use transformers. In the simple circuit of figure 20 we assume nonetheless that the 1–1 transformers are ideal, and that the tubes are identical.

In this circuit, which we analyze only in class A, we can adjust the amount of cathode degeneration, plate current and voltage gain by adjusting R_K, V_{CC} and R_L respectively. (In a real circuit, R_L would be given, and we would change the output transformer turns ratio to adjust the effective load resistance.) We set the voltage gain to 10; plate bias current, to 10 mA; and peak voltage in each input component to 0.1 volts. This is higher than we used in the other circuits, because the distortion of the triode circuit turned out to be very low.

In figure 21, we see that, allowing for the level difference in the input signal, the low-order part of the spectrum of the push-pull amplifier without feedback is very similar to the spectrum of the single-ended tube amplifier (figure 12) with the even-order components suppressed. However, there are some new high-order components (with frequency larger than 15). The spectrum with $R_K = 100\ \Omega$, a moderate amount of feedback, proved much better (figure 22).

In the class-A vacuum triode push-pull stage the relative distortion decreases with decreasing signal amplitude, with or without feedback. This is in stark contrast to the behavior we observe with the class-B FET complementary-pair stage. In that case the relative distortion increases as the signal amplitude decreases. This is because the size of the nasty region near zero is constant in the class-B stage. (On the other hand, the class-A FET pair had zero distortion!)

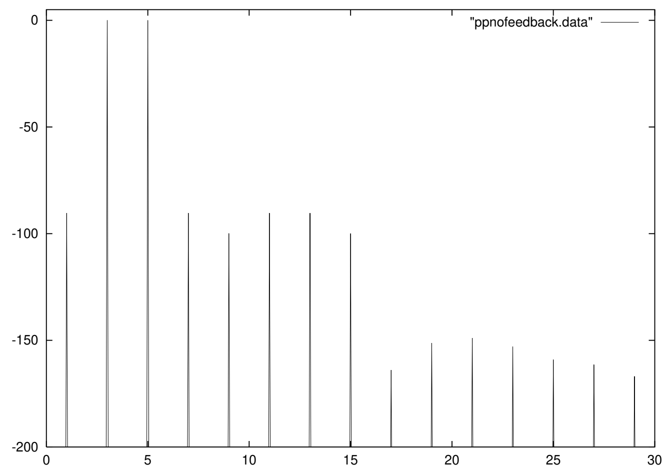

Figure 21: The spectrum of the push-pull vacuum triode amplifier without feedback; the two-tone input has 0.1 peak volts in each component.

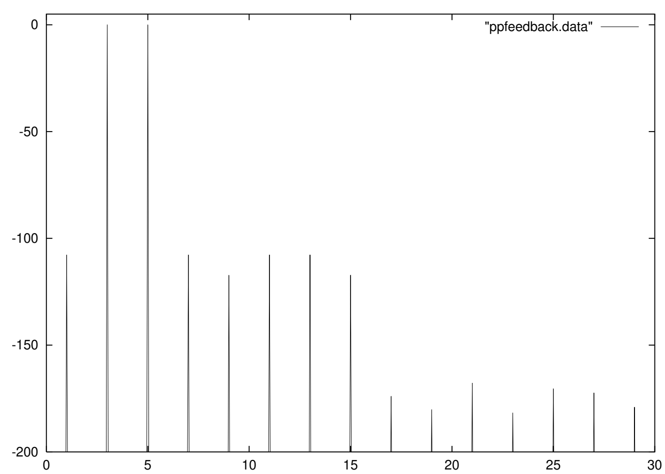

Figure 22: The spectrum of the push-pull vacuum triode amplifier with feedback; the two-tone input has 0.1 peak volts in each component.

Summary of output amplifiers

For all cases, input signals were at frequencies 3 and 5 (unscaled). Input voltage levels were always equal for the two components and were as given below. Unless otherwise stated, (a) Higher input level raises the relative distortion and (b) emphasizes higher-order distortion products. (c) Adding more feedback lowers all distortion products.

1. BJT: 0.005 peak volts in each input component

   Complementary pair analyzed in class A only. Distortion products of even-numbered frequencies are absent due to circuit symmetry. However, the pair without feedback, compared to the single-ended BJT stage without feedback, generates frequencies 17, 25, 27 and 29 at levels much higher than expected. And the pair with feedback, compared to the single-ended BJT with feedback, generates 17, 19, 21, 23 and 25 at much higher levels than expected.

2. FET: 0.06 peak volts in each input component

   Symmetrical source follower, with N-type pullup analyzed in class A and class B. In class A, the pair is distortion-free even without feedback, and feedback's only role is to set gain. In class B, even-numbered distortion products are absent due to circuit symmetry; but frequencies 23 and 25 get worse with feedback! Also in class B with or without feedback, relative distortion goes up as input level goes down.

3. Triode: 0.1 peak volts in each input component

   Push-pull circuit analyzed in class A only. Even-numbered distortion products are absent due to circuit symmetry. Feedback suppresses all products.

Differential input stages

The long-tailed differential pair appears as the input stage in many audio amplifiers. It can be used to provide common-mode rejection on a balanced input, or a convenient place to apply global DC feedback, allowing the designer to control the bias as well as the gain with the feedback network. This remarkable circuit can be realized with a matched pair of tubes, BJTs, or FETs, as in figure 23. If a single-ended output is needed then only one output need be used, at the cost of a factor of two in gain (though if current mirrors are available in the technology we can get a single-ended output without loss of gain).

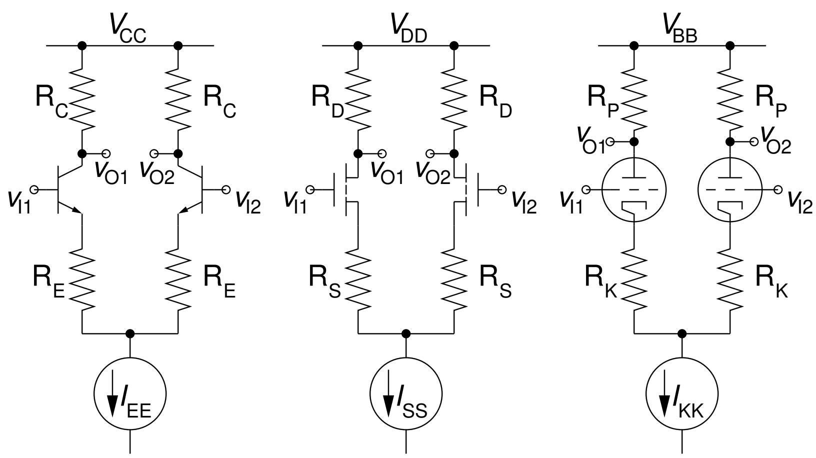

Figure 23: Differential pairs can be constructed using any transconductance device.

In the ideal case we drive the differential pair with a current source and the current is divided between the two branches. If the input voltages v_{I1} and v_{I2} are equal then the current is divided equally and the voltage drops across the load resistors are equal, so the output voltages v_{O1} and v_{O2} are equal. If the input voltages differ then more of the current is routed through the branch with the higher input voltage and thus that branch has a lower output voltage than the other one. If the differential input voltage $v_{I1} - v_{I2}$ is too large then all of the current is routed through one branch, pinning the pair. If the current source is perfect then the differential pair is insensitive to common-mode variations in the input voltages.

In the circuits of figure 23 we have included degeneration resistors (emitter, source, cathode) to allow us to introduce feedback. In a differential pair such degeneration can help us to compensate for the fact that the devices are not perfectly matched, and it allows us to exchange gain for an increase in the range of differential input voltages for which the pair has approximately linear operation.

The analysis of a differential pair with degeneration resistors is rather involved, requiring us to work with a nasty set of nonlinear equations, which must be solved numerically. Here we show the equations for the case of the vacuum triode pair; the other cases are similar, but a bit easier.

The plate currents in the triodes are

$$i_{P1} = K(v_{PK1} + \mu\, v_{GK1})^{3/2} \tag{51}$$

$$i_{P2} = K(v_{PK2} + \mu\, v_{GK2})^{3/2}, \tag{52}$$

where

$$v_{PK1} = (V_{BB} - R_P\, i_{P1}) - (v_{I1} - v_{GK1}) \tag{53}$$

$$v_{PK2} = (V_{BB} - R_P\, i_{P2}) - (v_{I2} - v_{GK2}). \tag{54}$$

The branches are connected by two facts: the plate currents must sum up to the current source

$$I_{KK} = i_{P1} + i_{P2}, \tag{55}$$

and Kirchoff's voltage law must hold

$$v_{I1} - v_{GK1} - R_K\, i_{P1} + R_K\, i_{P2} + v_{GK2} - v_{I2} = 0. \tag{56}$$

Finally, if we know the plate currents, we know the output voltages:

$$v_{O1} = V_{BB} - R_P\, i_{P1} \tag{57}$$

$$v_{O2} = V_{BB} - R_P\, i_{P2}. \tag{58}$$

We simulated differential pairs built with BJTs FETs and triodes, with and without feedback. The results are what we would expect. The spectra show only odd-order terms: the even-order distortions are cancelled by the symmetry of the system. By choosing the scale of the input signals and the amount of feedback, we could get the main distortion products to be approximately the same size for each circuit both with and without feedback.

For the BJT differential pair we set the current source at 1 mA, so the bias current for each BJT is 0.5 mA. This gives us a transconductance of about 19.3 mS. With no feedback (emitter resistors omitted) we obtain a gain of 10 with collector resistors of about 520 Ω. To add feedback we install the emitter resistors. With emitter resistors of 217 Ω we had to increase the collector resistors to 2700 Ω to maintain the amplifier gain of 10.

In the BJT spectra (figures 24 and 25) the strongest distortion terms are the third-order intermodulation products at frequencies 1, 7, 11, 13, followed by the third harmonics at frequencies 9 and 15. Without feedback the intermodulation is at about -53 dB and the third harmonics are at about -62 dB. With feedback these components are suppressed to -96 dB and -105 dB respectively. However, to keep the distortion this small the BJT differential pair must be operated with very small input signals. Here we use 5 mV peak in each component.

For the FET differential pair, we set the current source at 2 mA, so the bias current for each FET was 1 mA, giving a transconductance of 2 mS. With no feedback (source resistors omitted) we obtain a gain of 10 with drain resistors of 5000 Ω. We set the current source for the triode pair at 10 mA, so the bias current for each triode was 5 mA. (The transconductance was not computer here, because for triodes the plate current depends also on plate-cathode voltage.) With no feedback (cathode resistors omitted) we obtain a gain of 10 with plate resistors of 1530 Ω.

We adjusted the input signal levels for the FET and triode pairs so that, without feedback, the strengths of the strongest distortion products would be about the same as that of the corresponding lines in the BJT spectrum without feedback. Where the BJTs were running at 0.005 peak volts in each input component, the FETs could handle 0.15 peak volts and the triodes 0.35 peak volts (a mighty big input signal!)

Then we adjusted the amount of feedback in the FET and triode circuits so that, again, the distortion products would come as close in level as possible to those of the BJT with feedback. For the FETs, this was accomplished with source resistors of 2000 Ω; and the drain resistors were then chosen to be 25,000 Ω to give the desired gain of 10. For the triodes, this required cathode and plate resistors of 400 Ω and 7500 Ω, respectively.

The spectra for all three devices are remarkably similar, reflecting the good behavior of differential-pair topology with all three types of devices. The FET pair (figures 24 and 25) show minor differences from the BJTs in the second tier. The triodes produced a second tier with higher levels than

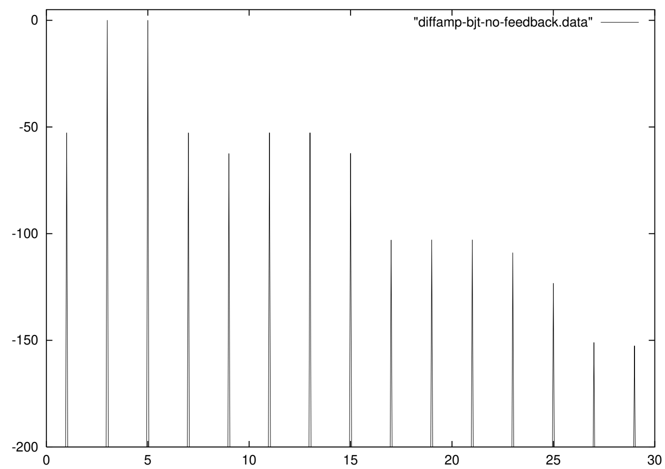

Figure 24: The spectrum of the BJT differential pair without feedback; the two-tone input has 0.005 peak volts in each component.

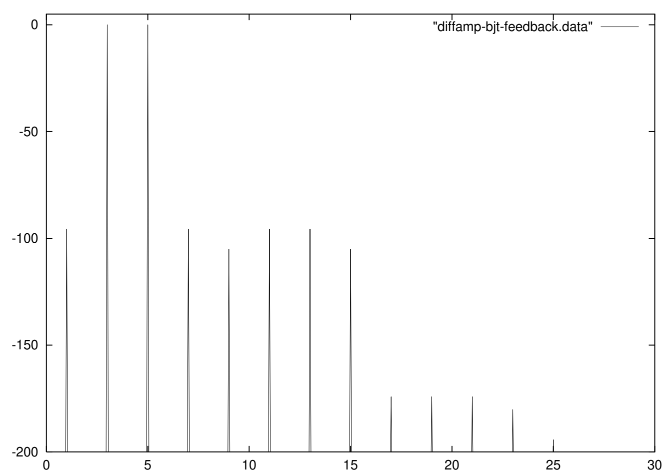

Figure 25: The spectrum of the BJT differential pair with feedback; the two-tone input has 0.005 peak volts in each component.

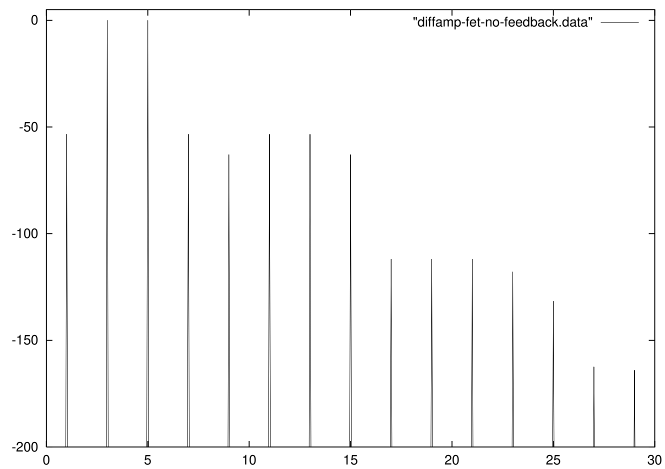

Figure 26: The spectrum of the FET differential pair without feedback; the two-tone input has 0.15 peak volts in each component.

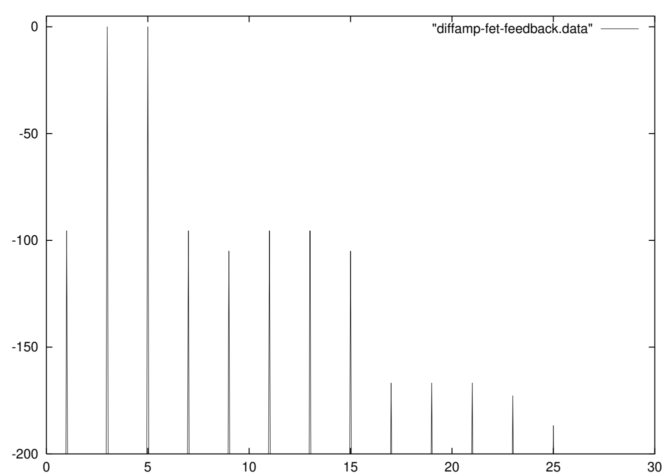

Figure 27: The spectrum of the FET differential pair with feedback; the two-tone input has 0.15 peak volts in each component.

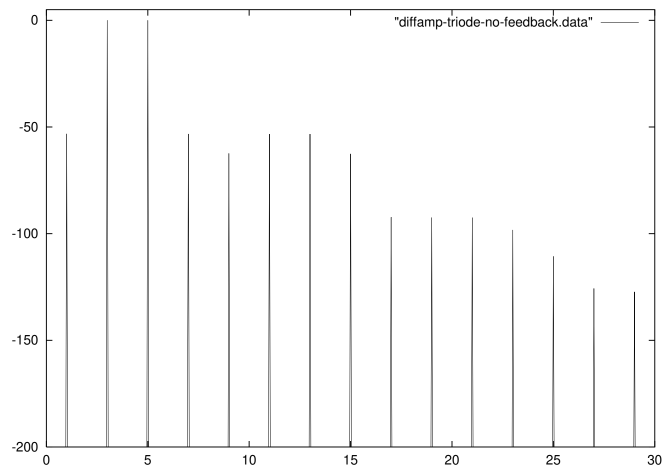

Figure 28: The spectrum of the triode differential pair without feedback; the two-tone input has 0.35 peak volts in each component.

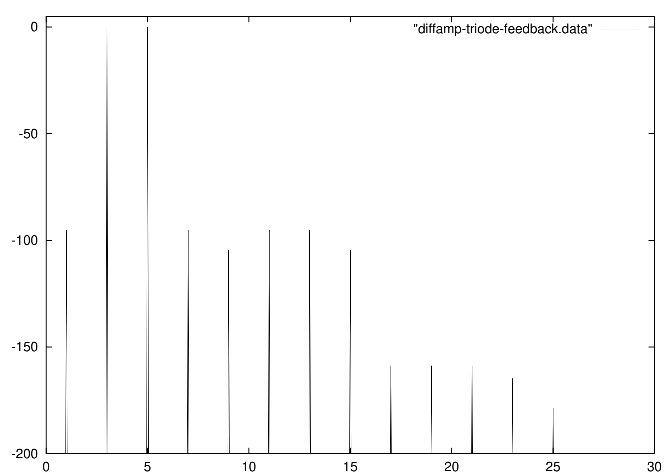

Figure 29: The spectrum of the triode differential pair with feedback; the two-tone input has 0.35 peak volts in each component.

the BJT or FETs, but with feedback they were all down more than 150 dB.

Further experiments indicate that the distortion only decreases with decreasing signal strength, and that feedback only helps.

Summary of differential-input amplifiers

For all cases, input signals were at frequencies 3 and 5 (unscaled). Input voltage levels were always equal for the two components and were as given below. Unless otherwise stated, (a) Higher input level raises the relative distortion and (b) emphasizes higher-order distortion products. (c) Adding more feedback lowers all distortion products.

1. BJT: 0.005 peak volts in each input component

2. FET: 0.15 peak volts in each input component

3. Triode: 0.35 peak volts in each input component

Discussion

In all cases, with and without feedback, intermodulation terms dominate the harmonic distortion terms of the same order. We all know this must be true, but we usually forget it.

Feedback generally improves the intermodulation behavior of the amplifier fragments we have examined, but the exceptions are interesting: the performance of a single-ended FET stage can be made significantly messier with feedback; and strange things happen when we apply feedback to a FET class-B complementary pair.

The tube amplifier fragments start off nicer than BJTs and FETs—and they keep the advantage of handling bigger signals for a given amount of distortion—but BJTs can be made very nice with lots of feedback. And a class-A FET complementary pair is distortionless, if you can find perfectly-matched FETs that perfectly follow the theoretical square law. Unfortunately, it is hard to find well-matched FETs, and the FET model that this conclusion is based on is rather crude. We do not know what happens with a more accurate model of any particular FET pair.

The spurious signals generated by feedback in the single-ended FET amplifier, and enhanced by feedback in the class-B FET pair, are correlated with the program material, since they are constructed from sums and differences of integer multiples of the program-material frequencies. We do not know whether or not this program-correlated noise is psychoacoustically significant, but its presence is certainly suggestive.

So high-order products can be enhanced by feedback in some simple amplifier stages. Although we have not investigated whether or not this happens in more complex amplifiers, it is tempting to relate this finding to the perception of feedback introducing what can sound like a badly-integrated "super-tweeter". Similarly, the fact that feedback can sometimes increase the relative distortion of very low-level signals makes it tempting to relate this finding to the loss of fine detail and room sounds. Whether or not these are appropriate attributions can be determined only by psychoacoustic experiments.

Acknowledgments

The authors thank Dan Boyk, Denny Freeman, Jerry Lettvin, Piotr Mitros, Lynn Olson, Jim Roberge, Joel Schindall, Peter Sutheim, Rick Walker, and Keith Winstein for useful discussions, arguments, and suggestions.

Bibliography

[1] Amperex specifications for tube type 6DJ8/ECC88, September 1960.

[2] James Boyk. "There's Life Above 20 Kilohertz! A Survey of Musical Instrument Spectra to 102.4 kHz," `http://www.cco.caltech.edu/~boyk/spectra/spectra.htm`.

[3] Daniel H. Cheever, *A New Methodology for Audio Frequency Power Amplifier Testing Based on Psychoacoustic Data that Better Correlates with Sound Quality*, S.M. Thesis, University of New Hampshire, December 2001.

[4] Norman Crowhurst, "Some Defects in Amplifier Performance Not Covered by Standard Specifications," in *Journal of the Audio Engineering Society*, October 1957, pp. 195–201.

[5] S. Holmes, G. J. Sussman, J. Boyk, *On the Characteristic Spectra of 28 Varieties of Feedback*, Cambridge: Beekeepers Press, 1899.

[6] W. Marshall Leach Jr, "Spice Models for Vacuum Tubes Amplifiers," in *Journal of the Audio Engineering Society*, March 1995, p. 117.

[7] Matti Otala, "Transient distortion in transistorised audio power amplifiers," in *IEEE Trans. AU-18*, **3**, 1970.

[8] T. Roddam, "Calculating transient response," in *Wireless World*, **58**, 8, August 1952, p. 292.

[9] D.E.L. Shorter, "The influence of High-Order Products in Non-Linear Distortion," in *Electrical Engineering*, April 1950, pp. 152–153.