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Relativistic Gravitational Effects in Pulsars

The most amazing feature of the pulsating radio sources (" pulsars ") is the long-term constancy of the interval between pulses. The best studied of the pulsars, CP 1919, emits a 30-millisecond-long burst of radio waves once every 1.33730113 seconds; and any long-term drift in its " period " must be less than $\sim 10^{-7}$ second per year. This corresponds to a fractional change in period during each pulse of

$$\left(\frac{1}{P}\frac{dP}{dt}\right)P = \frac{dP}{dt} \lesssim 10^{-14}. \tag{1}$$

This long-term stability of the period has led astronomers to separate the theoretical explanation of pulsars into two parts: (i) the " clock mechanism ", which regulates the timing of the pulses; and (ii) the " emission mechanism ". In this comment I shall describe why the clock mechanism probably involves relativistic gravitational effects.

To produce such phenomenal stability, the clock must have a structure which is very constant in time; i.e., it must be a star, or two stars orbiting each other, rather than an amorphous configuration of matter and magnetic fields. The periodicity of the clock could be due to the pulsation or rotation of a single star, or to the orbiting of two bodies. No other possibilities have been suggested—except " little green men ".

Normal stars have pulsation, rotation, and orbital periods of hours or days or longer. The only types of stars which have characteristic periods of ~ 1 second are types which have never been identified before in nature, but which *should* exist, according to firmly based theoretical calculations. These are highly compact white dwarfs (radius between 1000 and 4000 kilometers, mass between 0.9 and 1.2 solar masses), and neutron stars (radius between 8 and 300 kilometers, mass between 2 and 0.1 solar masses).

Thus it is that pulsar models generally make use of neutron stars or compact white dwarfs, which are pulsating, rotating, or orbiting. To lovers of general relativity theory like me, this is a very exciting state of affairs, because relativistic modifications of Newtonian theory should be important in all neutron stars and compact white dwarfs!

Consider, first, orbital models. Here the most important relativistic effect is gravitational radiation. If general relativity is the correct relativistic theory of gravitation, then a binary star system must radiate gravitational waves, which carry off energy and cause the two stars to spiral in toward each other. As a consequence, the orbital period P_0 must decrease at the rate[1]

$$\frac{dP_0}{dt} = -\frac{96}{5}(2\pi)^{8/3}\left[\frac{G^5 m_1^3 m_2^3/(m_1+m_2)}{c^{15}P_0^5}\right]^{1/3}\left[\frac{1+(73/24)e^2+(37/96)e^4}{(1-e^2)^{7/2}}\right]$$

$$\approx 3\times 10^{-6} \qquad (2)$$

for $P_0 \approx 1$ sec, $m_1 \approx m_2 \approx M_\odot$, and $e = 0$.

Here m_1 and m_2 are the masses of the two stars and e is the eccentricity of their orbit. For all reasonable choices of masses and eccentricity, this predicted change in period is much faster than the observed limit on changes in pulsar periods [Eq. (1)]![2]

Proponents of orbital models[3] have sought to circumvent this difficulty by appealing to Herman Bondi's 1962 criticism of the simple-minded analysis by which Eq. (2) is usually derived, and to his suggestion that bodies orbiting each other might not radiate at all.[4] However, Bondi's remarks are now six years old. Since he wrote them, improved analyses[5] have greatly strengthened our understanding of the coupling of gravitational waves to their sources, and have greatly strengthened our confidence that general relativity predicts gravitational radiation from binary star systems.

Other attempts to circumvent radiation damping in orbital pulsar models appeal to the shaky experimental foundations of general relativity. However, whatever may be the correct relativistic theory of gravitation—general relativity or some other theory—it will almost certainly predict the existence of gravitational radiation. Why? Experiment reveals that a sudden change in the shape of a body produces a change in its gravitational influence. Causality and local Lorentz invariance demand that this change in gravitational influence propagate with a speed less than or equal to the velocity of light—i.e., as a " wave ". In a wide variety of relativistic theories of gravity[6]—and perhaps even in *all* such theories—the wave carries off energy from the emitting system at a rate which, in order of magnitude, is the same as that predicted by general relativity.

The effects of gravitational radiation have forced attention away from orbital models of pulsars, to pulsational and rotational models. For pulsating, rotating, compact white dwarfs, relativity has a negligible ($\lesssim 0.1$

percent) effect on the stellar structure. However, relativistic gravitational effects are crucial in determining whether the star is stable against gravitational collapse and, if so, what its fundamental period of pulsation is. A compact white dwarf (density above 10^8 g/cm^3) has an adiabatic index very close to 4/3,

$$\Gamma_1 = 4/3 + \left(\frac{5.35 \times 10^5 A/Z \text{ g/cm}^3}{\text{density}}\right)^{2/3}. \tag{3}$$

(Here A/Z is the ratio of atomic weight to charge for the white-dwarf matter.) This means that when the star is compressed homologously ($\delta r/r$ independent of radius), the increased inward gravitational force on each fluid element is almost precisely balanced by the increased outward pressure force; and when the star is expanded, there is a similar near balance between the gravitational and pressure perturbations. In Newtonian theory the gravitational force is slightly less sensitive to compressions and expansions than is the pressure of white-dwarf

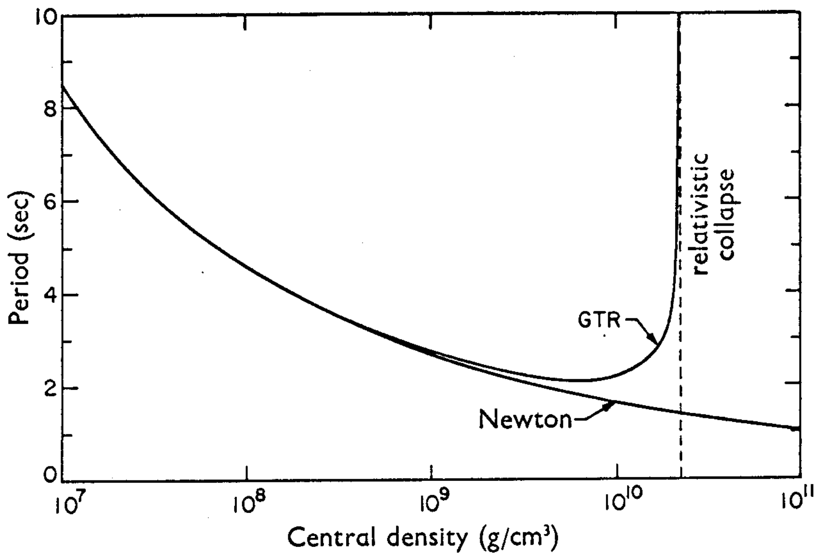

FIG. 1. Pulsation period as a function of central density for the fundamental radial mode of a white dwarf with $A/Z = 2$ (a white dwarf made of He^4, C^{12}, O^{16}, or Mg^{24}). The periods shown are predicted by Newton's theory of gravitation, and by the general theory of relativity. Beyond a central density of 2×10^{10} g/cm^3 (below a radius of ~1000 km) general relativistic effects produce an instability against gravitational collapse. For some chemical compositions collapse can also be triggered by pressure-induced electron capture in the stellar interior. [This figure is based on numerical studies by J. Faulkner and J. R. Gribben, and by W. J. Cocke and J. M. Cohen (see Ref. 7).]

matter, so the net force acts to restore equilibrium and produce stellar pulsations. In general relativity theory the sensitivity of gravity to compressions and expansions is a little greater than in Newtonian theory; consequently the restoring force (sum of gravitational and pressure perturbations) will be *markedly* less than in Newtonian theory, and the star will pulsate more slowly or even collapse. This effect is illustrated in Fig. 1 for nonrotating, homogeneous white dwarfs made of matter with $A/Z = 2$. Notice that the fundamental pulsation period for such stars can be arbitrarily small in Newtonian theory, but cannot be less than 2.14 seconds in general relativity theory. In other relativistic theories of gravitation the story might be different from either of these.

For a rotating white dwarf, centrifugal forces help to stabilize the pulsations, and permit fundamental pulsation periods as low as 0.6 second.[8] However, to obtain pulsation periods as low as 0.25 second (the period of pulsar CP 0950) or lower, one may have to turn from the fundamental mode to " overtone " modes.

The overtone periods of a given white-dwarf model are unaffected by relativity because the " nonhomologous " fluid displacements ($\delta r/r \neq$ constant) of the overtones produce gravitational perturbations which are markedly weaker than the pressure perturbations. However, there is a minimum possible period (usually less than 0.25 second) for each overtone, since no real star can be so compact that its fundamental mode is unstable.

The *rotational* periods of white dwarfs are limited by two factors: (i) the onset of collapse due to relativistic effects or nuclear reactions, which puts a lower limit on the radius of a white dwarf, and (ii) the demand that gravitational acceleration exceed centrifugal acceleration at the star's surface (no mass shedding). The point of onset of collapse is not known for rapidly, differentially rotating white dwarfs. However, calculations to date[9] suggest that white dwarfs can perhaps not have surface rotation periods as low as 0.25 second (except very near their poles).

Neutron-star models are much more compact than white dwarfs; and, hence, relativity affects their structure much more than the structure of a white dwarf. The relativistic effects on structure range from ~1 percent for the least massive, least compact neutron stars ($M \sim 0.1 M_\odot$, $R \sim 300$ km), to ~100 percent for the most massive, most compact ones ($M \sim 1 M_\odot$, $R \sim 10$ km).[10]

When pulsating, a neutron star experiences pressure and gravitational perturbations which do not balance so precisely as in a white dwarf. Consequently, the effect of relativity on the neutron-star pulsation is *not* magnified relative to its effect on the structure. Both structure and pulsation exhibit relativistic effects of the same magnitude— ~1 percent to ~100 percent, depending on the compactness of the star.

Unfortunately for lovers of relativity, the pulsation periods of highly compact, strongly relativistic neutron-star models are $\sim 10^{-3}$ to 2×10^{-4} second. The only neutron stars which should pulsate as slowly as the pulsars are those with low mass ($M \sim 0.1 M_\odot$), low central density ($\rho_c \sim 3 \times 10^{13}$ g/cm^3), large radius ($R \sim 200$ km), and small relativistic effects (~1 percent).

On the other hand, any neutron-star model—highly relativistic or not—can rotate with a period of one second. The minimum possible rotation period, corresponding to mass-shedding at the equator, is ~0.3 second for the least relativistic models and $\sim 3 \times 10^{-4}$ second for the most relativistic.

A neutron star—or white dwarf—pulsating nonradially, or deformed by rotation and pulsating radially, should emit gravitational waves. For the quadrupole pulsations of highly relativistic neutron stars (pulsation period $\sim 2 \times 10^{-4}$ sec), the radiation damps the pulsations in a time of ~1 second.[11] However, for the slower stellar pulsations used in pulsar models, the damping time is enormous ($\gtrsim 10^4$ years). Because the pulsation period—unlike an orbital period—is highly insensitive to damping, radiation reaction presents no problem for the constancy of the periods in pulsar models based on neutron stars. On the other hand, the gravitational waves from such pulsar models may be strong enough to be detectable at the Earth![12]

Recent observations of two pulsars by Drake and Craft[13] suggest quite strongly that, in addition to the highly stable period of ~1 second, there is a much less stable period of ~0.01 second. To account for this, Drake and Craft suggest that the "clock" is a pulsating, rotating neutron star with rotation period ~1 second and pulsation period ~0.01 second. Such a star, pulsating in its fundamental mode, would have the following properties[10]:

Central density $\sim 3 \times 10^{13}$ to 1×10^{14} g/cm^3;

Mass ~0.1 to 0.2 solar masses;

Radius ~50 to 200 km;

Gravitational binding: unbound relative to dispersed iron atoms, bound relative to dispersed hydrogen atoms;

Relativistic effects on structure ~2 or 3 percent at star's center;

Mass distribution: highly centrally condensed, with a large, diffuse envelope;

Radius of gyration ~8 to 12 km;

Quadrupole moment due to rotation $\sim (3 \text{ to } 4) \times 10^{-4} M_{\odot}$ km^2;

Energy of rotation $\sim 10^{46}$ ergs;

Energy of pulsation $\sim (10^{47}$ to 10^{48} ergs$) \times (\delta R/R)^2$;

Shape of pulsation eigenfunction: small amplitude at star's center, large at surface—$(\delta R/R) \sim (300 \text{ to } 3000) \times (\delta r/r)_{\text{center}}$;

Damping time for pulsations due to gravitational waves, caused by rotational coupling of quadrupole modes to fundamental radial mode: $\sim 10^5$ to 10^8 years.

In this article no attempt was made to give a fair treatment of the relative merits of various pulsar models. Rather, attention was concentrated only on some of the relativistic effects that permeate the models. Looking into the future, one can hope for two major steps in studies of relativistic aspects of pulsars: (i) the clear delineation of which clock mechanism is correct and of which relativistic gravitational effects (if any!) should be important for it; and (ii) observational verification that the relativistic effects are present. Although we might be fairly optimistic about step (i), it is far from clear whether step (ii) can be achieved in the near future. The observational unravelling of relativistic effects from effects due to equation of state, for example, might be a tremendous job.

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