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GRAVITATIONAL WAVES FROM COALESCING BLACK HOLE MACHO BINARIES

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ABSTRACT

If MACHOs are black holes of mass ∼0.5 $M_\odot$, they must have been formed in the early universe when the temperature was ∼1 GeV. We estimate that in this case in our Galaxy's halo out to ∼ 50 kpc there exist ∼5 × 10^8 black hole binaries the coalescence times of which are comparable to the age of the universe, so that the coalescence rate will be ∼5 × 10^{-2} events yr^{-1} per galaxy. This suggests that we can expect a few events per year within 15 Mpc. The gravitational waves from such coalescing black hole MACHOs can be detected by the first generation of interferometers in the LIGO/VIRGO/TAMA/GEO network. Therefore, the existence of black hole MACHOs can be tested within the next 5 yr by gravitational waves.

Subject headings: black hole physics — dark matter — gravitation — gravitational lensing — Galaxy: halo

1. INTRODUCTION

The analysis of the first 2.1 yr of photometry of 8.5 million stars in the Large Magellanic Cloud (LMC) by the MACHO collaboration (Alcock et al. 1996) suggests that $0.62^{+0.3}_{-0.2}$ of the halo consists of MACHOs of mass $0.5^{+0.3}_{-0.2}\ M_\odot$ in the standard spherical flat rotation halo model. The preliminary analysis of 4 yr of data suggests the existence of at least four additional microlensing events with $t_{\rm dur}$ ∼ 90 days in the direction of the LMC (Pratt 1997). The estimated masses of these MACHOs are just the mass of red dwarfs. However, the contribution of the halo red dwarfs to MACHO events should be small since the observed density of halo red dwarfs is too low (Bahcall et al. 1994: Graff & Freese 1996a, 1996b). As for white dwarf MACHOs, the mass fraction of white dwarfs in the halo should be less than 10% since, assuming the Salpeter initial mass function (IMF), the bright progenitors of more white dwarfs than this would be in conflict with the number counts of distant galaxies (Charlot & Silk 1995). If the IMF has a sharp peak around 2 $M_\odot$, then the fraction could be 50% or so (Adams & Laughlin 1996), sufficient to explain the MACHO observations. The existence of such a population of halo white dwarfs may or may not be consistent with the observed luminosity function (Gould 1997; Lidman 1997; Freese 1997). In any case, future observations of high-velocity white dwarfs in our solar neighborhood (Lidman 1997) will prove whether white dwarf MACHOs can exist or not.

If the number of high-velocity white dwarfs turns out to be large enough to explain the MACHOs, then stellar formation theory should explain why the IMF is sharply peaked at ∼2 $M_\odot$. If it is not, there arises a real possibility that MACHOs are absolutely new objects such as black holes of mass ∼0.5 $M_\odot$ which could only be formed in the early universe, or boson stars with the mass of the boson ∼10^{-10} eV. Of course, it is still possible that an overdense clump of MACHOs exists toward the LMC (Nakamura, Kan-ya, & Nishi 1996), MACHOs are brown dwarfs in the rotating halo (Spiro 1997), or MACHOs are stars in the thick disk (Turner 1997).

In this Letter we consider the case of black hole MACHOs (BHMACHOs). In this case, there must be a huge number (at least ∼4 × 10^{11}) of black holes in the halo, and it is natural to expect that some of them are binaries. In § 2 we estimate the fraction $f(a, e)da\,de$ of all BHMACHOs that are in binaries with semimajor axis a in range da and eccentricity e in de. We then use this distribution to estimate two observable event rates. First (at the end of § 2), the rate of microlensing events we should expect toward the LMC that is due to binaries with separation ≳2 × 10^{14} cm; our result is in accord with the observation of one such event thus far (Bennett et al. 1996). Second (§ 3), the rate of coalescence of BHMACHO binaries out to 15 Mpc distance. The gravitational waves from such coalescences should be detectable by the first interferometers in the LIGO/VIRGO/TAMA/GEO network (Barish 1997; Brillet 1996; Tsubono 1996; Hough 1996), and our estimated event rate is a few events per year. In § 4 we discuss some implications of our estimates.

2. FORMATION OF SOLAR MASS BLACK HOLE MACHO BINARIES

Since it is impossible to make a black hole of mass ∼0.5 $M_\odot$ as a product of stellar evolution, we must consider the formation of solar mass black holes in the very early universe (Yokoyama 1997; Jedamzik 1997). Our viewpoint here, however, is not to study detailed formation mechanisms but to estimate the binary distribution that results.

The density parameter of BHMACHOs, $\Omega_{\rm BHM}$, must be comparable to Ω_b (or $\Omega_{\rm CDM}$) to explain the number of observed MACHO events. For simplicity, we assume that BHMACHOs dominate the matter energy density, i.e., $\Omega = \Omega_{\rm BHM}$, although it is possible to consider other dark matter components in addition to BHMACHOs. To determine the mean separation of the BHMACHOs, it is convenient to consider it at the time of matter-radiation equality, $t = t_{\rm eq}$. At this time, the energy densities of radiation and BHMACHOs are approximately equal and are given by $\rho_{\rm eq} = 1.4 \times 10^{-15}(\Omega h^2)^4$ g cm^{-3}, where h is the Hubble parameter in units of 100 km s^{-1} Mpc^{-1}. Correspondingly, the mean separation of BHMACHOs with mass $M_{\rm BH}$ at this time is given by

$$\bar{x} = (M_{\rm BH}/\rho_{\rm eq})^{1/3} = 1.1 \times 10^{16}\,(M_{\rm BH}/M_{\odot})^{1/3}\,(\Omega h^2)^{-4/3}\ \text{cm}. \quad (1)$$

We set the scale factor R to unity at $t = t_{\rm eq}$, so $\bar{x}$ can also be regarded as the comoving mean separation. Note that the Hubble horizon scale at $t = t_{\rm eq}$ is $L_{\rm eq} \sim (3c^2/8\pi G\rho_{\rm eq})^{1/2} = 1.1 \times 10^{21}(\Omega h^2)^{-2}$ cm.

During the radiation-dominated era, the total energy inside the horizon increases as R^2. Since the Jeans mass in this era is essentially the same as the horizon mass, black holes are formed only at the time when the horizon scale is equal to the Schwarzschild radius of a BHMACHO. Thus, the scale factor at the formation epoch becomes

$$R_f = \sqrt{\frac{GM_{\rm BH}}{c^2 L_{\rm eq}}} = 1.2 \times 10^{-8}\left(\frac{M_{\rm BH}}{M_{\odot}}\right)^{1/2}(\Omega h^2). \quad (2)$$

The age of the universe and the temperature T_f at $R = R_f$ are $\sim 10^{-5}$ s and ~ 1 GeV, respectively.

As a foundation for computing the distribution function $f(a, e)$ for BHMACHO binaries, we assume that the BHMACHOs are created with a distribution of comoving separations x that is uniform over the range from an initial physical separation equal to the black hole size (which turns out to be so small that for the computations that follow we can approximate it as zero) to a maximum separation $x = \bar{x}$. We also assume that the BHMACHO motions, if any, relative to the primordial gas have been redshifted to negligible speeds by the time their mutual gravitational attractions become important.

Consider a pair of black holes with mass $M_{\rm BH}$ and a comoving separation $x < \bar{x}$. The masses of these black holes produce a mean energy density over a sphere the size of their separation given by $\rho_{\rm BH} \equiv \rho_{\rm eq}(\bar{x}/xR)^3$. This becomes larger than the radiation energy density $\rho_r = \rho_{\rm eq}/R^4$ for

$$R > R_m \equiv (x/\bar{x})^3. \quad (3)$$

This means that the binary decouples from the cosmic expansion and becomes a bound system when $R = R_m$. Note that the background universe is still radiation dominated at this stage.

If the motion of the two black holes is not disturbed, then the binary system cannot obtain any angular momentum, so they coalesce to a single black hole on the free-fall timescale. However, the tidal force from neighboring black holes gives the binary enough angular momentum to keep the holes from colliding with each other unless x is exceptionally small.

We refer to the semimajor axis and the semiminor axis of the binary as a and b, respectively, and we estimate a as

$$a = xR_m = x^4/\bar{x}^3. \quad (4)$$

We denote by y the comoving separation of the nearest neighboring black hole from the center of mass of the binary. Then b can be estimated as (tidal force) $\times$ (free-fall time)^2,

$$b = \frac{GM_{\rm BH}xR_m}{(yR_m)^3}\frac{(xR_m)^3}{GM_{\rm BH}} = \left(\frac{x}{y}\right)^3 a. \quad (5)$$

Hence, the binary's eccentricity e is given by

$$e = \sqrt{1-(x/y)^6}. \quad (6)$$

Since (by assumption) x and y have uniform probability distributions in the range $x < y < \bar{x}$, the probability distribution of a and e is

$$\begin{aligned} f(a, e)da\,de &= 18x^2y^2\bar{x}^{-6}dx\,dy \\ &= (3/2)a^{1/2}\bar{x}^{-3/2}e(1-e^2)^{-3/2}da\,de. \end{aligned} \quad (7)$$

From the condition that $y < \bar{x}$, the maximum value of the eccentricity for a fixed a is given by $e_{\rm max} = [1-(a/\bar{x})^{3/2}]^{1/2}$. Integrating $f(a, e)$ with respect to e, we obtain the following distribution of the semimajor axis:

$$f_a(a)da = \frac{3}{2}\left[\left(\frac{a}{\bar{x}}\right)^{3/4} - \left(\frac{a}{\bar{x}}\right)^{3/2}\right]\frac{da}{a}. \quad (8)$$

From equation (8), it is found that the fraction of BHMACHOs that are in binaries with $a \sim 2 \times 10^{14}$ cm is ~ 8% and ~ 0.9% for $\Omega h^2 = 1$ and 0.1, respectively. This estimated fraction of ~ 10 AU size BHMACHO binaries is slightly smaller than the observed rate of binary MACHO events (one binary event in eight observed MACHOs), but the agreement is good given the statistics of small numbers.

3. GRAVITATIONAL WAVES FROM COALESCING BHMACHO BINARIES

We consider here short-period BHMACHO binaries. Their coalescence times that result from the emission of gravitational waves are approximately given by (Peters & Mathews 1963; Peters 1964)

$$t = t_0\left(\frac{a}{a_0}\right)^4(1-e^2)^{7/2}, \quad (9)$$

where $t_0 = 10^{10}$ yr and

$$a_0 = 2 \times 10^{11}\left(\frac{M_{\rm BH}}{M_{\odot}}\right)^{3/4}\ \text{cm} \quad (10)$$

is the semimajor axis of a binary with circular orbit which coalesces in t_0. Integrating equation (7) for a fixed t with the aid of equation (9), we obtain the probability distribution for the coalescence time $f_t(t)$ as

$$f_t(t)dt = \frac{3}{29}\left[\left(\frac{t}{t_{\rm max}}\right)^{3/37} - \left(\frac{t}{t_{\rm max}}\right)^{3/8}\right]\frac{dt}{t}, \quad (11)$$

where $t_{\rm max} = t_0(\bar{x}/a_0)^4$. If the halo of our Galaxy consists of BHMACHOs of mass $\sim 0.5\ M_{\odot}$, $\sim 10^{12}$ BHMACHOs exist out to the LMC. The number of coalescing binary BHMACHOs with $t \sim t_0$ then becomes $\sim 5 \times 10^8$ for $\Omega h^2 = 0.1$, so that the event rate of coalescing binaries becomes $\sim 5 \times 10^{-2}$ events yr^{-1} per galaxy. If, however, the BHMACHOs extend up to halfway to M31, the number of coalescing binary BHMACHOs with $t \sim t_0$ can be $\sim 3 \times 10^9$, and the event rate becomes ~ 0.3 events yr^{-1} per galaxy. Both of these estimates are much larger than the best estimate of the event rate of coalescing neutron stars based on the statistics of binary pulsar searches in our Galaxy, $\sim 1 \times 10^{-5}$ events yr^{-1} per galaxy (Phinney 1991; Narayan, Piran, & Shemi 1991; van den Heuvel & Lorimer 1996).

The detectability of these waves by interferometers is most easily discussed in terms of the "characteristic amplitude" h_c

of the waves (eq. [46b] of Thorne 1987, with a well-known factor of 2 correction):

$$h_c = 4 \times 10^{-21} \left(\frac{M_{\rm chirp}}{M_\odot}\right)^{5/6} \left(\frac{\nu}{100\ {\rm Hz}}\right)^{-1/6} \left(\frac{r}{20\ {\rm Mpc}}\right)^{-1} . \quad (12)$$

Here $M_{\rm chirp} = (M_1 M_2)^{3/5}/(M_1 + M_2)^{1/5}$ is the "chirp mass" of the binary for which the components have individual masses M_1 and M_2. This h_c is to be compared with an interferometer's "sensitivity to bursts" $h_{\rm SB} = 11[fS_h(f)]^{1/2}$, where $S_h(f)$ is the spectral density of the strain noise of the interferometer (see eq. [111] of Thorne 1987, where $h_{\rm SB}$ is denoted $h_{3/{\rm yr}}$). This $h_{SB}(f) \equiv h_{3/yr}(f)$ is plotted in various publications, e.g., Abramovici et al. (1989) and Thorne (1995). It has a minimum (optimal sensitivity) at a frequency $f \simeq 100$ Hz. For the first LIGO and VIRGO interferometers, which are expected to be operational in 2001, that minimum is $h_{\rm SB\ min} \simeq 3 \times 10^{-21}$. The GEO600 and TAMA interferometers, with their somewhat shorter armlengths, will have $h_{\rm SB\ min}$ a little worse than this in 2001. LIGO/VIRGO should be able to detect coalescing binaries, with high confidence, out to the distance for which $h_c = h_{\rm SB\ min}$ at the optimal frequency $f \simeq 100$ Hz. Inserting $M_1 = M_2 \simeq 0.5\ M_\odot$ for BHMACHOs into the above equations, we see that *the first LIGO/VIRGO interferometers in 2001 should be able to see BHMACHO coalescences out to about* 15 Mpc *distance*, i.e., out to the Virgo Cluster, where our estimates (~1/100 yr in each galaxy like our own) suggest an event rate of several per year.

LIGO R&D for the first interferometers is now nearing completion and is beginning to be redirected toward interferometer enhancements, for which the sensitivity goal is a factor of 10 improvement, to $h_{\rm SB\ min} \simeq 3 \times 10^{-22}$ (B. Barish et al. 1997, unpublished). In the mid 2000s, with these enhancements in place, LIGO should be able to see BHMACHO coalescences out to about 150 Mpc, which would give a few events per year even if the event rate is 1000 times smaller than our estimates, $\sim 10^{-5}$ events yr^{-1} per galaxy.

4. DISCUSSION

In this Letter we have estimated the distribution function of binary BHMACHOs in order of magnitude. It is possible to compute the distribution function more accurately by *N*-body numerical simulations. This is an important, challenging numerical problem.

Our estimated event rate for coalescing BHMACHO binaries is comparable to or greater than the most optimistic upper limit for binary neutron star coalescences (Phinney 1991), which are one of the most important sources of gravitational waves. Coalescing neutron stars are also regarded as possible sources of the gamma-ray bursts (Mészáros 1995). If so, then the detection of gravitational waves should be accompanied by a gamma-ray burst. If we consider the fireball model (Mészáros 1995), the time delay between the gravitational waves and the gamma rays should be ~1 s. By contrast, in the coalescence of binary BHMACHOs the emission of gamma-rays is not expected. This may enable us to distinguish coalescing binary BHMACHOs from coalescing binary neutron stars.

If gamma rays are not emitted by coalescing binary neutron stars, we may still use their observed chirp masses $M_{\rm chirp}$ to distinguish them from BHMACHO binaries. The chirp mass can be measured from the gravitational waves to a fraction of a percent accuracy, which is much less than the expected spreads of BHMACHO masses and neutron star masses. The masses of neutron stars in binaries are expected to be ~1.4 $M_\odot$, corresponding to a chirp mass of 1.2 $M_\odot$. This is supported observationally as well as theoretically since the mass of the iron core before the collapse is ~1.4 $M_\odot$.

Therefore if the chirp masses of BHMACHO binaries are much smaller than 1 $M_\odot$, it is possible to identify them. However, if the mass of a BHMACHO is ~1 $M_\odot$, coalescing BHMACHO binaries and coalescing neutron star binaries may be indistinguishable from the detected gravitational waves. In principle, however, black holes absorb some of the gravitational waves so that the evolution of the binary in the last few minutes (Cutler et al. 1993) is slightly different from that of a neutron star binary of the same mass. The difference arises in the 2.5th post-Newtonian order for the Kerr black hole case (H. Tagoshi 1997, private communication) and fourth order for the Schwarzschild black hole case (Poisson & Sasaki 1995). This difference might be detectable, although it will present a challenging problem for gravitational wave data analysis.

It is known that the Silk damping of density perturbations on small scales causes distortions of the CMB spectrum by dumping acoustic energy into heat and thence into the CMB (Daly 1991; Barrow & Coles 1991; Hu, Scott, & Silk 1994). Thus it is important to examine whether the large primordial density perturbations that are needed for BHMACHO formation are compatible with the observed upper limit of CMB spectral distortions (Mather et al. 1994).

For a rms amplitude of density perturbations $\delta(l)$ on a comoving scale l in the radiation-dominated era, the fraction of the universe that turns into black holes is given by $f(l) \approx \delta(l) \exp[-1/18\delta(l)^2]$ (Carr 1975). In the present case, $f(l)$ is equal to R_f given by equation (2) and l is equal to the comoving scale $l_{\rm BH}$ of the density perturbations that give rise to BHMACHOs. Recalling that we fixed the scale factor R to unity at the time $t_{\rm eq}$ of matter-radiation equality, the present comoving scale corresponding to $l_{\rm BH}$ is given by $(1 + z_{\rm eq})l_{\rm BH} = (1 + z_{\rm eq})R_f^{-1}\, GM_{\rm BH}c^{-2} = 5 \times 10^{17}(M_{\rm BH}/M_\odot)^{1/2}$ cm, where $z_{\rm eq}$ is the redshift at $t = t_{\rm eq}$. Since $R_f \sim 10^{-8}$, we need $\delta(l_{\rm BH})^2 \sim 4 \times 10^{-3}$.

On the other hand, Hu et al. (1994) showed that the observational limit constrains the rms amplitude of density perturbations at horizon entry to be $\delta(l)^2 \lesssim 10^{-4}$ for $l > l_D$, where l_D is the comoving scale of the Silk damping at the double Compton thermalization time, which at present is given by $(1 + z_{\rm eq})l_D = 3 \times 10^{20}$ cm. Hence the scale of interest $l_{\rm BH}$ is approximately 3 orders of magnitude smaller than the scale l_D to which the observational constraint applies.

Correspondingly, if we assume the primordial density perturbation spectrum to have a power-law shape, $P(k) \propto k^n$, on scales greater than $l_{\rm BH}$, the above result implies that $n \gtrsim 1.6$. Interestingly, this value is consistent with the one suggested by the observed CMB anisotropies (Bennett et al. 1994), and it is marginally consistent with the *COBE* normalization (Hu et al. 1994). It may be that this rather blue spectrum can be produced by a variant of the so-called hybrid model of inflation (Garcia-Bellido, Linde, & Wands 1996).

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Note added in proof.—After this Letter was accepted for publication, we posted it on the Los Alamos preprint server (astro-ph/9708060). L. S. Finn (gr-qc/9609027 [1996]) informed us that his paper also discussed gravitational waves from BHMACHO binaries. He just assumed that 30% of MACHOs are binaries and that the event rate is 15 yr^{-1} per galaxy. From our estimate, this is too large by a factor of 300.