Reconstructing the neutron-star equation of state from gravitational-wave observations

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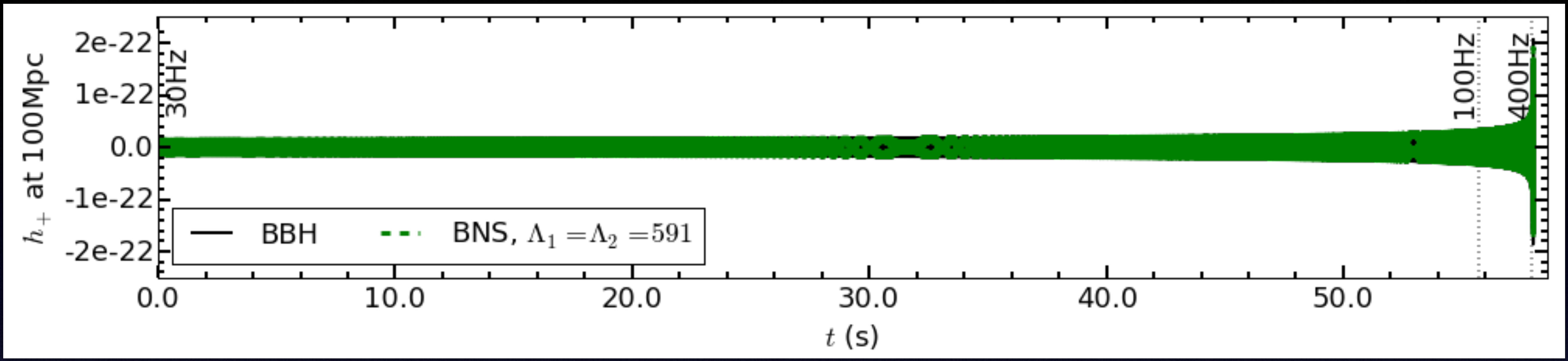
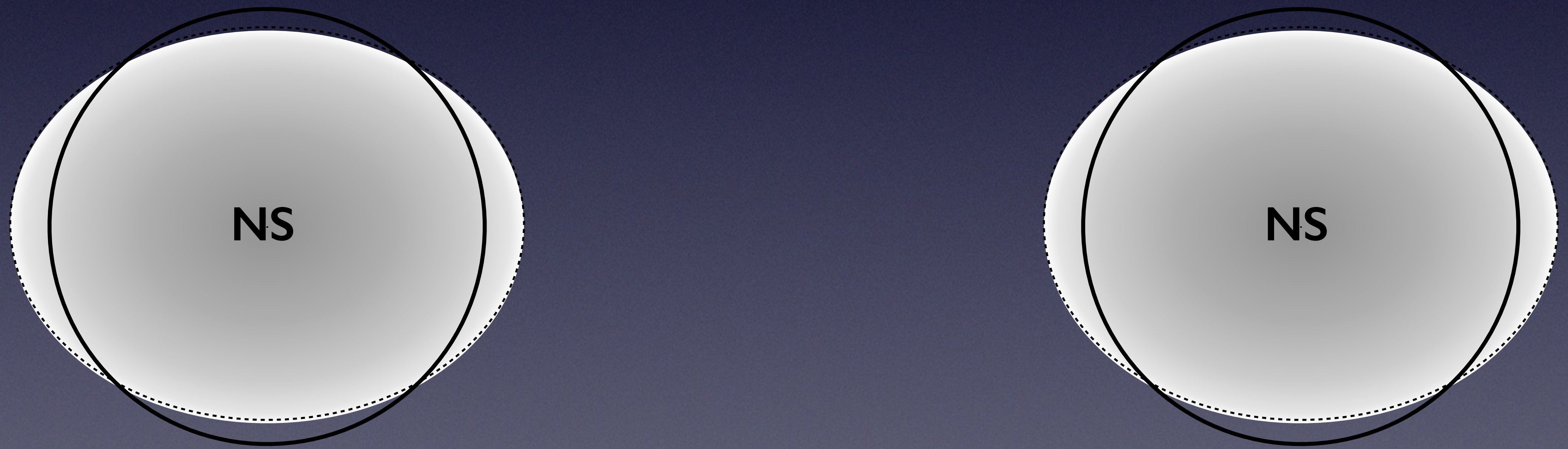
Second generation gravitational-wave detectors

- Will reach design sensitivity around end of decade
- Sensitive to gravitational-waves between ~10 Hz and a few kHz

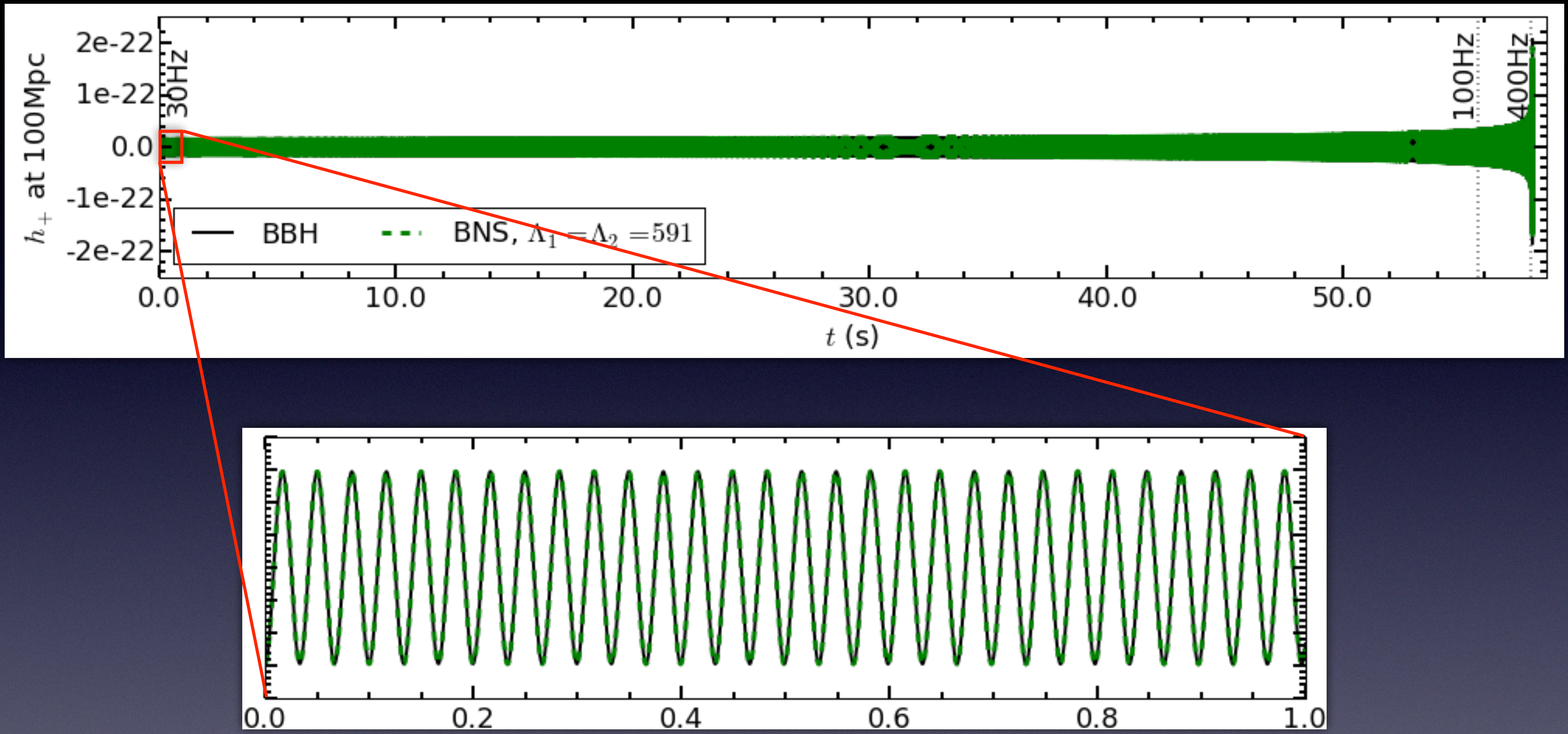
Stages of BNS coalescence





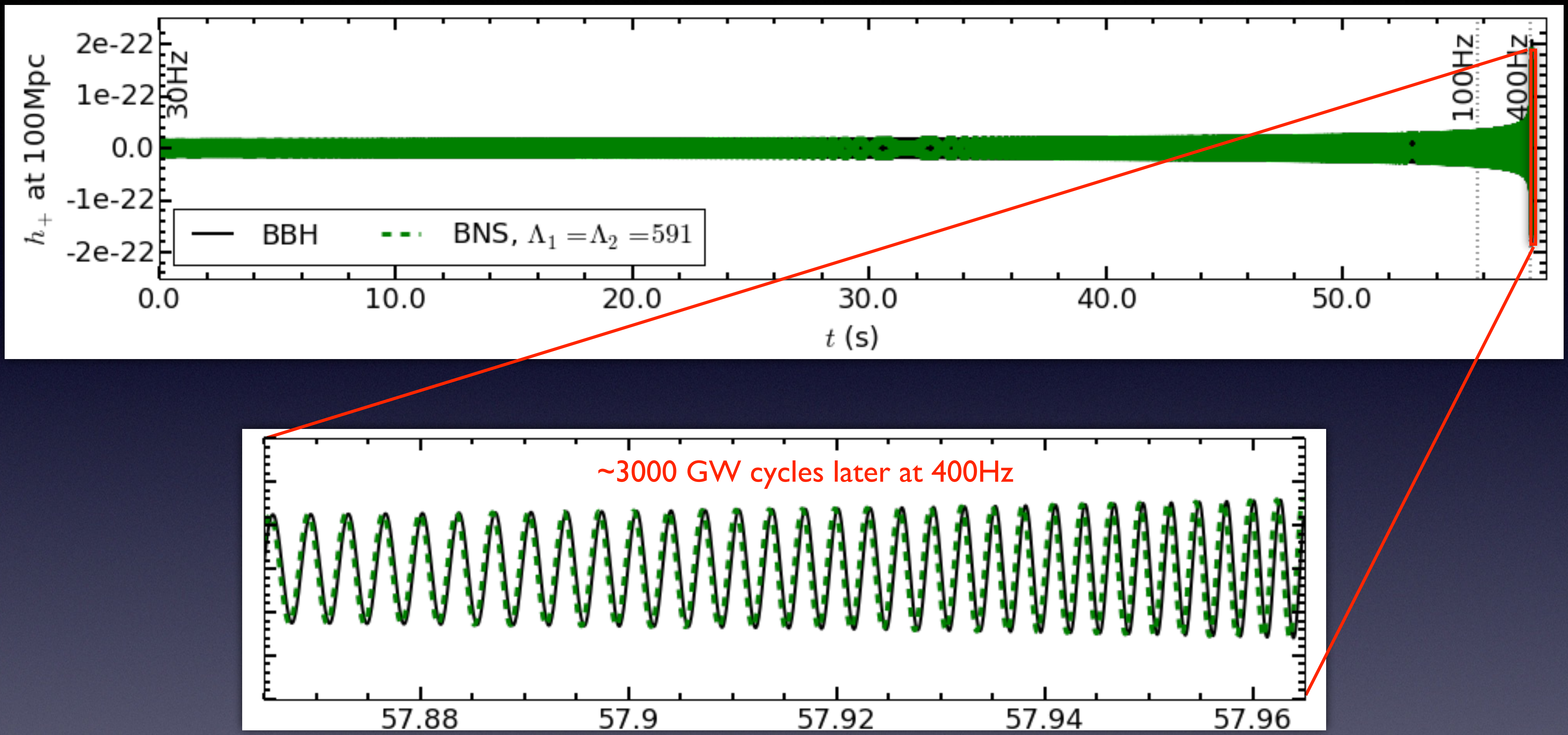
- Advanced LIGO sensitive to last few minutes of inspiral
- ~10^4 gravitational-wave cycles

Stages of BNS coalescence



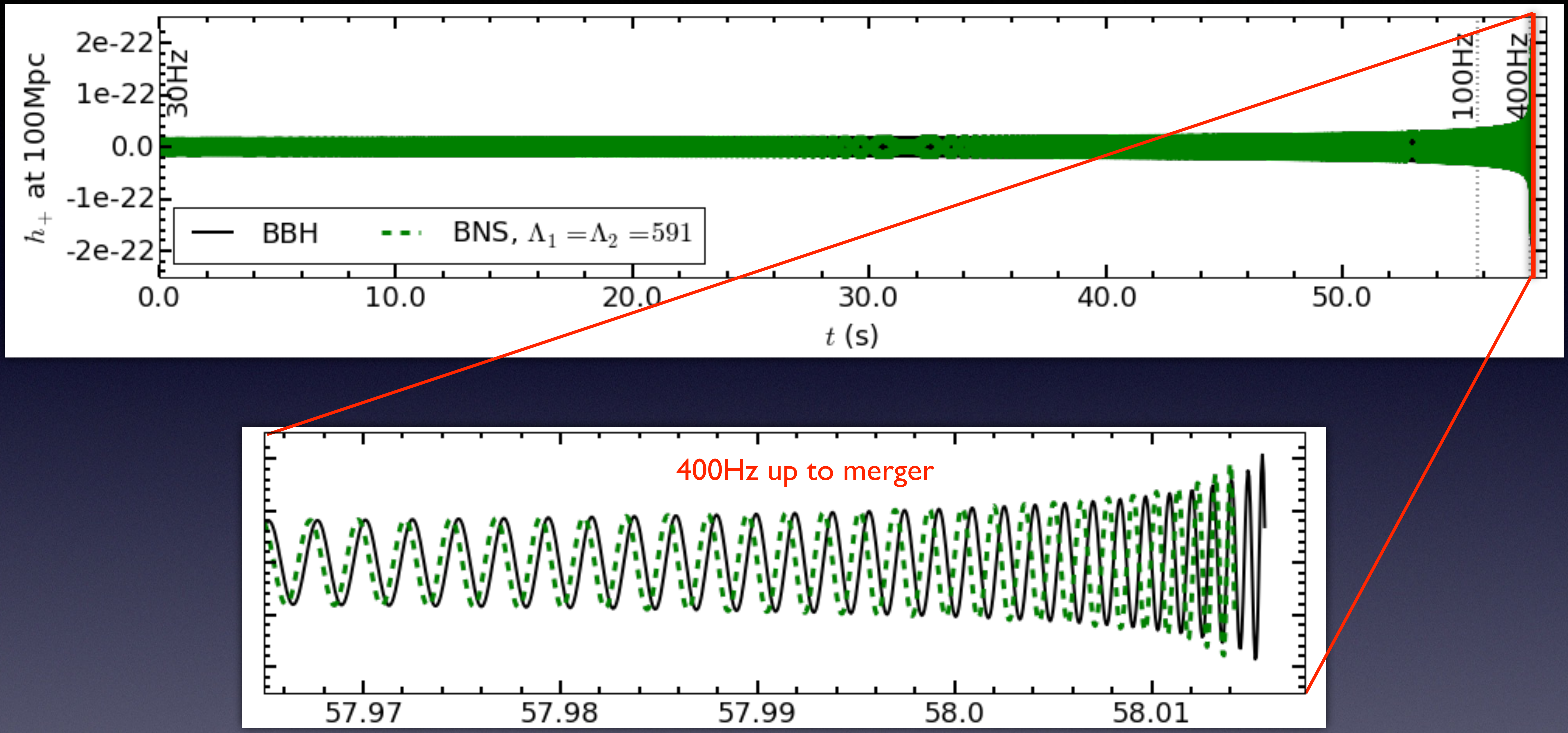
- <u>Early inspiral</u>: Evolution depends on chirp mass $\mathcal{M} = \frac{(m_1 m_2)^{3/5}}{(m_1+m_2)^{1/5}}$ and symmetric mass ratio $\eta = \frac{m_1 m_2}{(m_1+m_2)^2}$

Stages of BNS coalescence



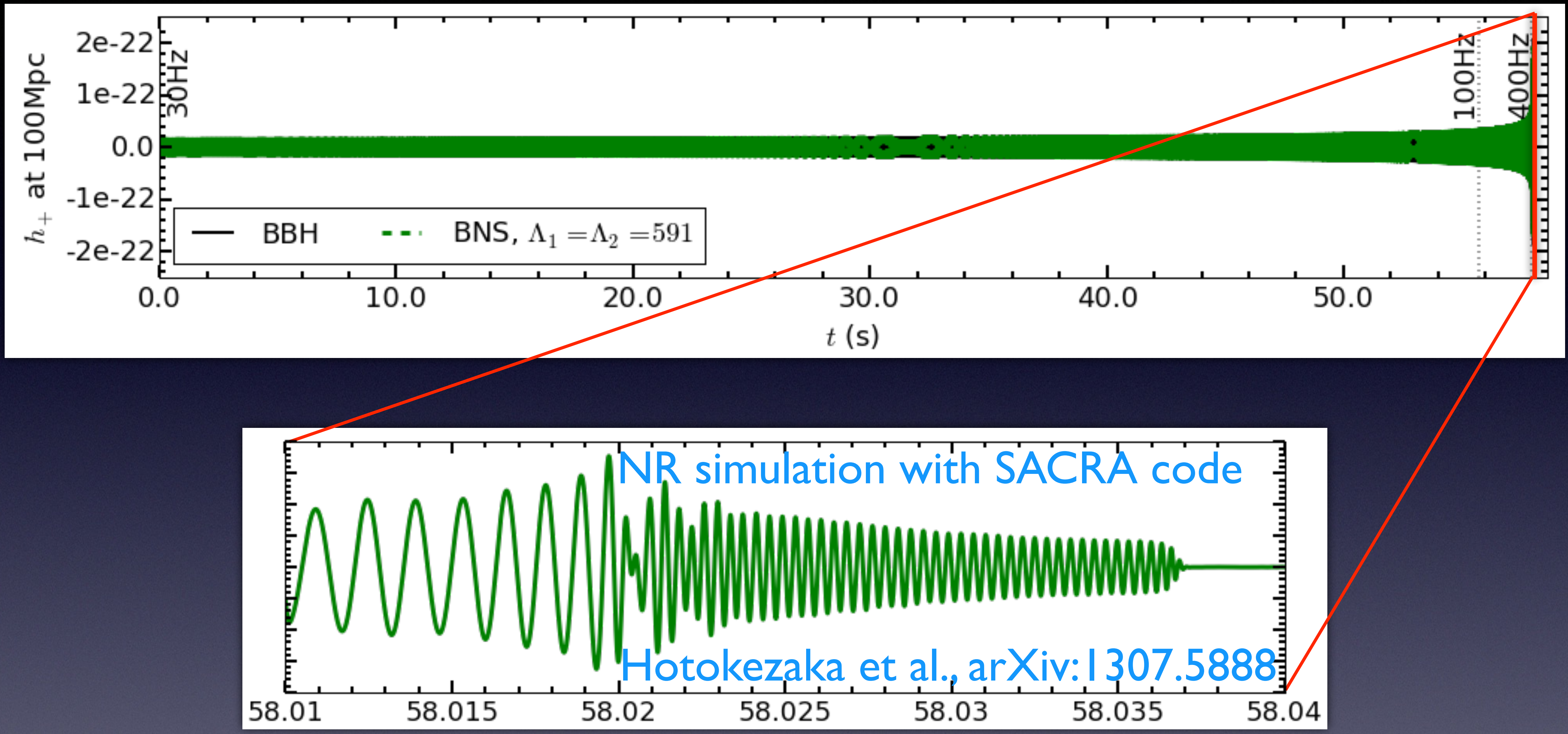
- Late inspiral: EOS-dependent tidal interactions lead to phase shift of ~1 radian up to 400Hz

Stages of BNS coalescence



- Last 20-30 cycles: Tidal interactions lead to phase shift of ~1 GW cycle

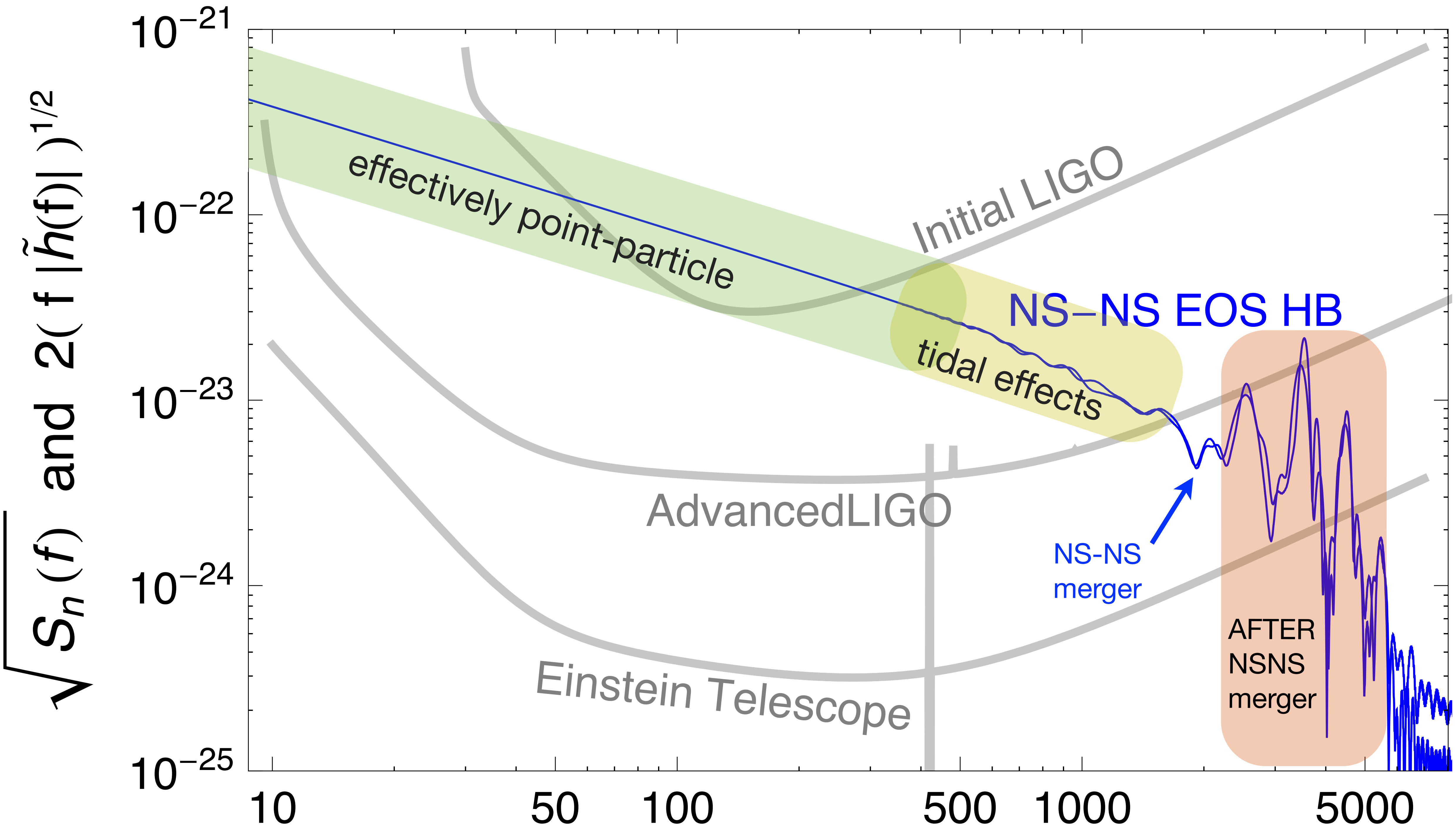
Stages of BNS coalescence



- <u>Post-merger</u>: Frequencies are a few kHz and depend sensitively on EOS

Stages of BNS coalescence

Fourier transform of waveform:



Waveforms from SACRA and WHISKY codes
(Credit: Jocelyn Read, arXiv:1306.4065)

Tidal interactions during inspiral

- Tidal field $\mathcal{E}_{ij}$ of each star induces quadrupole moment Q_{ij} in other star
- Amount of deformation depends on the stiffness of the EOS via the tidal deformability λ

$$\begin{aligned} Q_{ij} &= -\lambda(\mathrm{EOS}, m)\mathcal{E}_{ij} \\ &= -\Lambda(\mathrm{EOS}, m)m^5\mathcal{E}_{ij} \end{aligned}$$

- Interaction increases binding energy
- Additional quadrupole moments increase gravitational radiation

$$\frac{dE}{dt} = -(1/5)\langle \dddot{Q}_{ij}^{\mathrm{total}} \dddot{Q}_{ij}^{\mathrm{total}} \rangle$$

Tidal interactions during inspiral

- Post-Newtonian approximation expands solution to Einstein equations in powers of speed of bodies and compactness of the system:

$$x \equiv \left(\frac{GM\Omega}{c^3}\right)^{2/3} \sim \left(\frac{v}{c}\right)^2 \sim \frac{GM}{c^2 d}$$

- Energy and gravitational-wave luminosity expansions:

1PN–4PN 5PN, 6PN

$$E = -\frac{1}{2}c^2 M\eta x\left[1 + e_{\text{PP-PN}}(x;\eta) + e_{\text{Tidal}}(x;\eta,\Lambda_1,\Lambda_2)\right]$$

1PN–3.5PN 5PN, 6PN

$$\mathcal{L} = \frac{32}{5}\frac{c^5}{G}\eta^2 x^5\left[1 + l_{\text{PP-PN}}(x;\eta) + l_{\text{Tidal}}(x;\eta,\Lambda_1,\Lambda_2)\right]$$

- Orbital evolution found with energy balance:

$$\frac{dx}{dt} = \frac{dE/dt}{dE/dx} = \frac{-\mathcal{L}}{dE/dx}$$

$$\frac{d\phi}{dt} \equiv \Omega = \frac{c^3 x^{3/2}}{GM}$$

- Waveform is then:

$$h_+ + ih_\times \propto \frac{\eta M}{d_L} x(t) e^{2i\phi(t)}$$

Tidal interactions during inspiral

- Tidal parameters encoded in phase evolution of waveform

Amplitude

Phase

$$\tilde{h}(f) = \frac{A(\alpha, \delta, \iota, \psi)}{d_L} \mathcal{M}^{5/6} f^{-7/6} e^{i\psi(f)}$$

Newtonian | 1PN-3.5PN | 5PN | 6PN

$$\psi(f) = 2\pi f t_c - \phi_c - \frac{\pi}{4} + \frac{3}{128(\pi \mathcal{M} f)^{5/3}} \left[1 + \hat{\psi}_{(\mathrm{PP-PN})}(x;\eta) - \frac{39}{2}\tilde{\Lambda} x^5 + \left(-\frac{3115}{64}\tilde{\Lambda} + \frac{6595}{364}\sqrt{1-4\eta}\,\delta\tilde{\Lambda}\right) x^6\right]$$

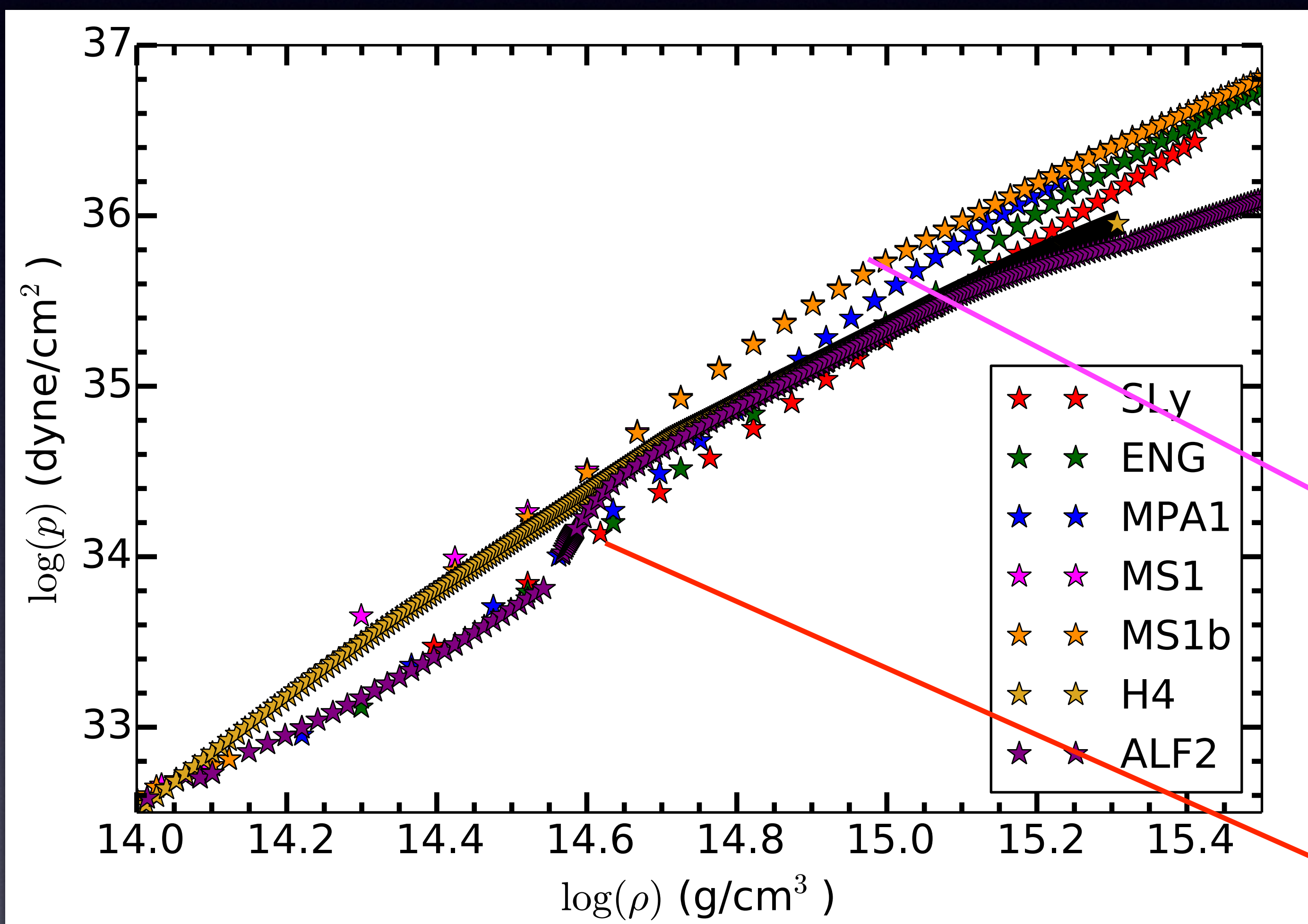
$$x = (\pi M f)^{2/3} \sim \left(\frac{v}{c}\right)^2$$

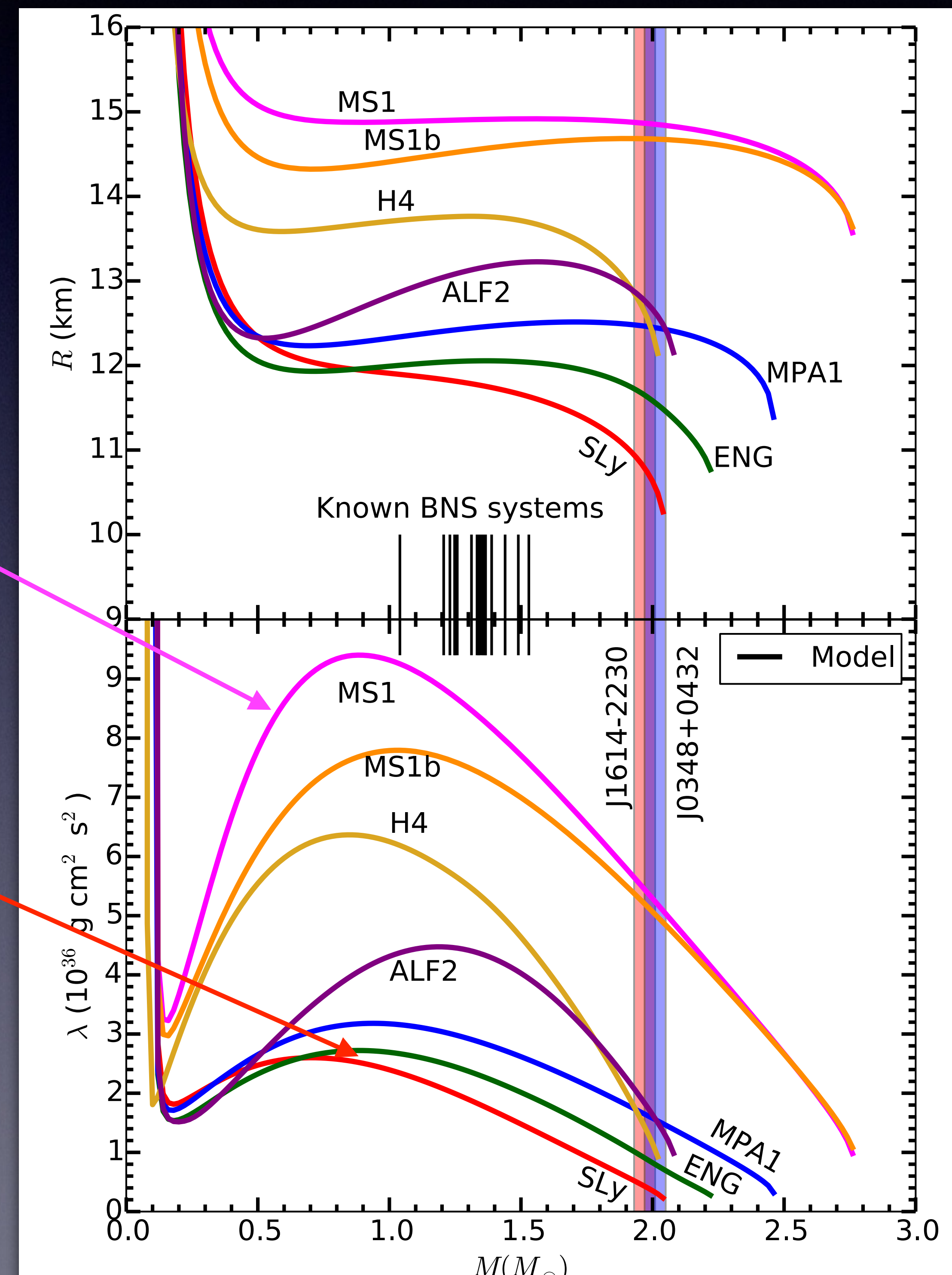
$$\tilde{\Lambda} = \frac{8}{13}\left[(1 + 7\eta - 31\eta^2)(\Lambda_1 + \Lambda_2) + \sqrt{1-4\eta}(1 + 9\eta - 11\eta^2)(\Lambda_1 - \Lambda_2)\right]$$

$$\delta\tilde{\Lambda} = \frac{1}{2}\left[\sqrt{1-4\eta}\left(1 - \frac{13272}{1319}\eta + \frac{8944}{1319}\eta^2\right)(\Lambda_1 + \Lambda_2) + \left(1 - \frac{15910}{1319}\eta + \frac{32850}{1319}\eta^2 + \frac{3380}{1319}\eta^3\right)(\Lambda_1 - \Lambda_2)\right]$$

EOS fit

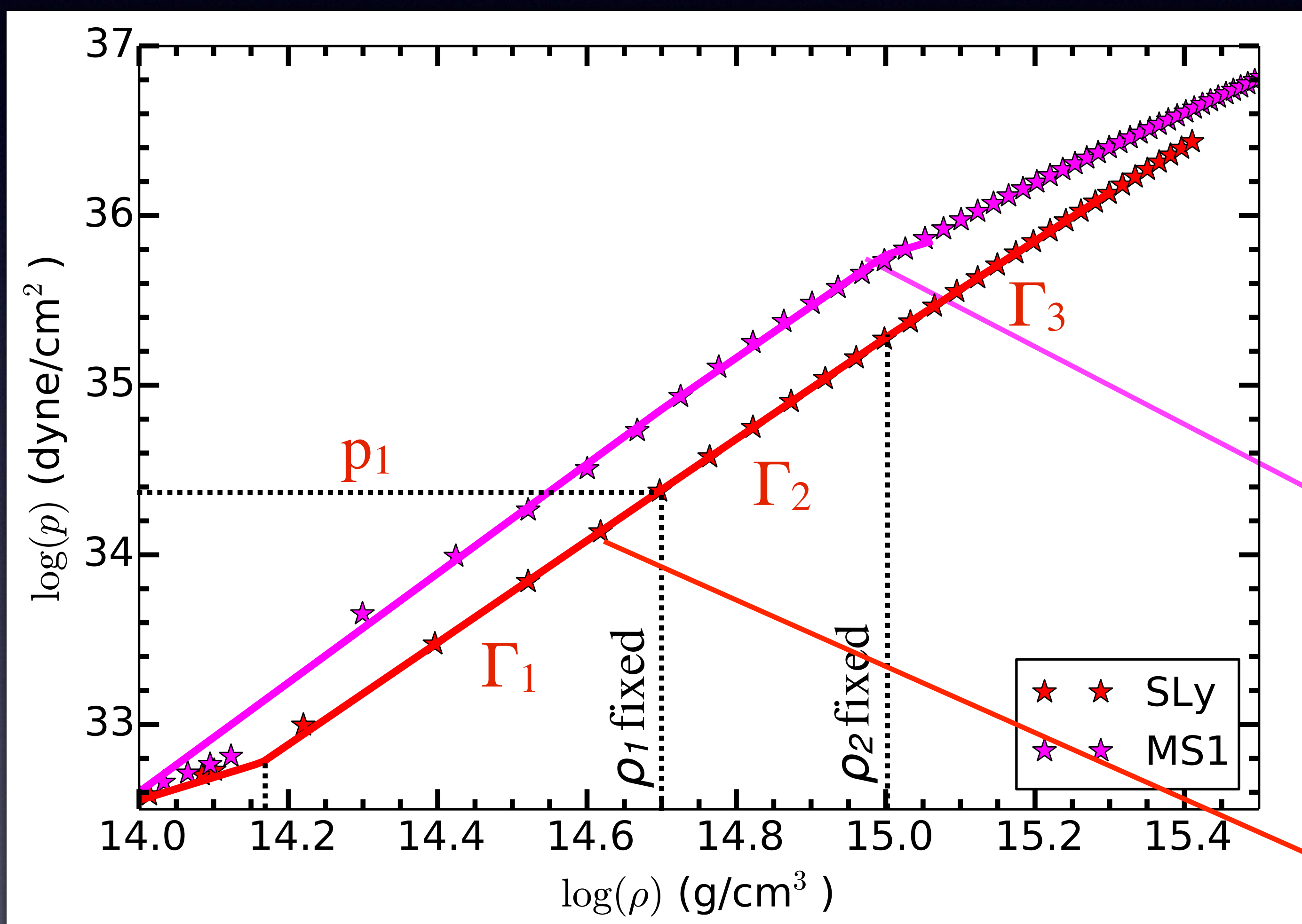
- One-to-one relation between EOS and radius-mass curves
- As well as between EOS and tidal deformability-mass curves

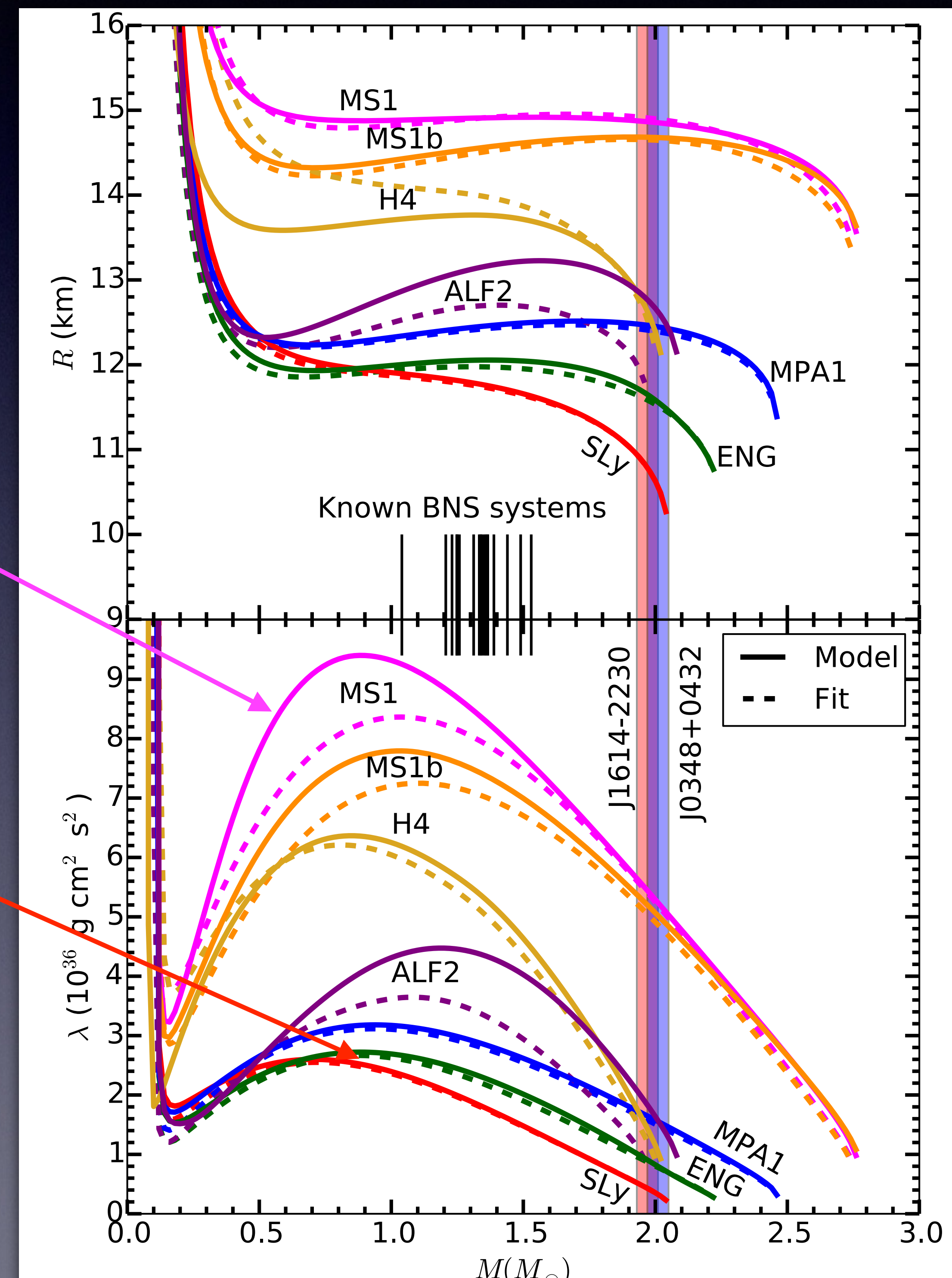




EOS fit

- Purely phenomenological EOS with 4 free parameters
- Methods apply to any EOS with free parameters





$$p(\rho) = \begin{cases} K_1 \rho^{\Gamma_1}, & \rho_0 < \rho < \rho_1 \\ K_2 \rho^{\Gamma_2}, & \rho_1 < \rho < \rho_2 \\ K_3 \rho^{\Gamma_3}, & \rho > \rho_2 \end{cases}$$

Step 1: Estimate masses and tidal deformability

- Can estimate parameters of each BNS inspiral from Bayes' Theorem:

Posterior Prior Likelihood

$$p(\vec{\theta}|d_n) = \frac{p(\vec{\theta})p(d_n|\vec{\theta})}{p(d_n)}$$

Evidence

- $\vec{\theta} = \{d_L, \alpha, \delta, \psi, \iota, t_c, \phi_c, \mathcal{M}, \eta, \tilde{\Lambda}, \delta\tilde{\Lambda}\}$
- d_n: data from nth BNS event

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$$p(\vec{\theta}|d_n) = \frac{p(\vec{\theta})p(d_n|\vec{\theta})}{p(d_n)}$$

Evidence

- Time series of stationary, Gaussian noise has the distribution

$$p_n[n(t)] \propto e^{-(n,n)/2} \qquad (a,b) = 4\mathrm{Re}\int_0^\infty \frac{\tilde{a}(f)\tilde{b}(f)}{S_n(f)}\,df$$

- Likelihood of observing data d for gravitational wave model $m(t;\vec{\theta})$ with parameters $\vec{\theta}$

$$p(d|\vec{\theta}) \propto e^{-(d-m,d-m)/2}$$

  - where (data) = (noise) + (GW signal)

Step 1: Estimate masses and tidal deformability

- Can estimate parameters of each BNS inspiral from Bayes' Theorem:

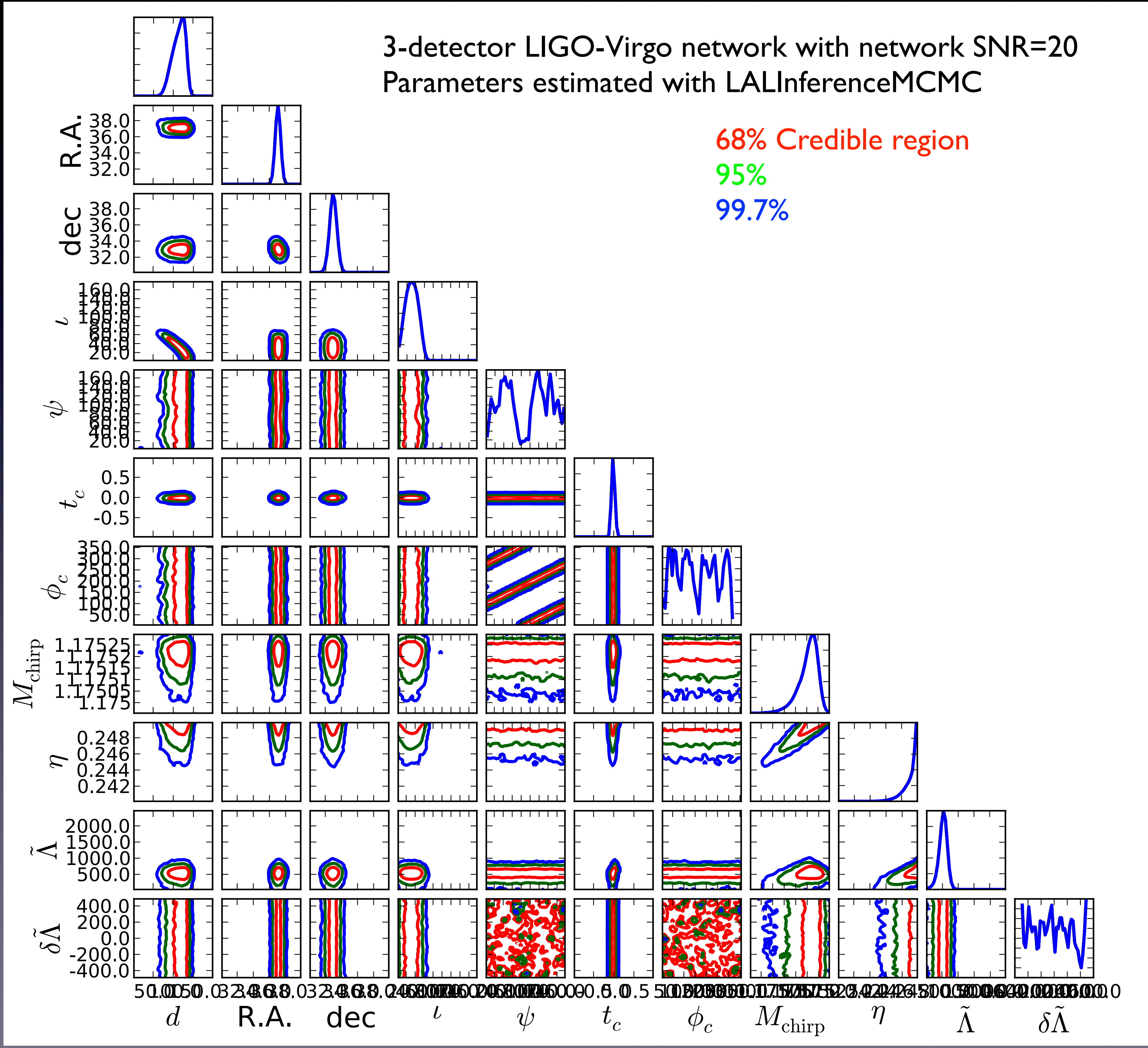
Posterior / Prior / Likelihood / Evidence

$$p(\vec{\theta}|d_n) = \frac{p(\vec{\theta})p(d_n|\vec{\theta})}{p(d_n)}$$

- Use Markov Chain Monte Carlo (MCMC) to sample posterior and marginalize over nuisance parameters

$$p(\mathcal{M}, \eta, \tilde{\Lambda}|d_n) = \int p(\vec{\theta}|d_n) d\vec{\theta}_{\text{nuisance}}$$

Step 1: Estimate masses and tidal deformability



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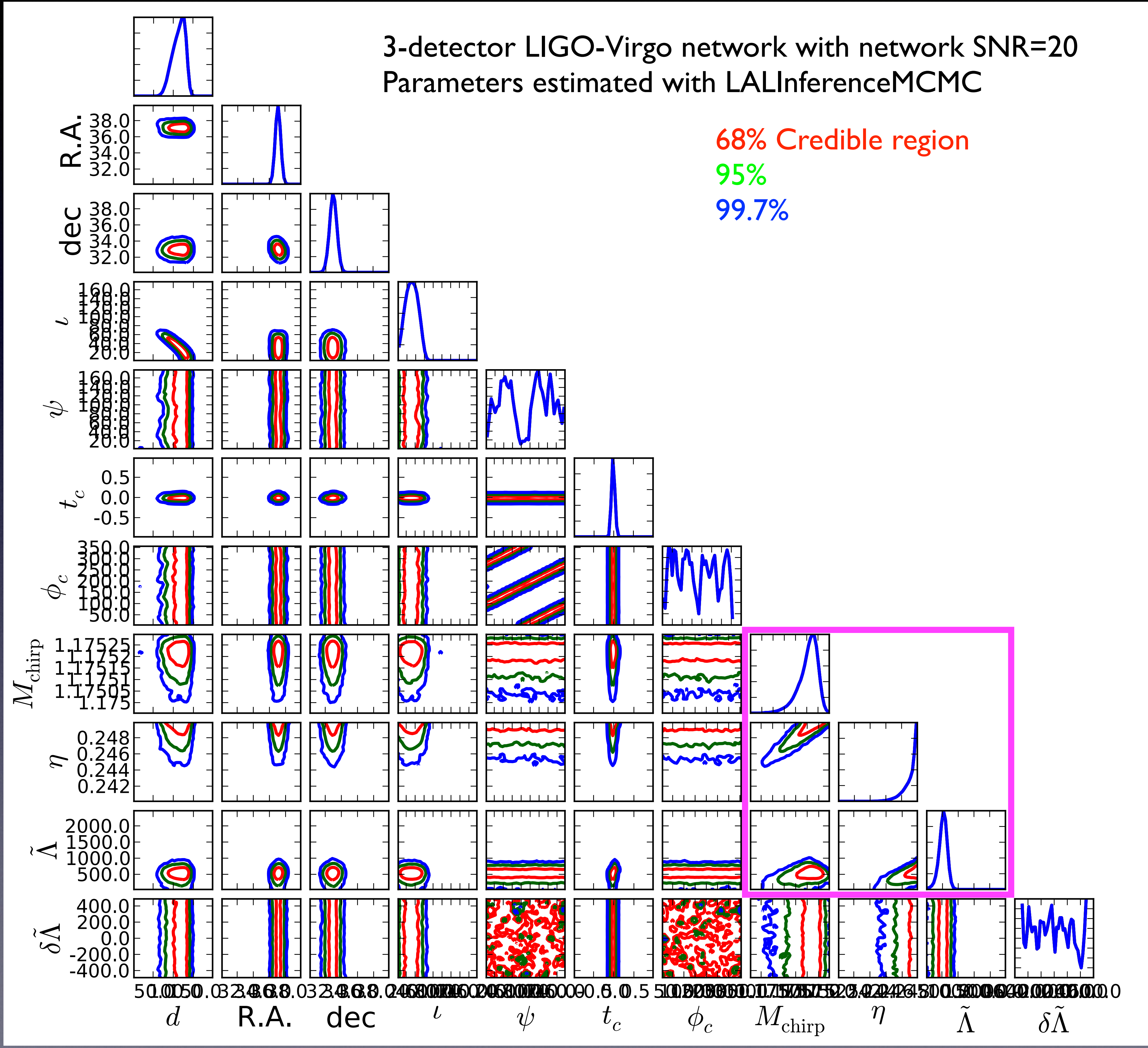

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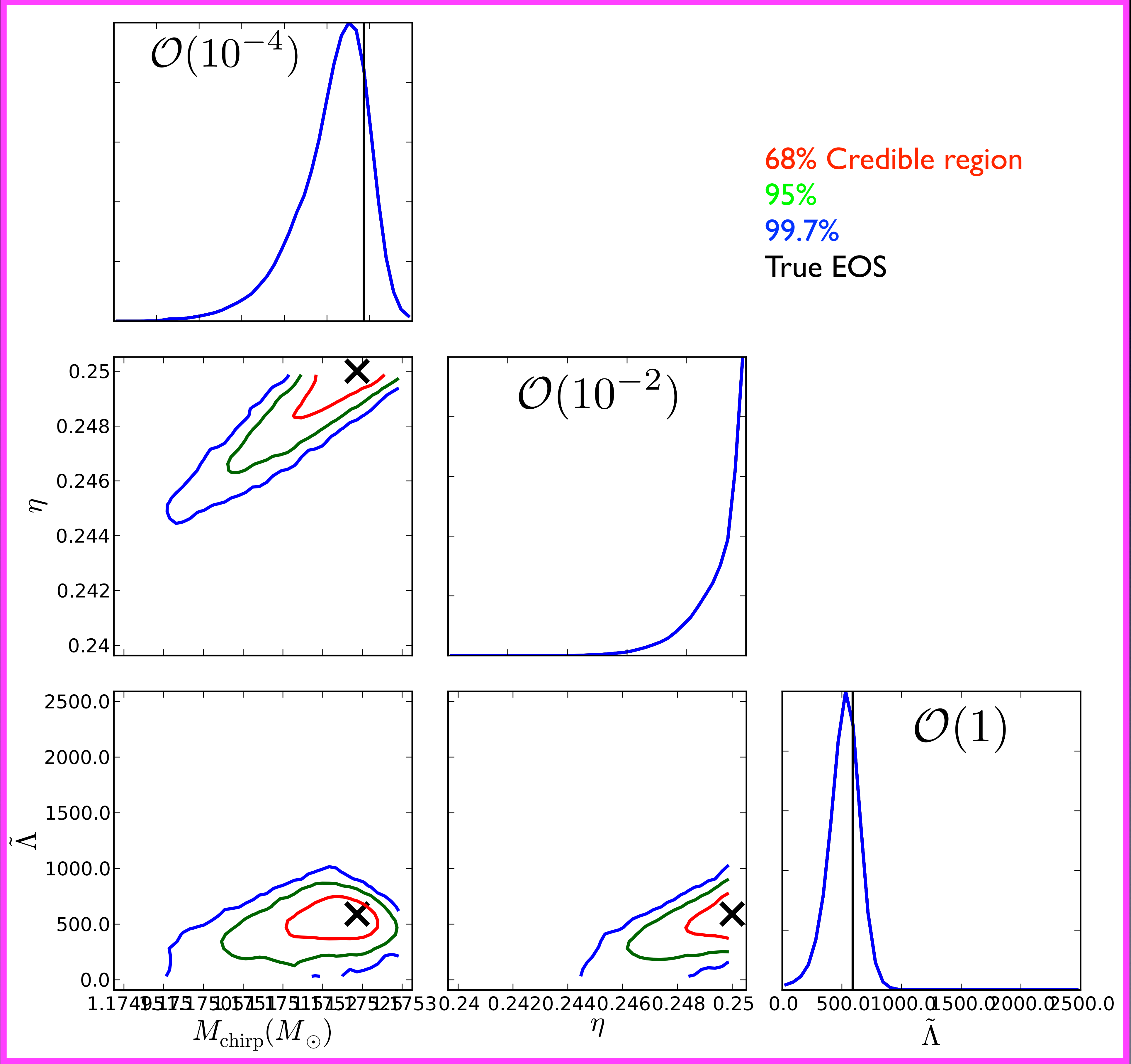


Step 2: Estimate EOS parameters

- Use Bayes' theorem again to estimate masses and EOS parameters:

Posterior — Prior — Likelihood

$$p(\vec{x}|d_1 \ldots d_N) = \frac{p(\vec{x})p(d_1 \ldots d_N|\vec{x})}{p(d_1 \ldots d_N)}$$

Evidence

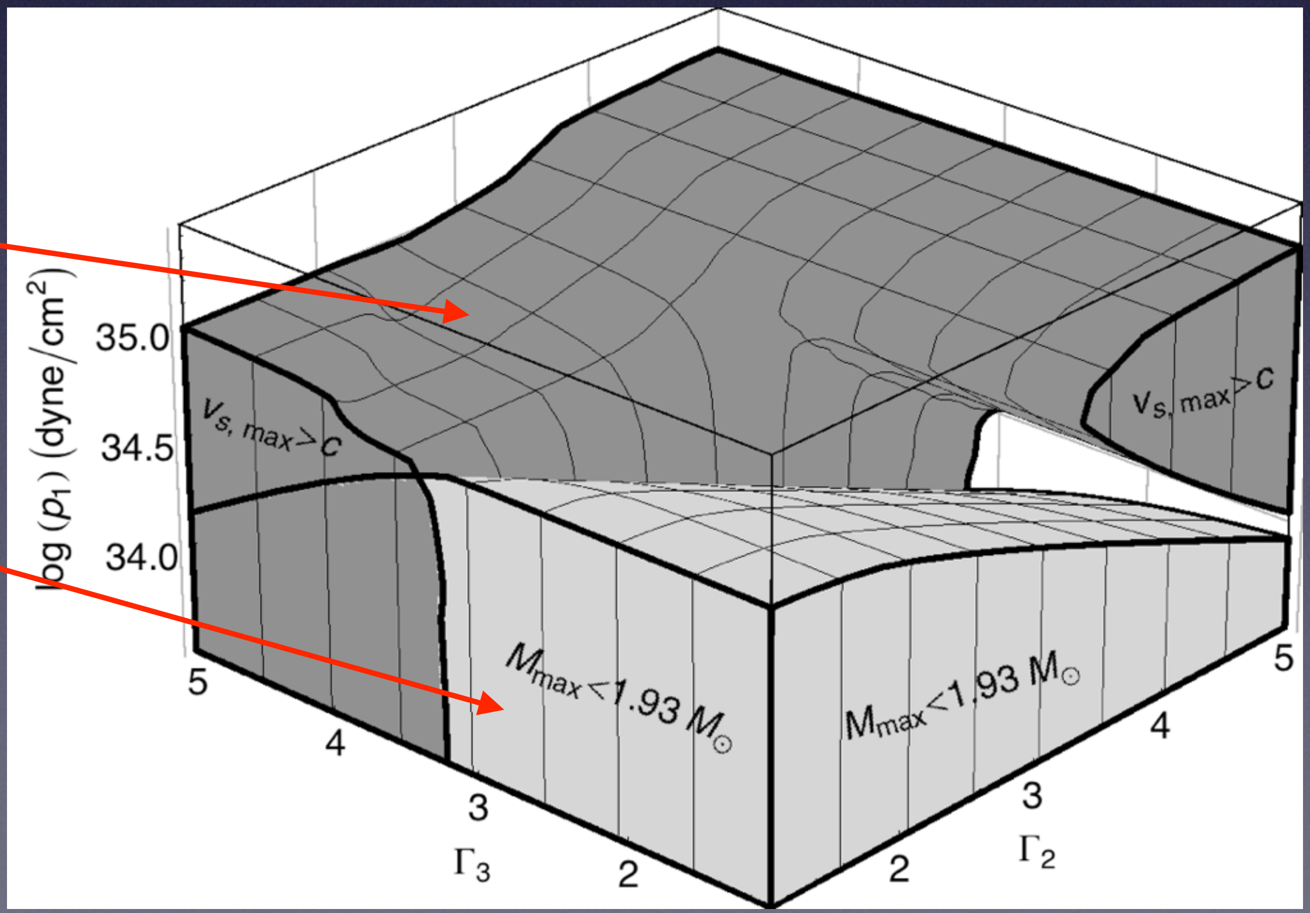
$$\vec{x} = \{\log(p_1), \Gamma_1, \Gamma_2, \Gamma_3, \mathcal{M}_1, \eta_1, \ldots, \mathcal{M}_N, \eta_N\}$$

Step 2: Estimate EOS parameters

- Use Bayes' theorem again to estimate masses and EOS parameters:

$$\underbrace{p(\vec{x}|d_1 \ldots d_N)}_{\text{Posterior}} = \frac{\overbrace{p(\vec{x})}^{\text{Prior}}\overbrace{p(d_1 \ldots d_N|\vec{x})}^{\text{Likelihood}}}{\underbrace{p(d_1 \ldots d_N)}_{\text{Evidence}}}$$

- Causality: Speed of sound must be less than the speed of light $v_s = \sqrt{dp/d\epsilon} < c$
- Maximum mass: EOS must support observed stars with masses greater than $1.93 M_\odot$



Step 2: Estimate EOS parameters

- Use Bayes' theorem again to estimate masses and EOS parameters:

Posterior — Prior — Likelihood — Evidence

$$p(\vec{x}|d_1 \ldots d_N) = \frac{p(\vec{x})p(d_1 \ldots d_N|\vec{x})}{p(d_1 \ldots d_N)}$$

- Total likelihood is product of likelihoods for each independent event
- Rewritten in terms of the EOS parameters instead of tidal deformability

Marginalized posterior for single event

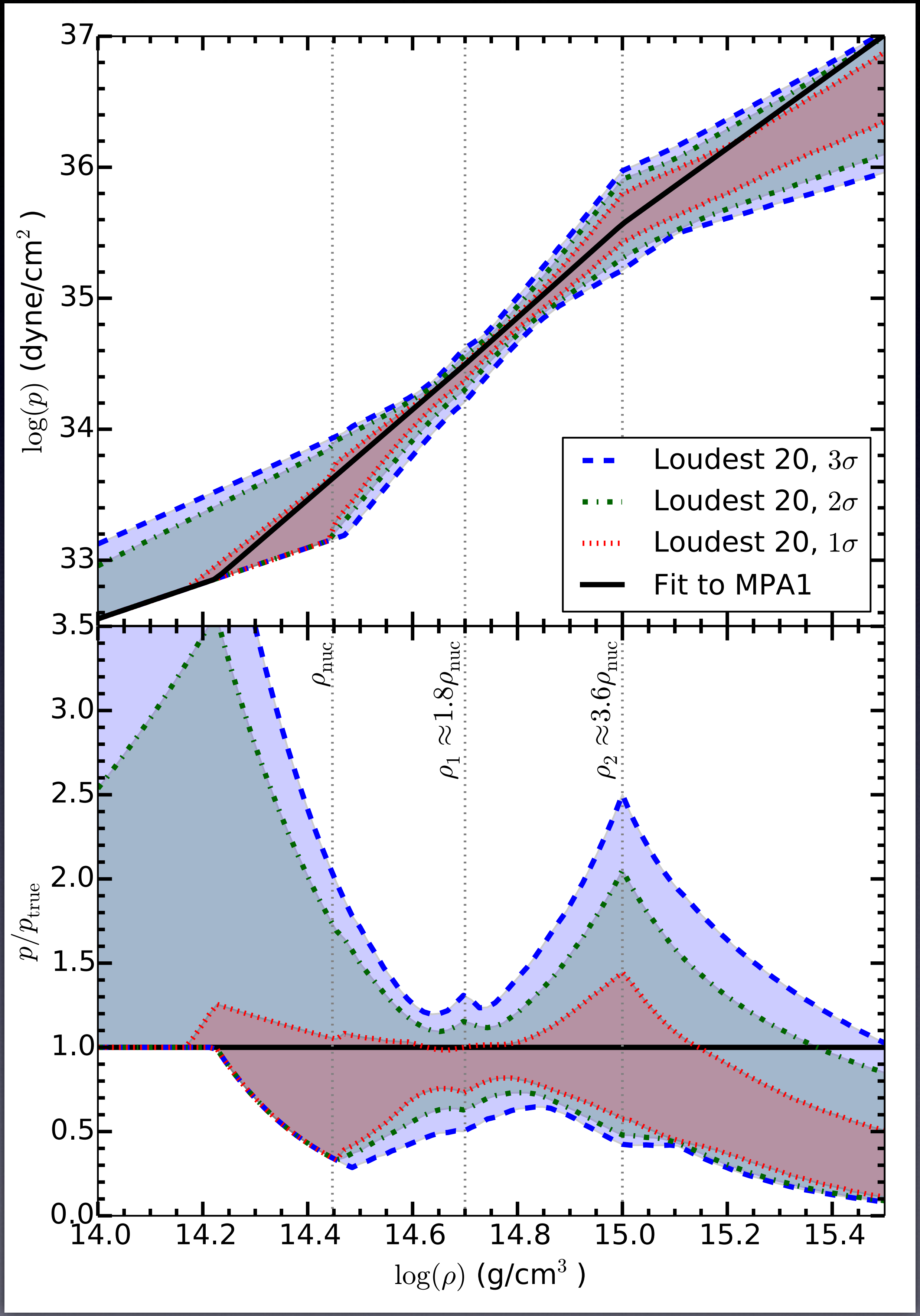
$$p(d_1, \ldots, d_N|\vec{x}) = \prod_{n=1}^{N} p(\mathcal{M}_n, \eta_n, \tilde{\Lambda}_n|d_n)|_{\tilde{\Lambda}_n = \tilde{\Lambda}(\mathcal{M}_n, \eta_n, \mathrm{EOS})}$$

- EOS parameters found from MCMC simulation for 4+2N parameters by marginalizing over the 2N mass parameters

Simulating a population of BNS events

- Sampled a year of data using the standard “realistic” event rate
    - ~40 BNS events/year for single detector with SNR>8
- Masses sampled uniformly in $[1.2M_{\odot}, 1.6M_{\odot}]$
- Chose MPA1 to be “true” EOS when calculating tidal parameters for these events
- Injected waveforms into simulated noise for the 3-detector LIGO-Virgo network

Results for 1 year of data



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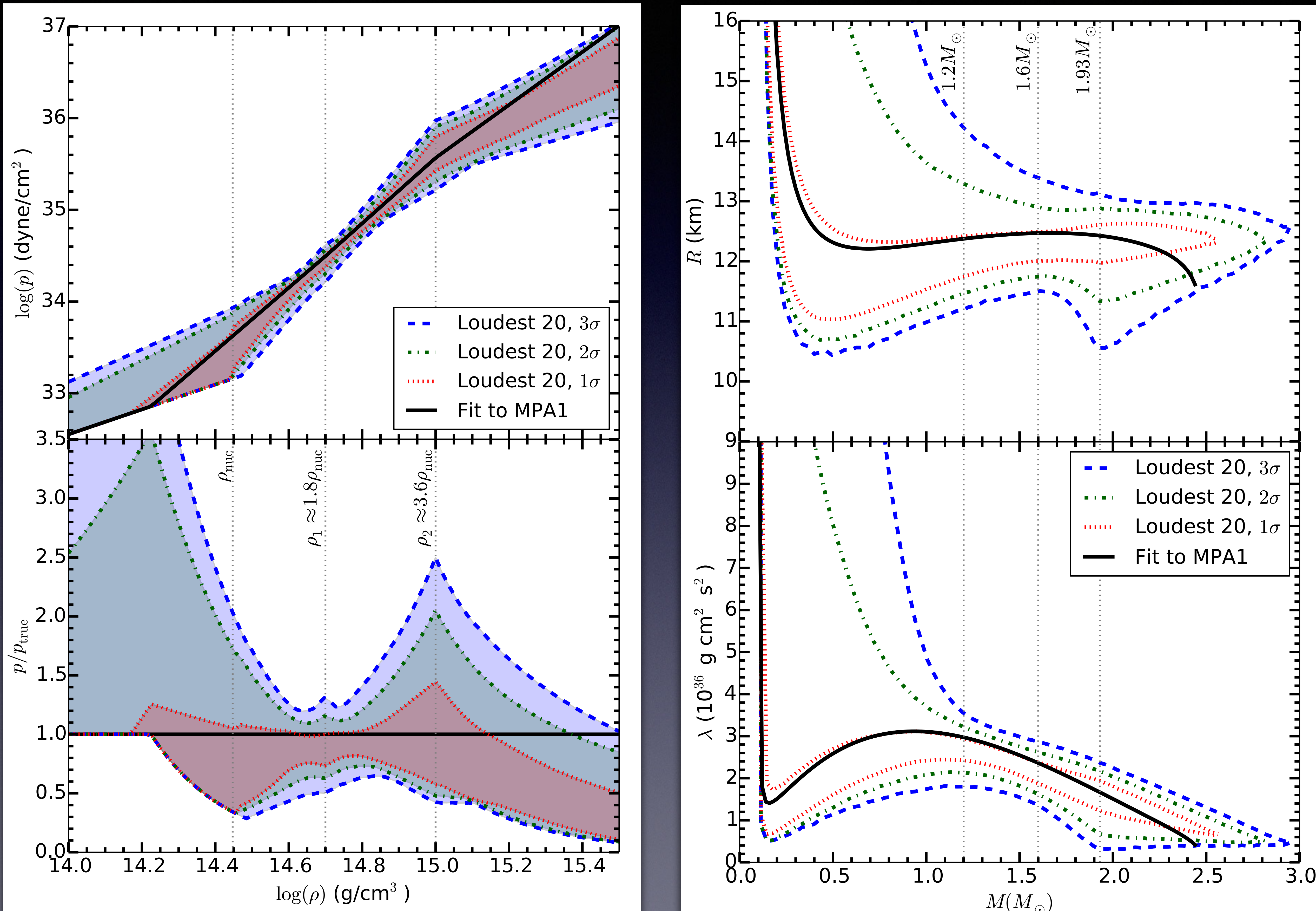
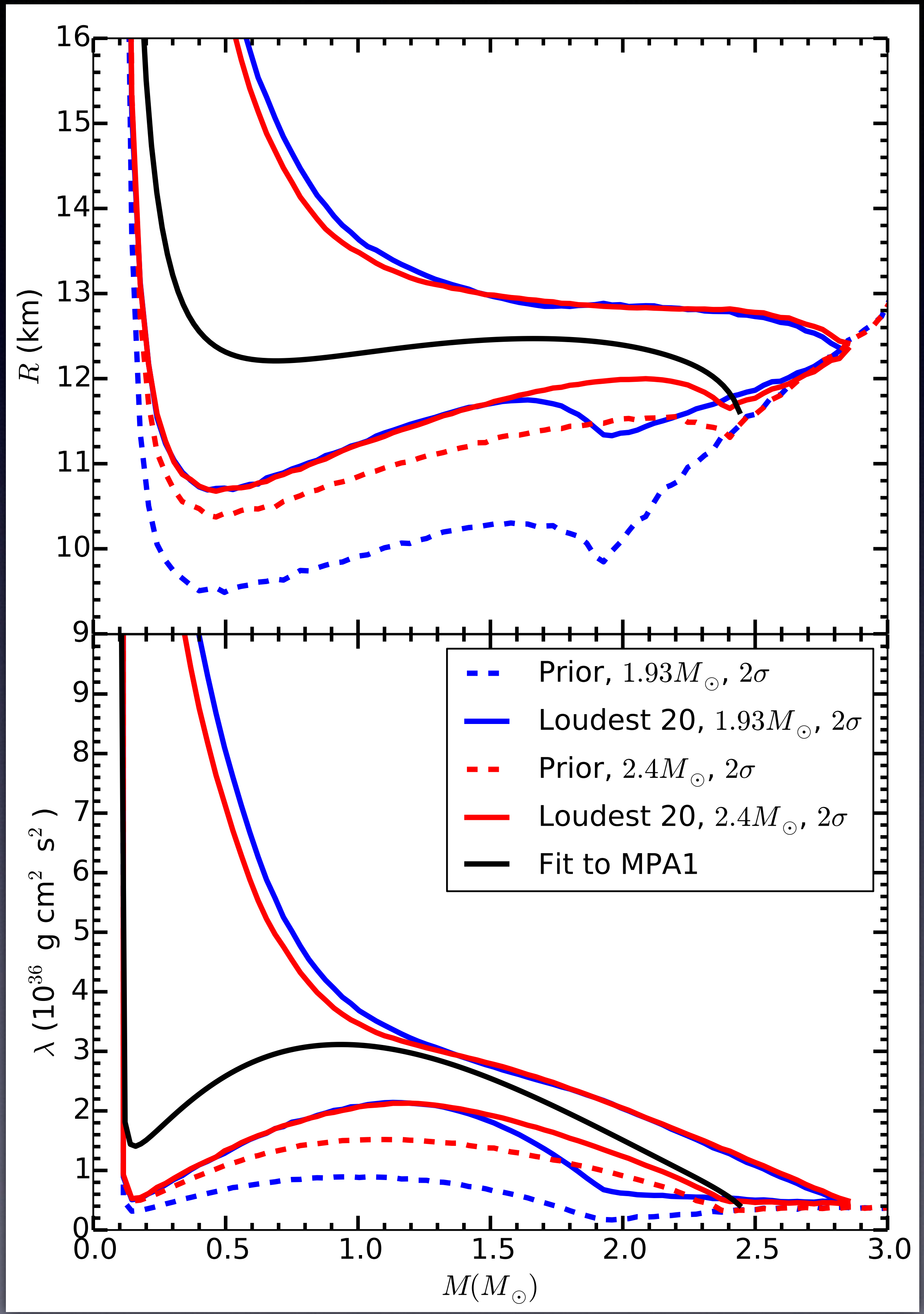


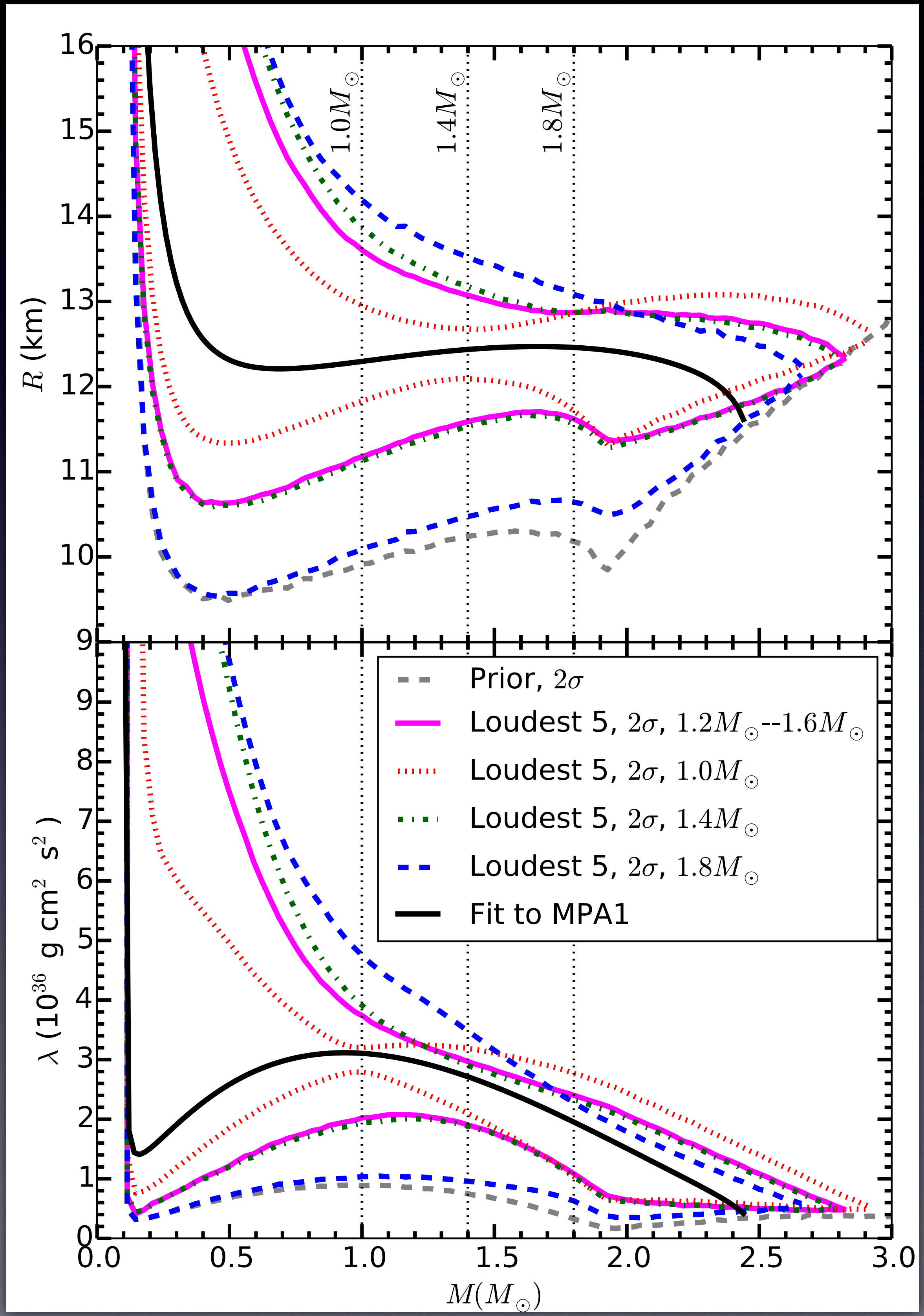
Higher mass NS observations

- Black widow pulsars may have particularly high masses, but large systematic uncertainties
    - PSR B1957+20: $2.40 \pm 0.12 M_\odot$
    - PSR J1311-3430: $2.68 \pm 0.14 M_\odot$
- Higher mass NS observations improve the measurability at higher masses

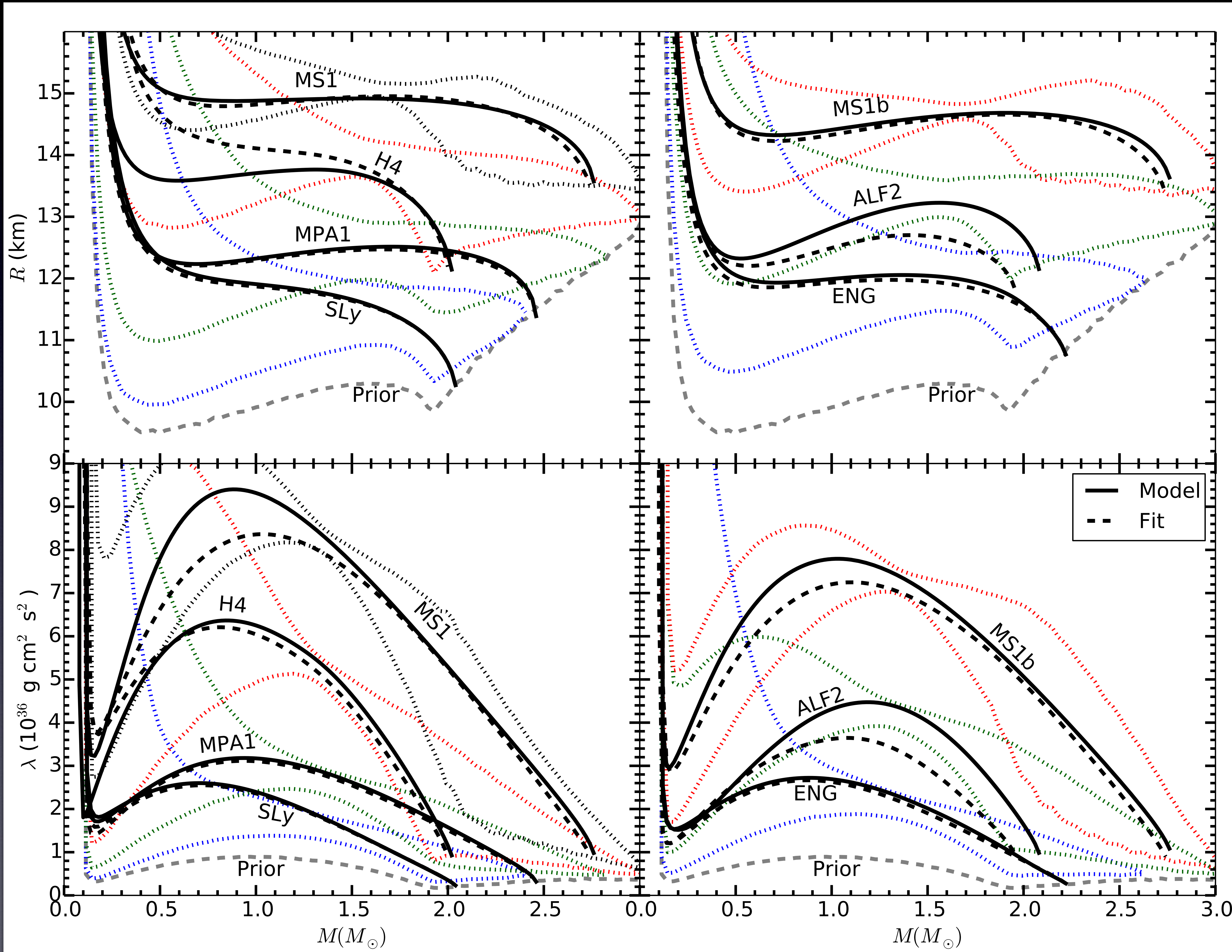


Range of sampled BNS masses

- Simulated BNS populations where all the masses were fixed at $1.0M_{\odot}$, $1.4M_{\odot}$, or $1.8M_{\odot}$
- Errors are smallest near the masses of the simulated population
- Can still measure NS properties at other masses due to prior constraints on the equation of state

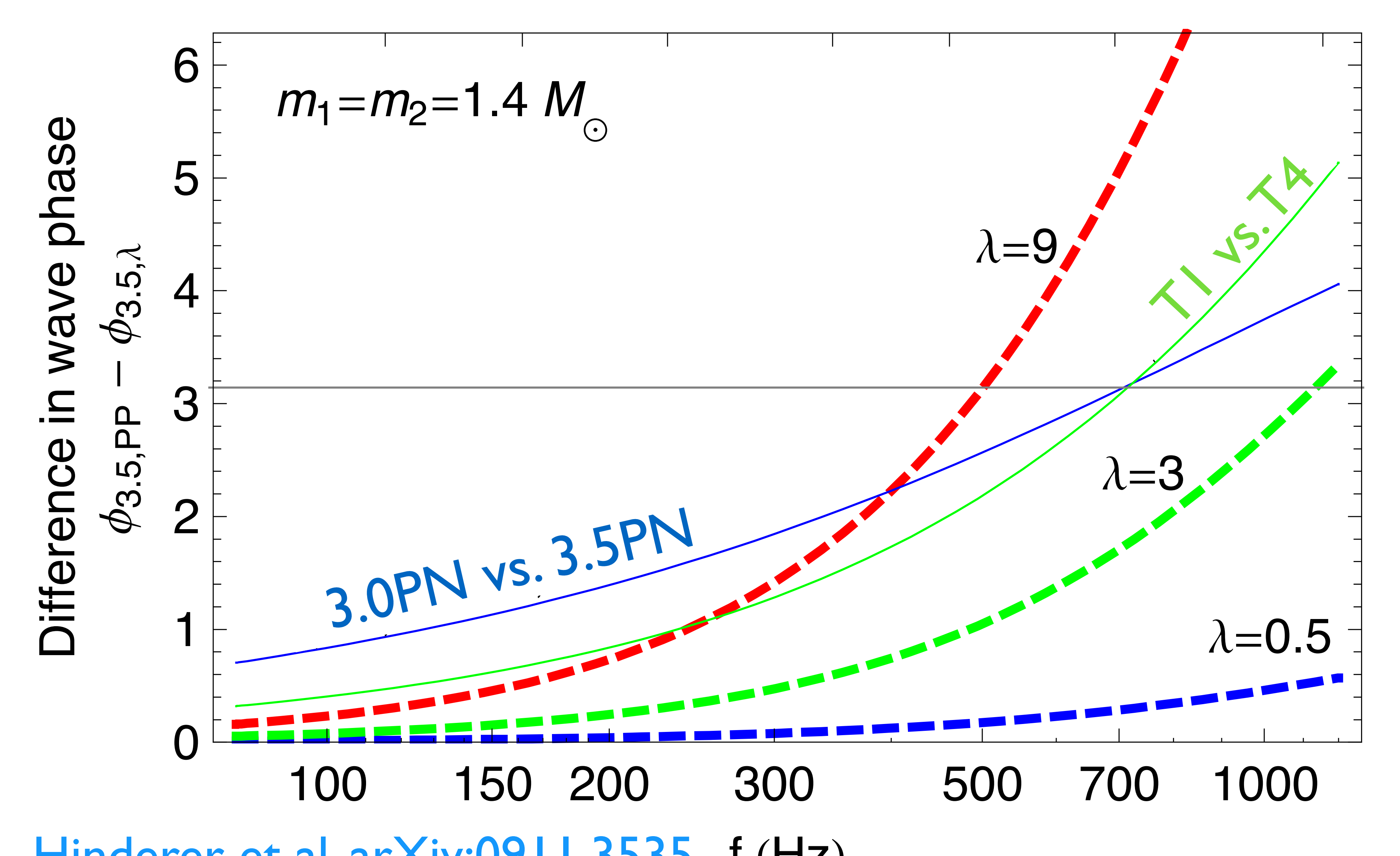


Other EOS models



Systematic errors

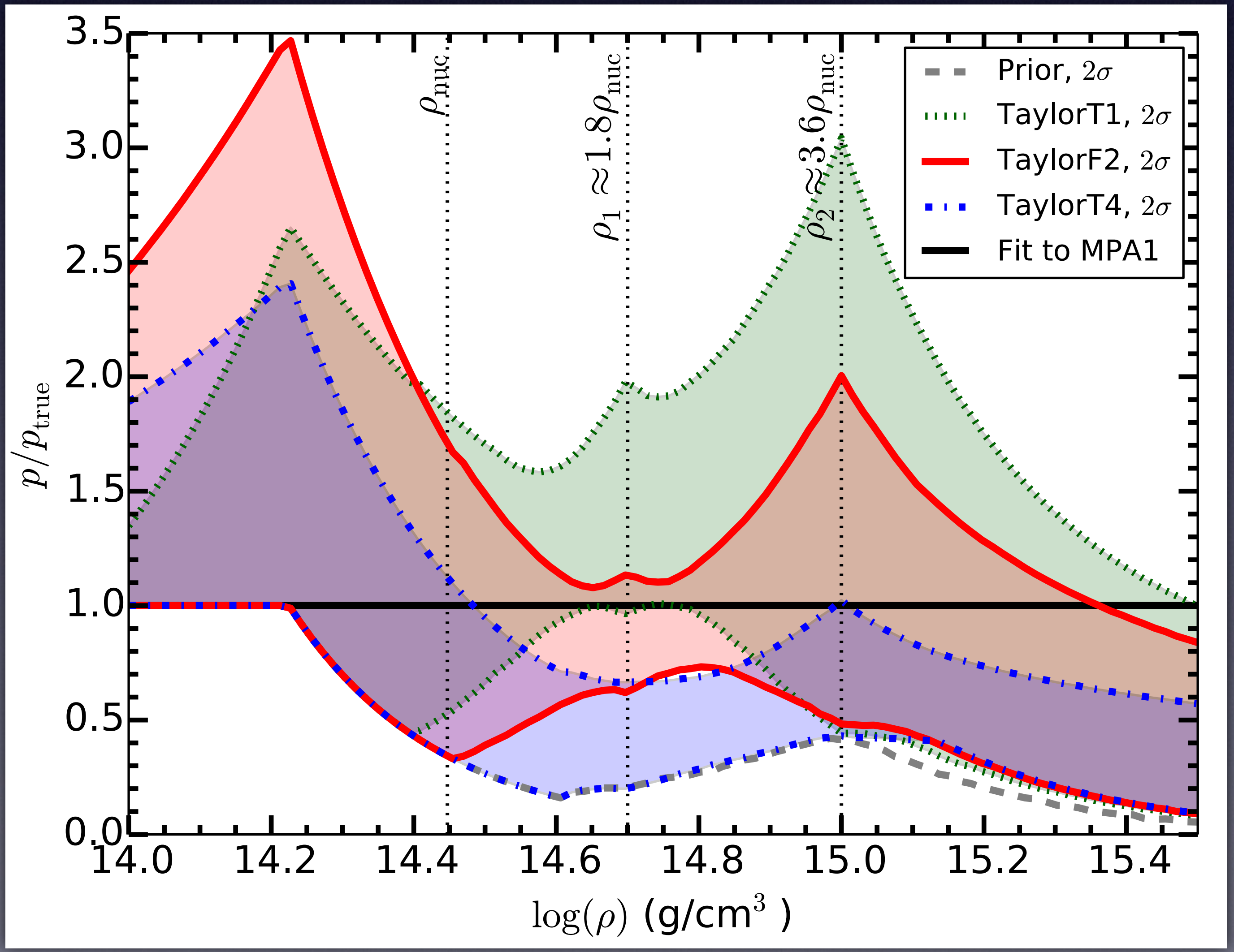
- Several ways to calculate waveform phase from energy and luminosity expressions
- Phase difference between 3PN and 3.5PN as big as tidal effect
- Phase difference between TaylorT1 and TaylorT4 as big as tidal effect

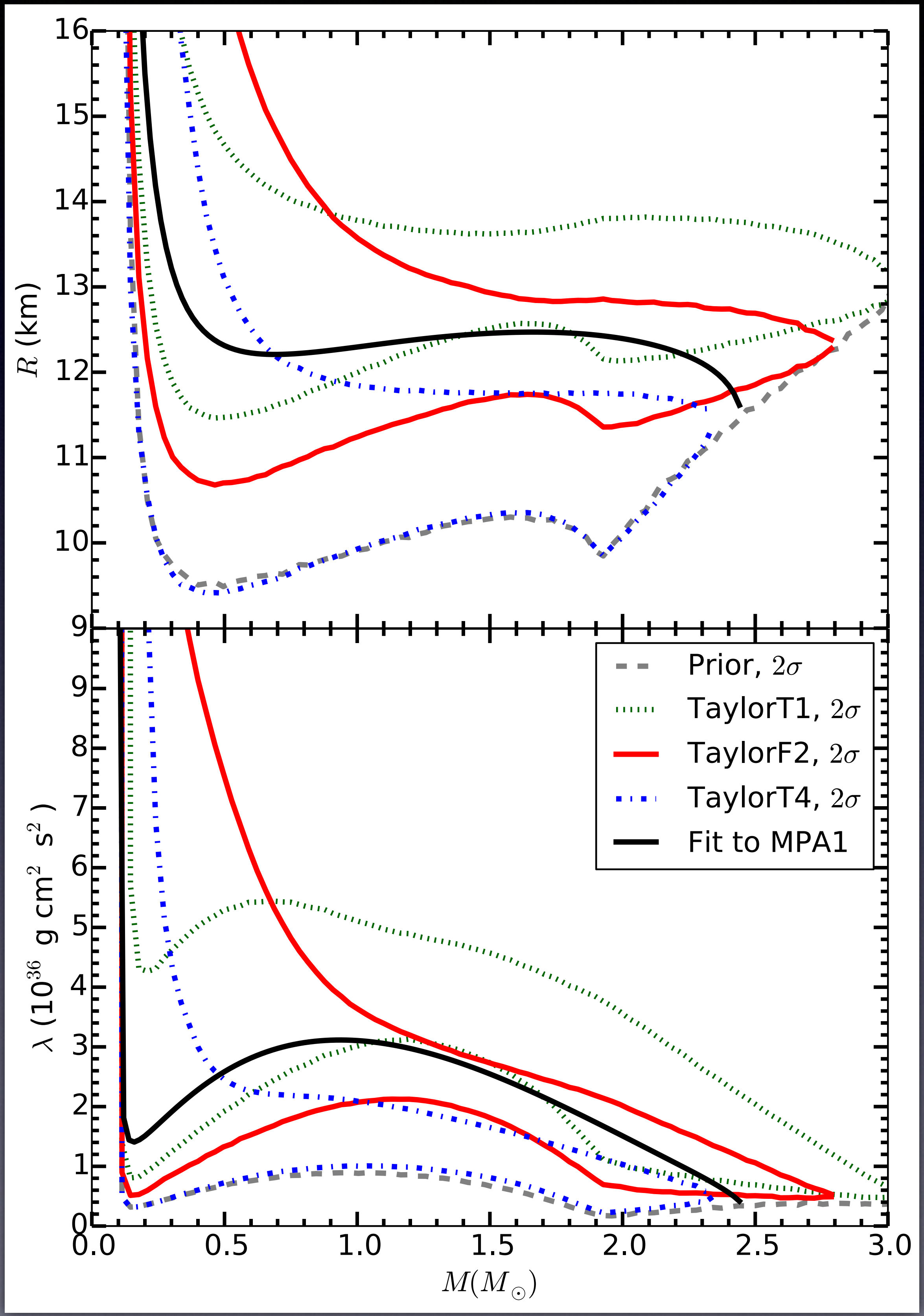


Hinderer et al. arXiv:0911.3535

Systematic errors

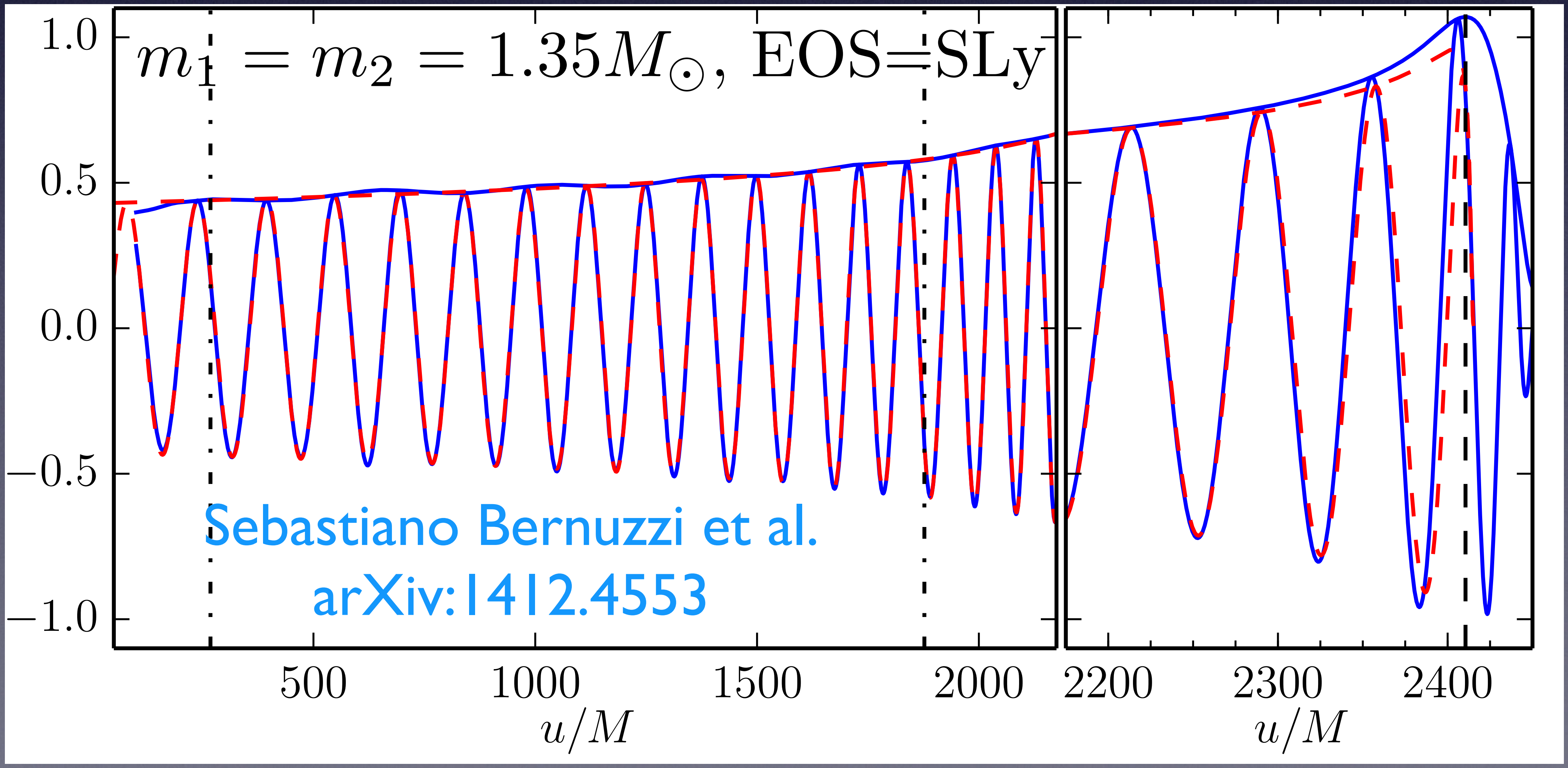
- Injected TaylorF2, TaylorT1, TaylorT4 waveform models
- Used TaylorF2 as template





Systematic errors

- Several ways to improve the waveform model
  - Effective one body waveforms
    - Reproduce BBH waveforms to high accuracy
    - Recent comparisons with BNS simulations are promising
  - Numerical simulations are the only solution once NSs are in contact



Conclusions

- The BNS inspiral waveform provides detailed EOS information
- 1 year of data will be sufficient to measure (statistical error):
  - Pressure to less than a factor of 2
  - Radius to +/- 1 km
- Systematic errors from inexact waveform templates will be primary difficulty in measuring the EOS
  - Will be reduced in the near future with improved waveform models

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Thank you