Gravity and the Unseen Sky

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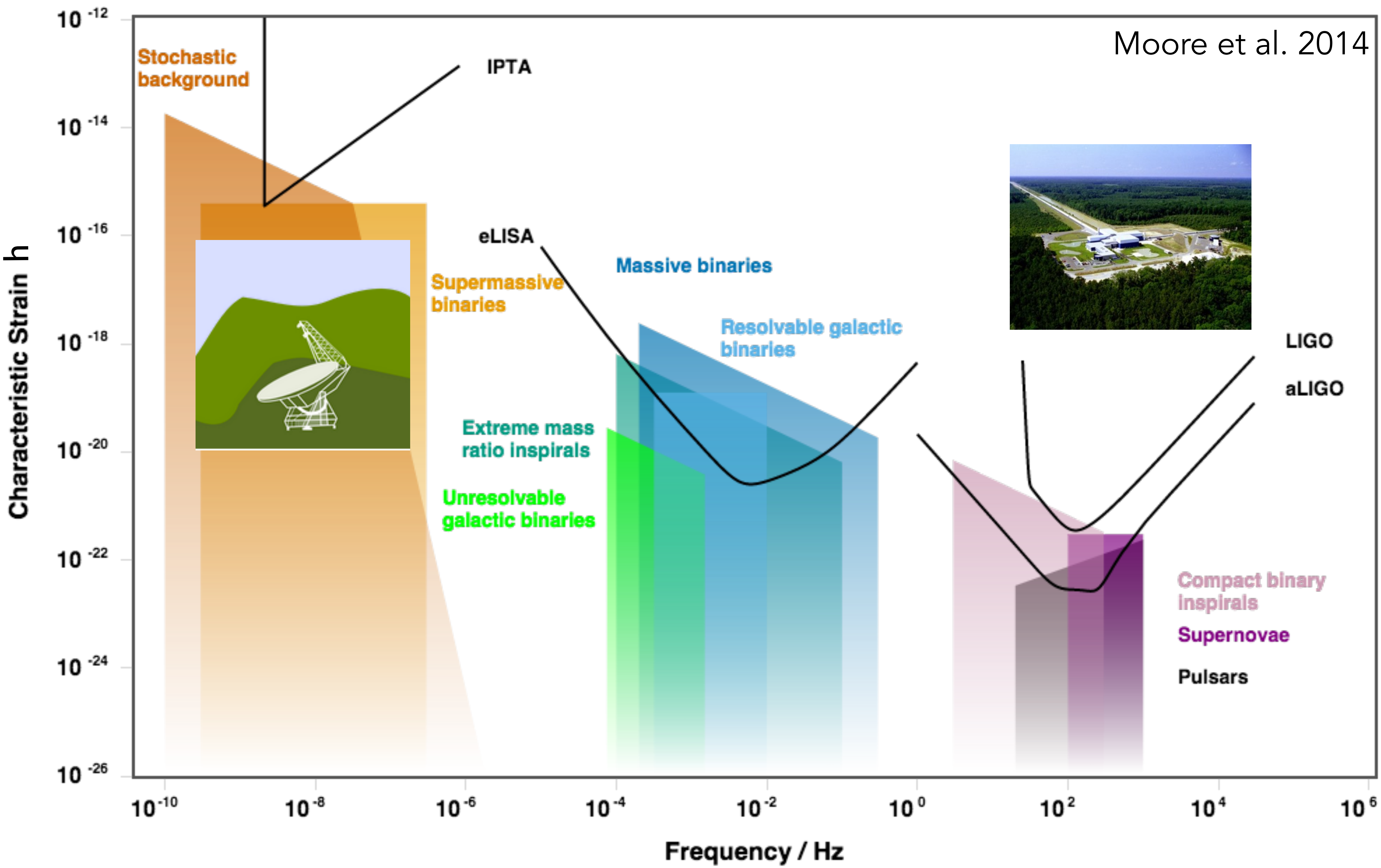
Outline

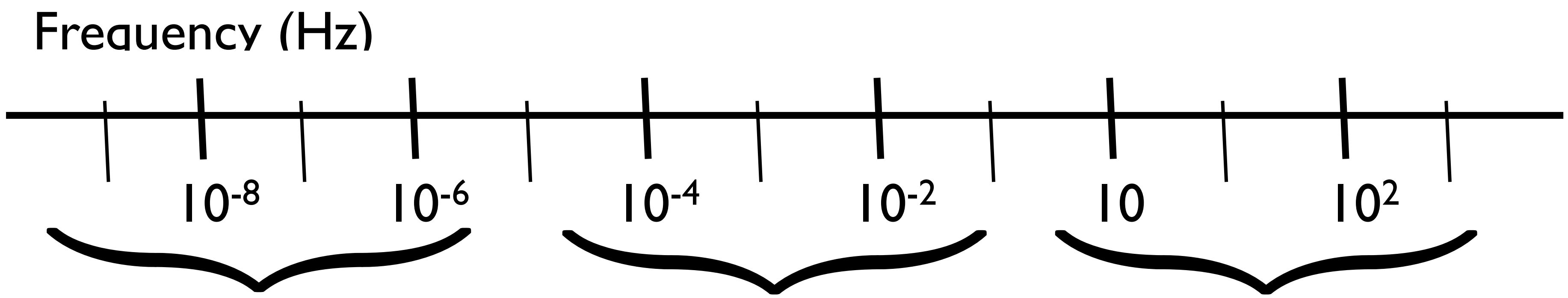
* Gravitational-wave detectors
* The road to detecting stochastic backgrounds with pulsar timing arrays
* Testing general relativity with pulsar timing arrays

Gravitational wave spectrum and sources

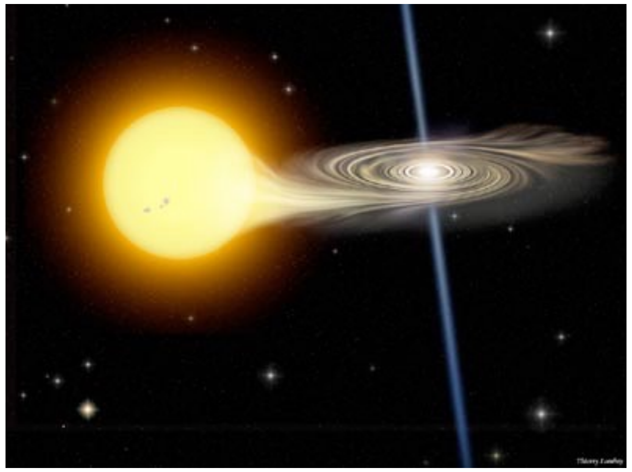


Gravitational wave (GW) experiments

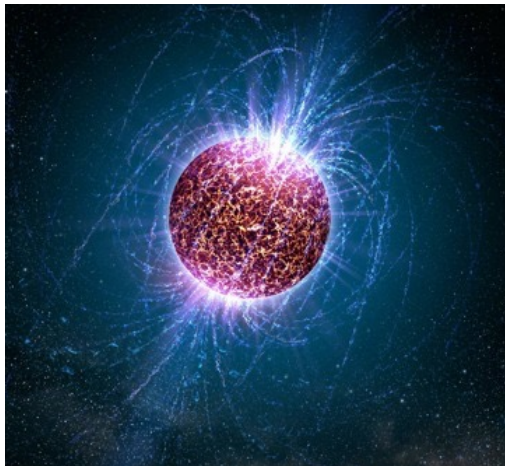
The (detectable) gravitational wave spectrum



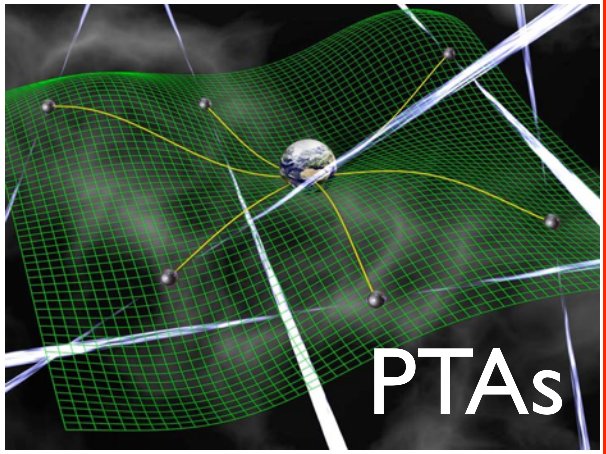
supermassive black hole binaries



neutron star/black hole binaries



compact binary coalescence, supernovae, pulsars



e.g.:

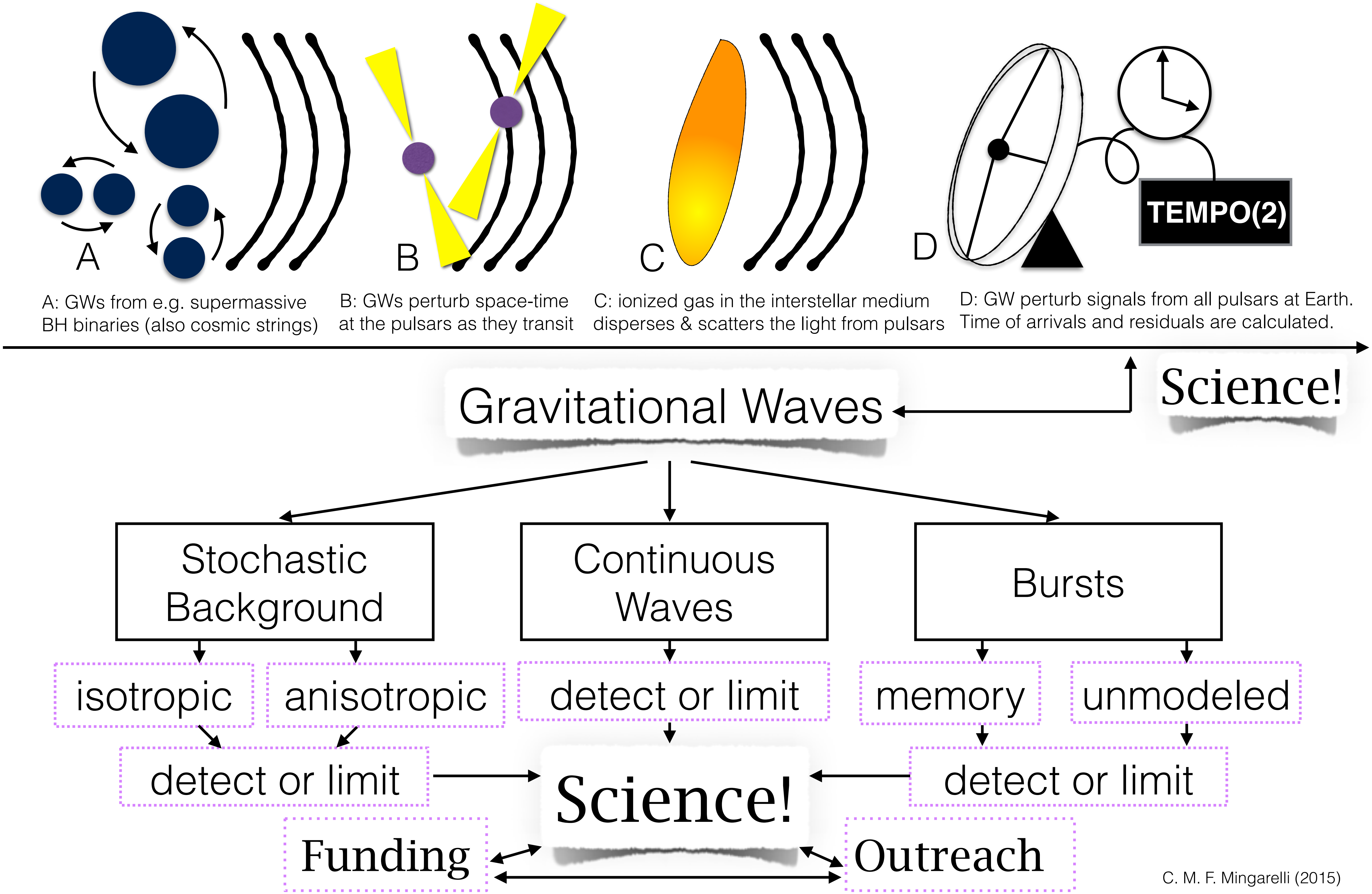
European Pulsar Timing Array

Parkes Pulsar Timing Array

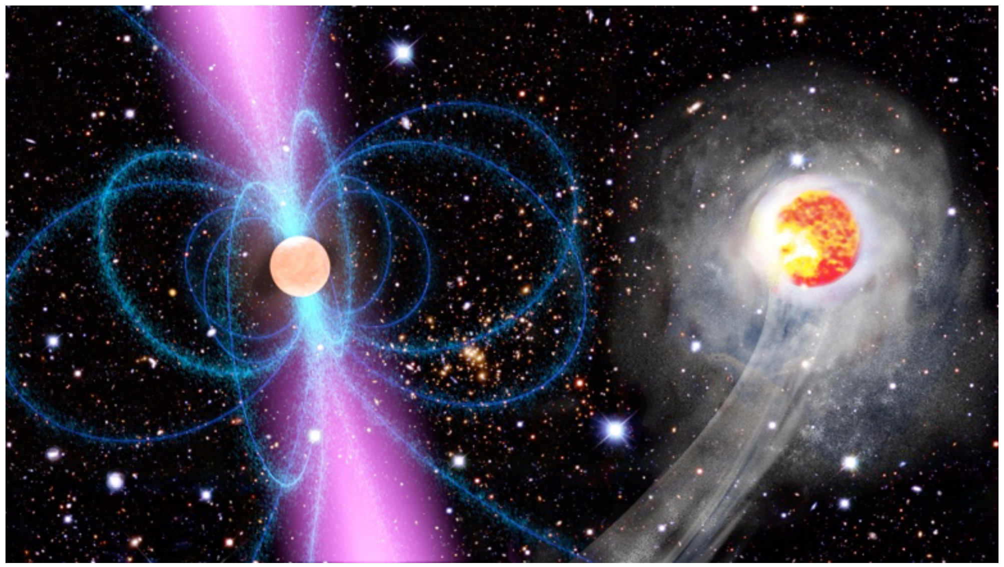
North American Nanohertz GW Observatory (NANOGrav)

All together: *International Pulsar Timing Array*

Pulsar Timing Array

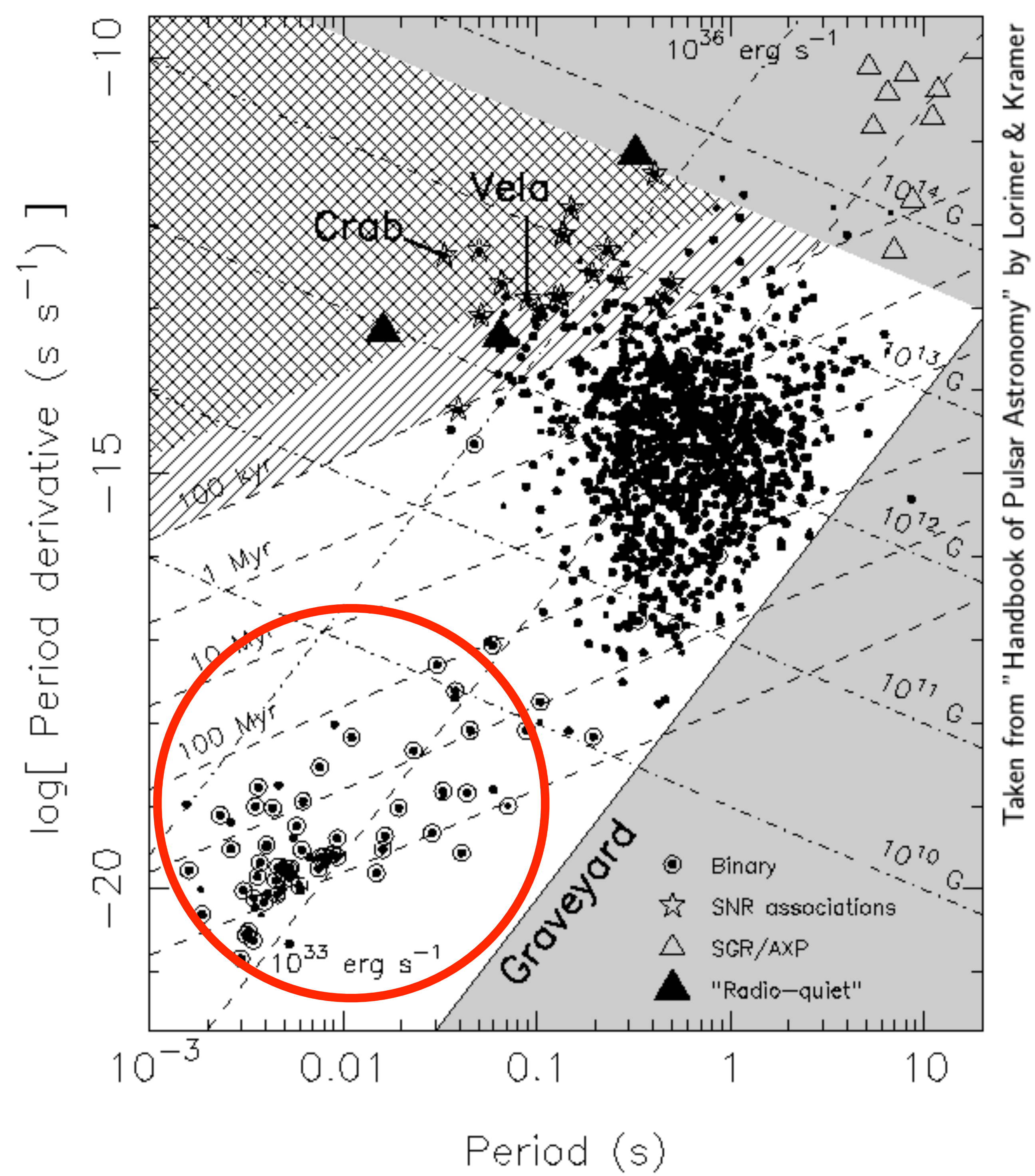


Pulsar timing preliminaries

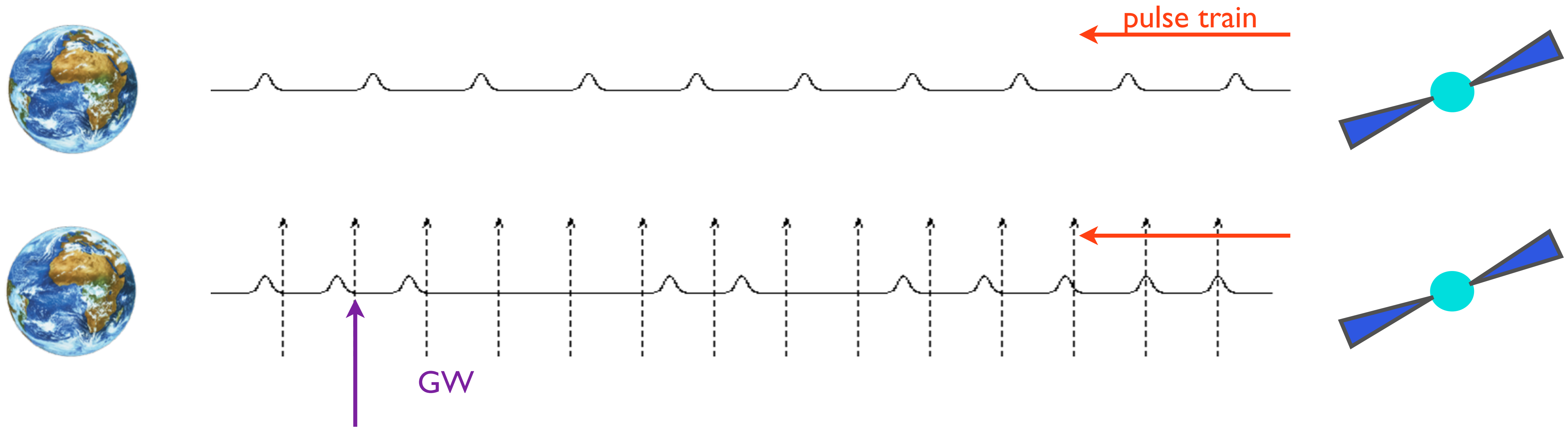


Millisecond Pulsars (MSPs) are used for GW detection because they are very stable (on the level of atomic clocks!!)

MSPs are thought to be “recycled” pulsars — spun up through accretion in binary systems



Pulsar timing preliminaries



- A passing GW induces a redshift in the pulse train from the pulsar. This redshift can be calculated with the geodesic equation.

But important to note that the physical observable is not the redshift: is the *timing residual*

$$r(t) = \mathrm{TOA}_{\mathrm{actual}} - \mathrm{TOA}_{\mathrm{expected}}$$

$$r(t) = \int_0^t z(t')\,dt'$$

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Time-domain Implementation of the Optimal Cross-Correlation Statistic for Stochastic Gravitational-Wave Background Searches in Pulsar Timing Data

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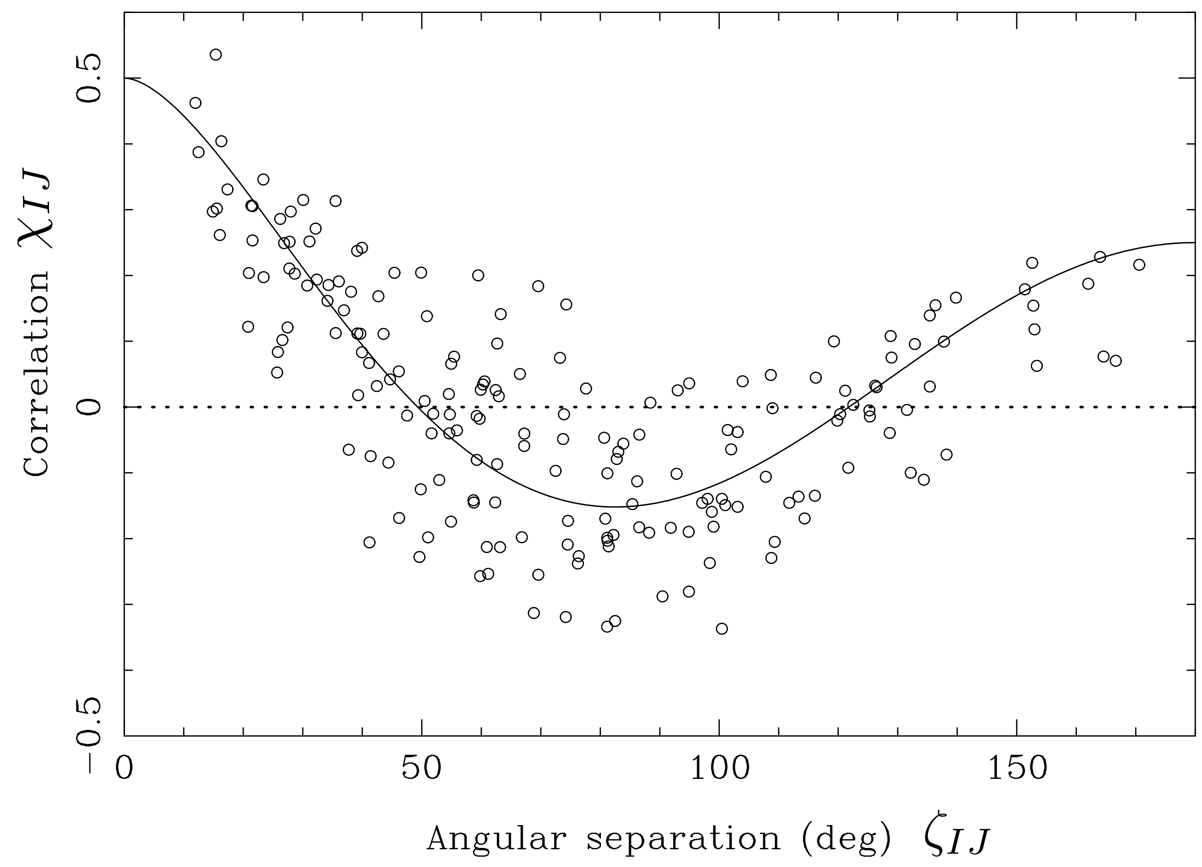
(Dated: October 31, 2014)

Stochastic GW searches with PTAs

- First showed by Hellings and Downs in 1983 that a GW produces *uniquely correlated* variations in the timing residuals of a set of pulsars



- So can search for GWs by fitting this curve to the cross-correlated data (Jenet et al. 2005)

- Or, **find an optimized cross correlation that takes into account the GW spectrum and noise power spectra:**

$$\langle \tilde{r}_i^*(f)\tilde{r}_j(f')\rangle \propto \Omega_{gw}(f)\,\delta(f-f')\,\chi_{ij}$$

Correlated residuals — GW source spectrum — Hellings-Downs coefficient

Stochastic GW searches with PTAs

- Stochastic background generated by a large # of independent, individually unresolvable sources:
  - cosmic string bursts
  - colliding bubbles from CQD phase transitions
  - **supermassive black hole binaries**
- Sources provide spectrum for (isotropic) stochastic background:

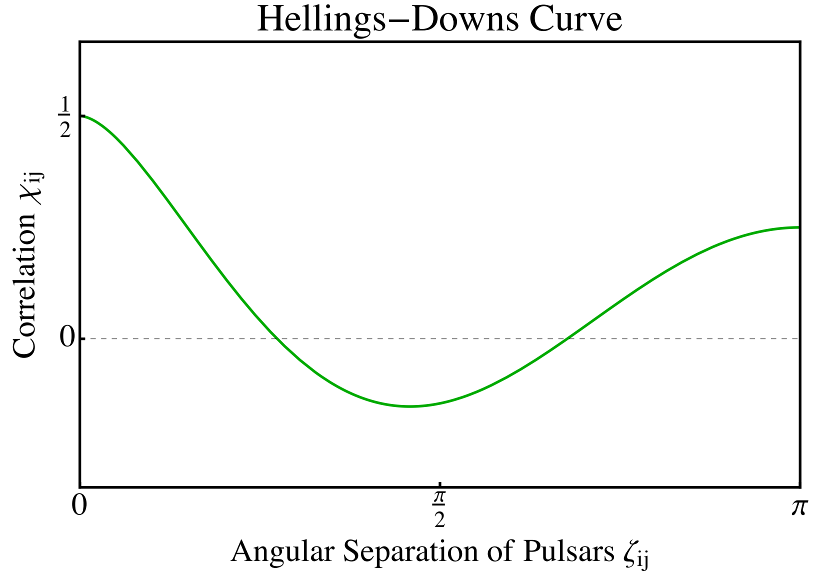
$$\langle \tilde{r}_i^*(f)\tilde{r}_j(f')\rangle \propto \Omega_{gw}(f)\,\delta(f-f')\,\chi_{ij}$$

Correlated residuals → **How do we get timing residuals?**

GW source spectrum ↓

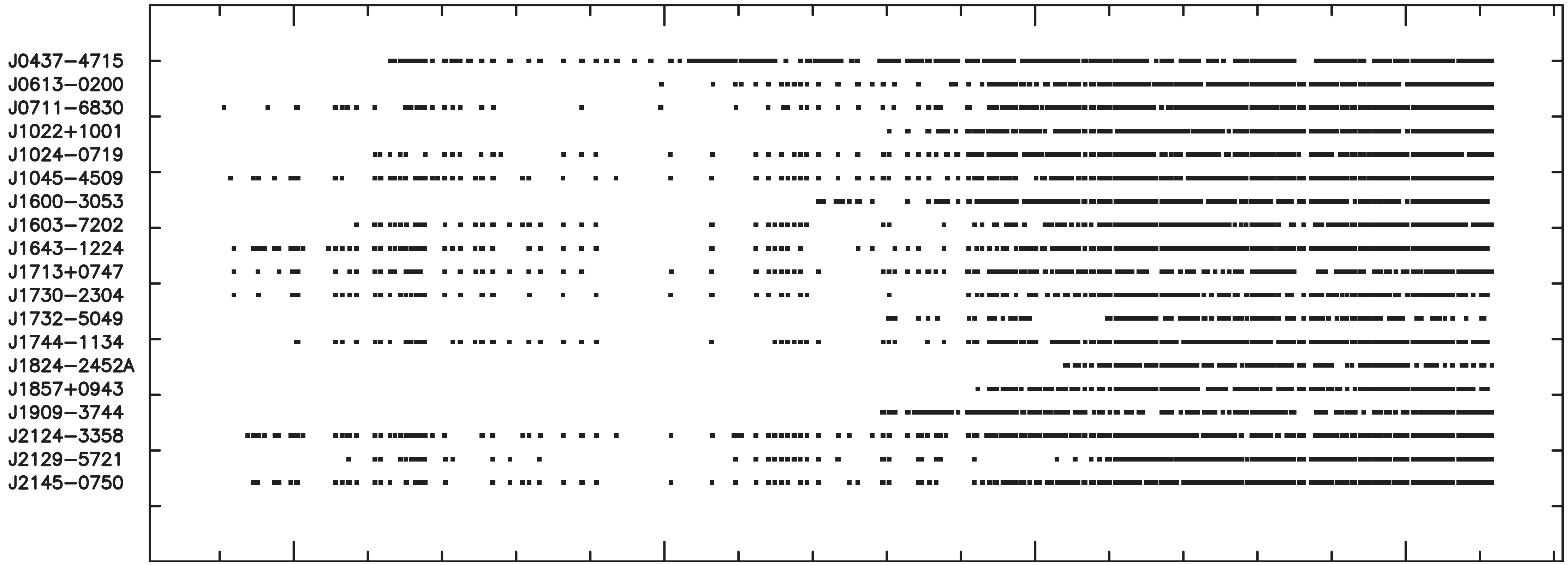
$$\Omega_{\rm gw}(f) \propto A_{\rm gw}^2\, f^2 \left(\frac{f}{f_{\rm 1yr}}\right)^{2\alpha}$$

Hellings-Downs coefficient



Stochastic GW searches with PTAs

- Such an optimized cross-correlation (or optimal statistic) first probed by Anholm et al. 2009, but focused on frequency domain implementation
- But PTA data is very irregularly sampled…



- … and we need a timing model to predict TOAs.

What follows will develop the optimal statistic in the **time-domain**

Time-domain optimal statistic

- At radio telescope, measure TOAs (essentially phase of pulsar)
- TOAs contain many terms of known functional form: intrinsic pulsar parameters, pulse phase jitter, possible red noise from interstellar medium effects, stochastic GWB (?)...

$$\phi = \underbrace{\phi_0 + \nu(t - t_0) + \frac{1}{2}\dot{\nu}(t - t_0)^2 + \text{sky location terms + binary terms (if appropriate) + ...}}$$

$$\mathrm{TOA}_{\mathrm{observed}} = \mathrm{TOA}_{\mathrm{modeled}} + n$$

Gaussian process: Intrinsic red and white noise + GWs +...

- Timing model is projected out of TOA data with linear operator R

$$r = R(\mathrm{TOA}_{\mathrm{modeled}} + n) = Rn$$

all information about noise sources/stochastic GWB is contained in n, but we can't actually measure n directly because we subtract out the timing model. Instead, work with observable quantity r.

Time-domain optimal statistic

- Assume that we have a PTA with M pulsars, each with some intrinsic noise

- Since we assumed that the intrinsic pulsar noise was Gaussian, the likelihood function for the PTA is just be a standard multivariate Gaussian

$$p(\mathbf{n}|\vec{\theta}) = \frac{1}{\sqrt{\det(2\pi\boldsymbol{\Sigma}_n)}} \exp\left(-\frac{1}{2}\mathbf{n}^T\boldsymbol{\Sigma}_n^{-1}\mathbf{n}\right)$$

where

$$\mathbf{n} = \begin{bmatrix} n_1 \\ n_2 \\ \vdots \\ n_M \end{bmatrix}$$

$\boldsymbol{\Sigma}_{\mathbf{n}}$: pre-fit noise covariance matrix

$\vec{\theta}$: parameters that characterize the noise

- Just one problem: we don't measure n. The observable is r.

Time-domain optimal statistic

- We *want* the likelihood for r. Some math happens…
- … and we get the likelihood for our observable, r:

$$p(\mathbf{r}|\vec{\theta}) = \frac{1}{\sqrt{\det(2\pi\mathbf{\Sigma})}} \exp\left(-\frac{1}{2}\mathbf{r}^T\mathbf{\Sigma}^{-1}\mathbf{r}\right)$$

where now

$$\mathbf{\Sigma} = \langle \mathbf{r}\mathbf{r}^T \rangle = \begin{bmatrix} P_1 & S_{12} & \dots & S_{1M} \\ S_{21} & P_2 & \dots & S_{2M} \\ \vdots & \vdots & \ddots & \vdots \\ S_{M1} & S_{M2} & \dots & P_M \end{bmatrix}$$

Diagonal terms depend on red noise power spectrum
(pulsar's intrinsic red noise + GW)

$$\mathcal{P}_I(f) = \mathcal{P}_I^{\mathrm{int}}(f) + \mathcal{P}_g(f) \quad \text{where} \quad P_g(f) \propto A_{\mathrm{gw}}^2 f^{-\gamma}$$

Time-domain optimal statistic

- We *want* the likelihood for r. Some math happens…
- … and we get the likelihood for our observable, r:

$$p(\mathbf{r}|\vec{\theta}) = \frac{1}{\sqrt{\det(2\pi\boldsymbol{\Sigma})}} \exp\left(-\frac{1}{2}\mathbf{r}^T\boldsymbol{\Sigma}^{-1}\mathbf{r}\right)$$

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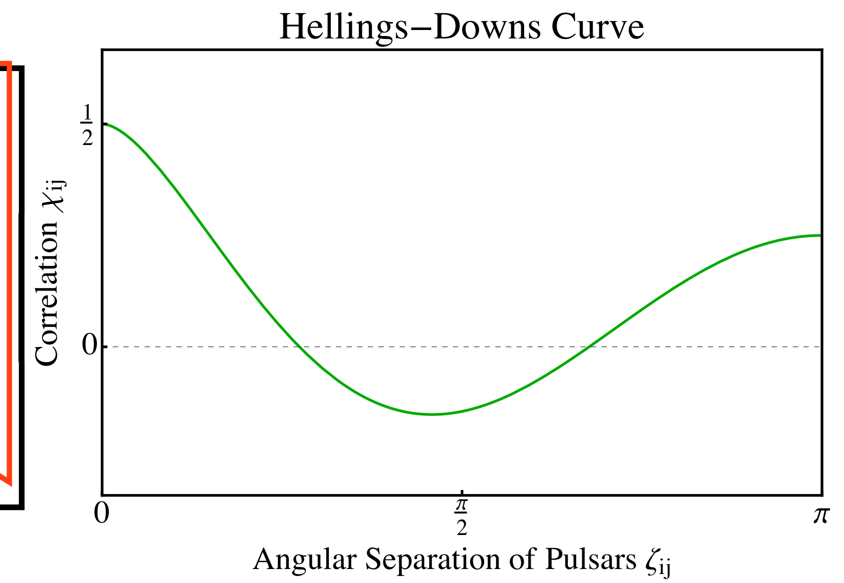

Off-diagonal terms given by cross-covariance matrices, depend on Hellings & Downs curve and GW power spectrum

i.e. $S_{IJ} = S_{IJ}(\chi_{IJ}, \mathcal{P}_g(f)), \quad \mathcal{P}_g(f) \propto A_{\rm gw}^2\, f^{-\gamma}$

Time-domain optimal statistic

* Since we are in the weak signal limit (noise > signal), we obtain the **optimal statistic** by maximizing the likelihood ratio over GW amplitudes (for a fixed GW spectral index α)

$$\hat{A}^2 = \frac{\sum_{IJ} r_I^T P_I^{-1} \tilde{S}_{IJ} P_J^{-1} r_J}{\sum_{IJ} \mathrm{Tr}\left[P_I^{-1} \tilde{S}_{IJ} P_J^{-1} \tilde{S}_{JI}\right]}$$

* The normalization factor is chosen so that the optimal statistic is also the maximum likelihood estimator for the amplitude of the stochastic GWB

(want to quantify significance of the expected GW correlations between pulsars, but **give larger weight to timing residuals with lower noise levels**!)

* The SNR for this statistic is

$$\hat{\rho} = \frac{\sum_{IJ} r_I^T P_I^{-1} \tilde{S}_{IJ} P_J^{-1} r_J}{\left(\sum_{IJ} \mathrm{Tr}\left[P_I^{-1} \tilde{S}_{IJ} P_J^{-1} \tilde{S}_{JI}\right]\right)^{1/2}}$$

(strength of correlated signal compared to uncorrelated signal of same strength)

Limitations and uses of this method

* The time-domain implementation of the optimal statistic allows us to deal more naturally with irregular data sampling and noise modeling

* But, it has some drawbacks: because we model the noise in individual pulsars, end up with potential biases in amplitude estimates

* So… not a substitute for more robust Bayesian analyses — but it still has some advantages
    - Computationally inexpensive (esp. compared to Bayesian inference)
    - SNR as defined here turns out to be an excellent approximation to the Bayes factor in Bayesian analysis
    - Can use time-domain optimal statistic to probe scaling laws for PTAs
    - Can also be used to create signal injections

For all the details, see Chamberlin et al. 2014, arXiv: 1410.8256

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- Testing general relativity with pulsar timing arrays

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Stochastic backgrounds in alternative theories of gravity: Overlap reduction functions for pulsar timing arrays

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In the next decade gravitational waves might be detected using a pulsar timing array. In an effort to develop optimal detection strategies for stochastic backgrounds of gravitational waves in generic metric theories of gravity, we investigate the overlap reduction functions for these theories and discuss their features. We show that the sensitivity to nontransverse gravitational waves is greater than the sensitivity to transverse gravitational waves and discuss the physical origin of this effect. We calculate the overlap reduction functions for the current NANOGrav pulsar timing array and show that the sensitivity to the vector and scalar-longitudinal modes can increase dramatically for pulsar pairs with small angular separations. For example, the J1853 + 1303 − J1857 + 0943 pulsar pair, with an angular separation of about 3°, is about 10^4 times more sensitive to the longitudinal component of the stochastic background, if it is present, than the transverse components.

Alternative theories of gravity?

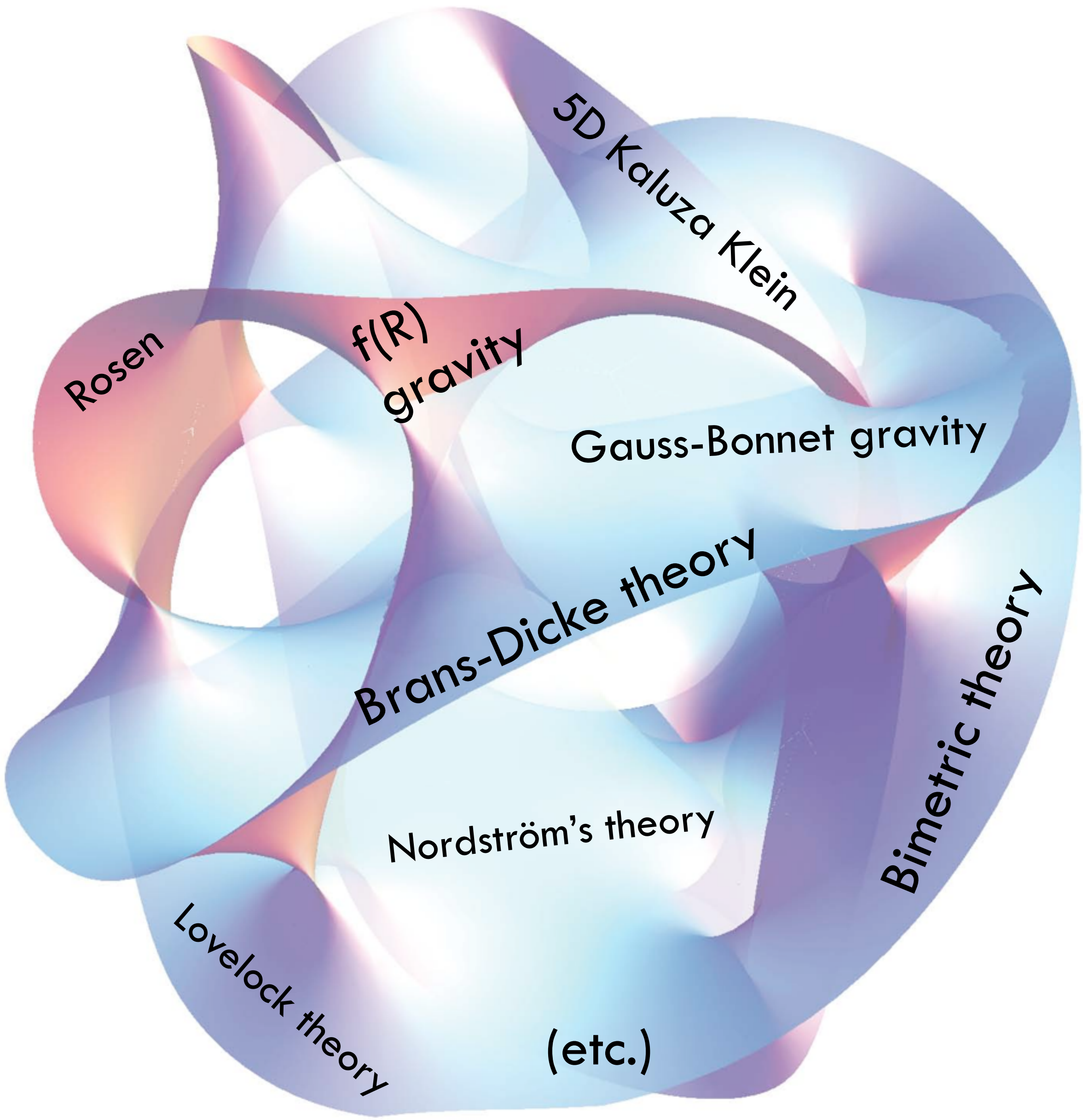
- Direct observation of gravitational waves will provide mechanism to test GR against other viable theories

- By "viable", mean theories that obey the Einstein Equivalence Principle:

  metric theories of gravity

  - Other gravitational fields (besides the metric) can exist, but only the metric itself interacts with matter.

  - Distinguish metric theories from one another by # and type of other fields involved

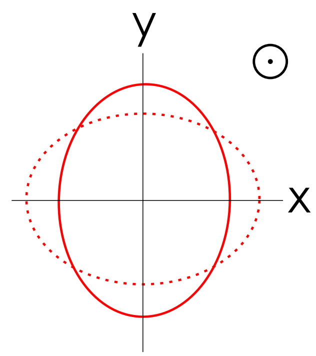


GWs in metric theories of gravity

* What's different from GR? Now (up to) six possible GW polarizations:

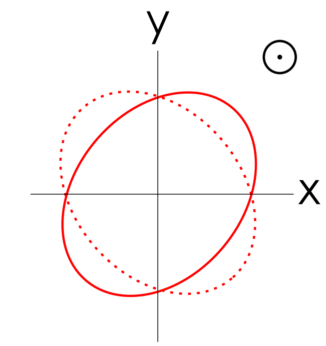
tensor

$$h_{ij}^{(+)} = h_+ \epsilon_{ij}^{(+)}$$



(a) plus mode

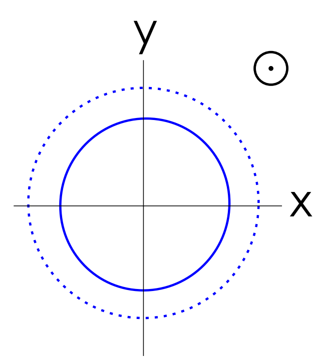
$$h_{ij}^{(\times)} = h_\times \epsilon_{ij}^{(\times)}$$



(b) cross mode

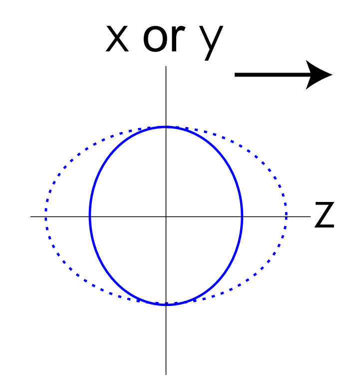
scalar

$$h_{ij}^{(b)} = h_b \epsilon_{ij}^{(b)}$$



(c) breathing mode

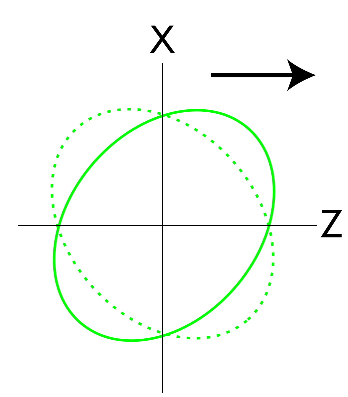
$$h_{ij}^{(l)} = h_l \epsilon_{ij}^{(l)}$$



(d) longitudinal mode

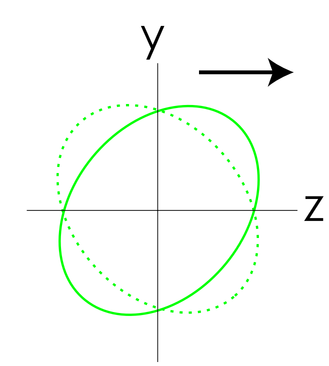
vector

$$h_{ij}^{(x)} = h_x \epsilon_{ij}^{(x)}$$



(e) vector-x mode

$$h_{ij}^{(y)} = h_y \epsilon_{ij}^{(y)}$$

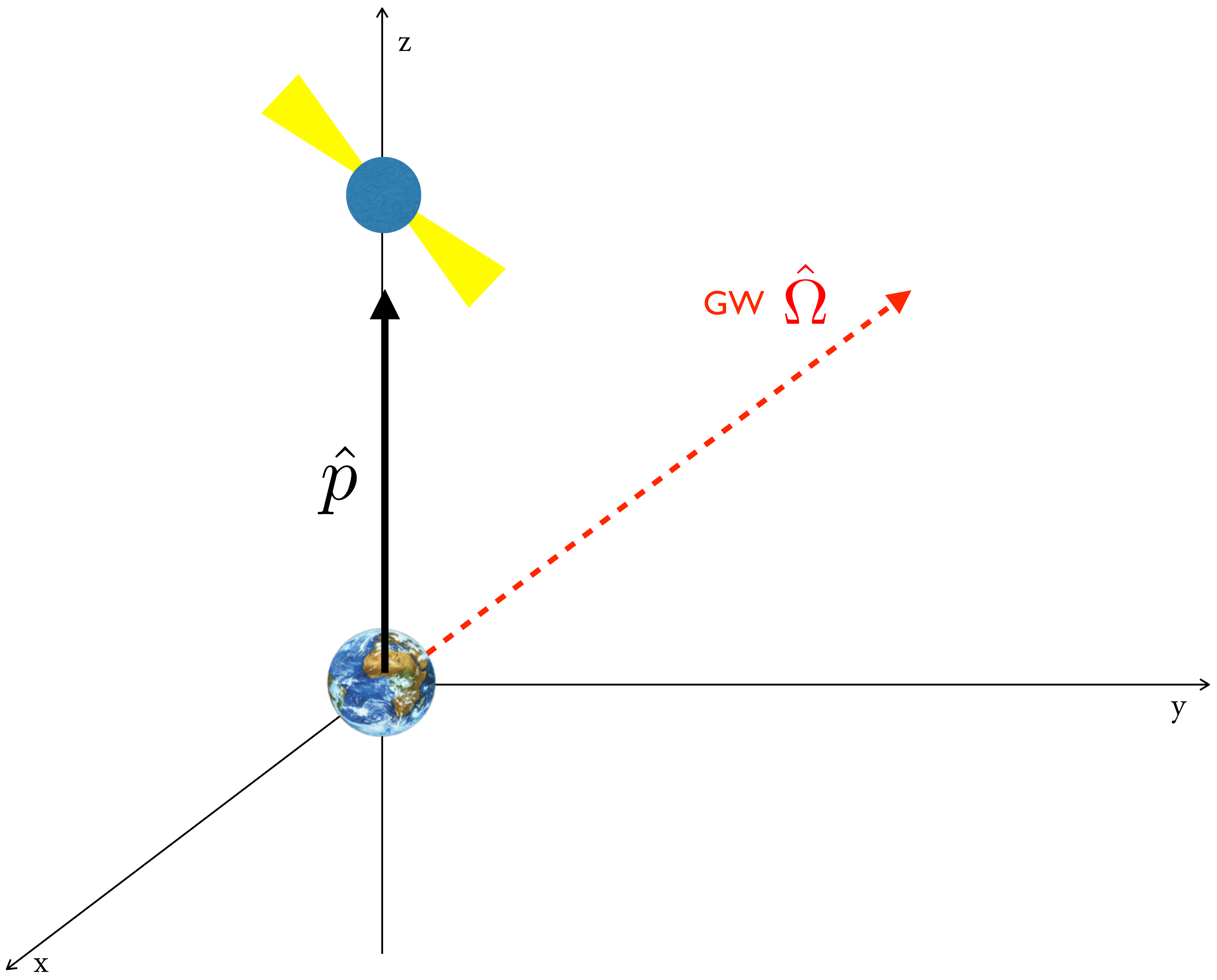


(f) vector-y mode

* What happens to stochastic background analysis from earlier slides when other polarizations possible?

Geometry of the pulsar-Earth system

* Let's begin by considering a single pulsar & GW propagating nearby



GWs in metric theories of gravity

* By definition, the redshift induced in the pulsar's signal is:

$$z(t, \hat{\Omega}) = \frac{\nu_p - \nu_e}{\nu_e}$$

which ends up being (Detweiler, 1979):

$$z(t, \hat{\Omega}) = \frac{p^i p^j}{2(1 + \hat{\Omega} \cdot \hat{p})} \left[h_{ij}(t_p, \hat{\Omega}) - h_{ij}(t_e, \hat{\Omega}) \right]$$

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Terms describing pulsar-Earth-GW geometry

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Terms describing pulsar-Earth-GW geometry

"Earth" term: correlated for all pulsars, independent of pulsar-Earth distance

GWs in metric theories of gravity

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Terms describing pulsar-Earth-GW geometry

"Earth" term: correlated for all pulsars, independent of pulsar-Earth distance

"Pulsar" term: not correlated for all pulsars, dependent on pulsar-Earth distance

GW redshift on the pulsar-Earth system

- By definition, the redshift induced in the pulsar's signal is:

$$z(t,\hat{\Omega}) = \frac{\nu_p - \nu_e}{\nu_e}$$

which ends up being (Detweiler, 1979):

$$z(t,\hat{\Omega}) = \frac{p^i p^j}{2(1+\hat{\Omega}\cdot\hat{p})}\left[h_{ij}(t_p,\hat{\Omega}) - h_{ij}(t_e,\hat{\Omega})\right]$$

turns out to be more useful to work in frequency domain…

$$\tilde{z}_A(f,\hat{\Omega}) = \left(e^{-2\pi i f L(1+\hat{\Omega}\cdot\hat{p})} - 1\right)\frac{p^i p^j}{2(1+\hat{\Omega}\cdot\hat{p})}\epsilon_{ij}^A(\hat{\Omega})\tilde{h}_A$$

$$A \in \{+,\times,b,l,x,y\}$$

GW redshift on the pulsar-Earth system

- When GW direction $\hat{\Omega}$ and pulsar direction $\hat{p}$ are anti-parallel, things get tricky…

$$\tilde{z}_A(f, \hat{\Omega}) = \left(e^{-2\pi i f L(1+\hat{\Omega}\cdot\hat{p})} - 1\right) \frac{p^i p^j}{2(1+\hat{\Omega}\cdot\hat{p})} \epsilon_{ij}^A(\hat{\Omega}) \tilde{h}_A$$

- Two physical scenarios that allow a Taylor series to be done on the pulsar term:

Case 1: *Long-wavelength limit*: metric perturbation same at pulsar and Earth

i.e., $fL << 1$

Case 2: "*Surfing*": metric perturbation at pulsar when pulse is emitted, and on Earth when pulse is received, also nearly the same

i.e., $1 + \hat{\Omega}\cdot\hat{p} << \dfrac{1}{fL}$

$$z(t, \hat{\Omega}) \propto \left[h_{ij}(t_p, \hat{\Omega}) - h_{ij}(t_e, \hat{\Omega})\right]$$

In the "surfing" regime, it is tempting to go back to the time domain and conclude that the effect of the GW cancels... but this is not correct!

GW redshift on the pulsar-Earth system

* To see delicate interaction between pulsar term and geometric term, do series expansion in the frequency domain.

Let $\hat{\Omega} \cdot \hat{p} = -1 + \delta \,, \quad \delta << 1:$

$$\tilde{z}_A(f, \hat{\Omega}) = (e^{-2\pi i f L(1+\hat{\Omega}\cdot\hat{p})} - 1)\frac{\hat{p}^i \hat{p}^j}{2(1+\hat{\Omega}\cdot\hat{p})}\epsilon_{ij}^A(\hat{\Omega})\tilde{h}_A$$

$$\downarrow$$

$$\tilde{z}_A(f, \hat{\Omega}) \propto f L\, p^i\, p^j\, \epsilon_{ij}^A\, \tilde{h}_A$$

doesn't vanish for non-transverse polarizations!

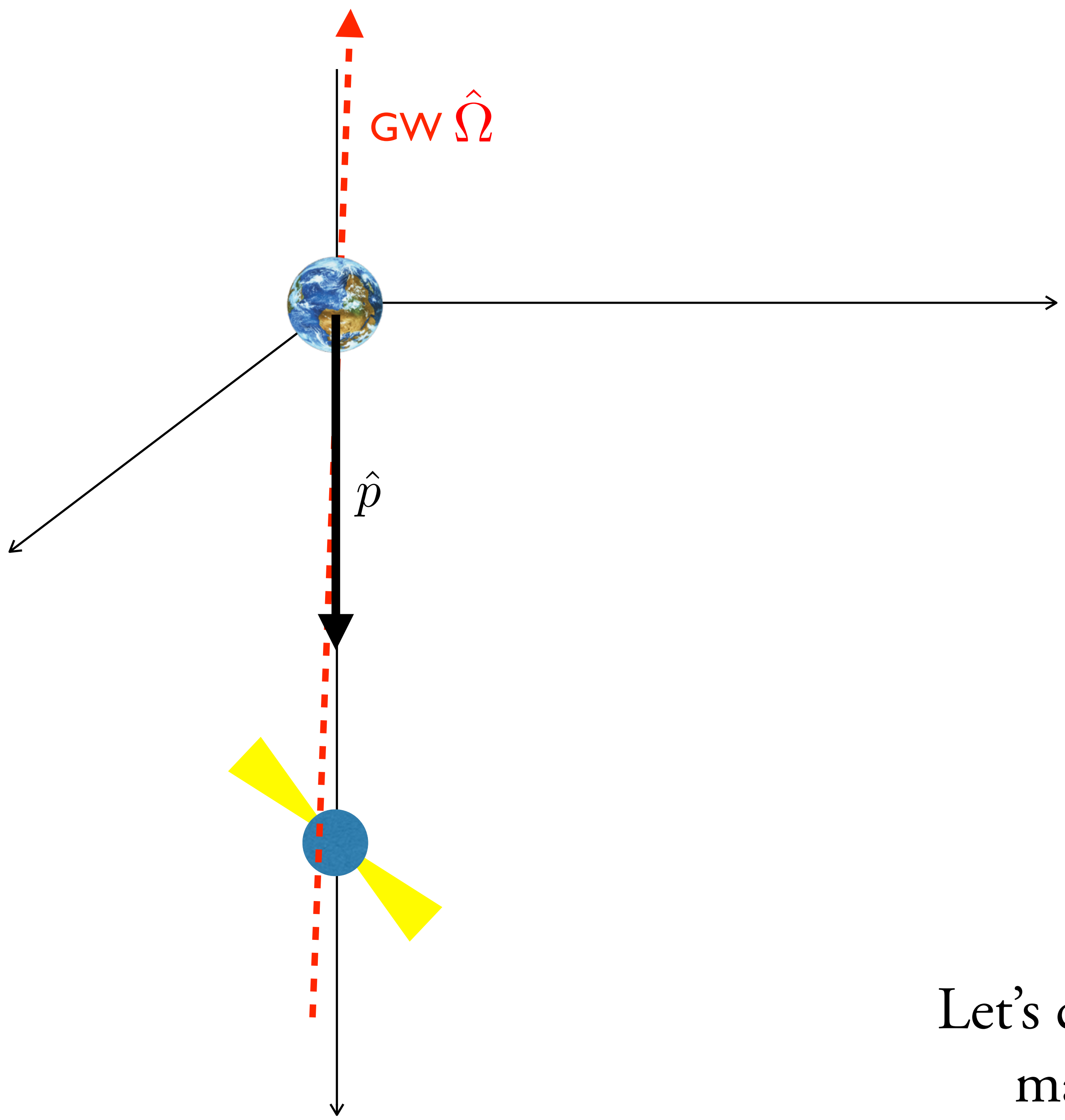
* Does redshift diverge or remain finite as $\delta \to 0$?

When $\hat{\Omega} \cdot \hat{p} \approx -1$, redshift can increase monotonically

(up to point some limiting frequency where
Taylor series no longer valid)

What does this mean for PTA observations?

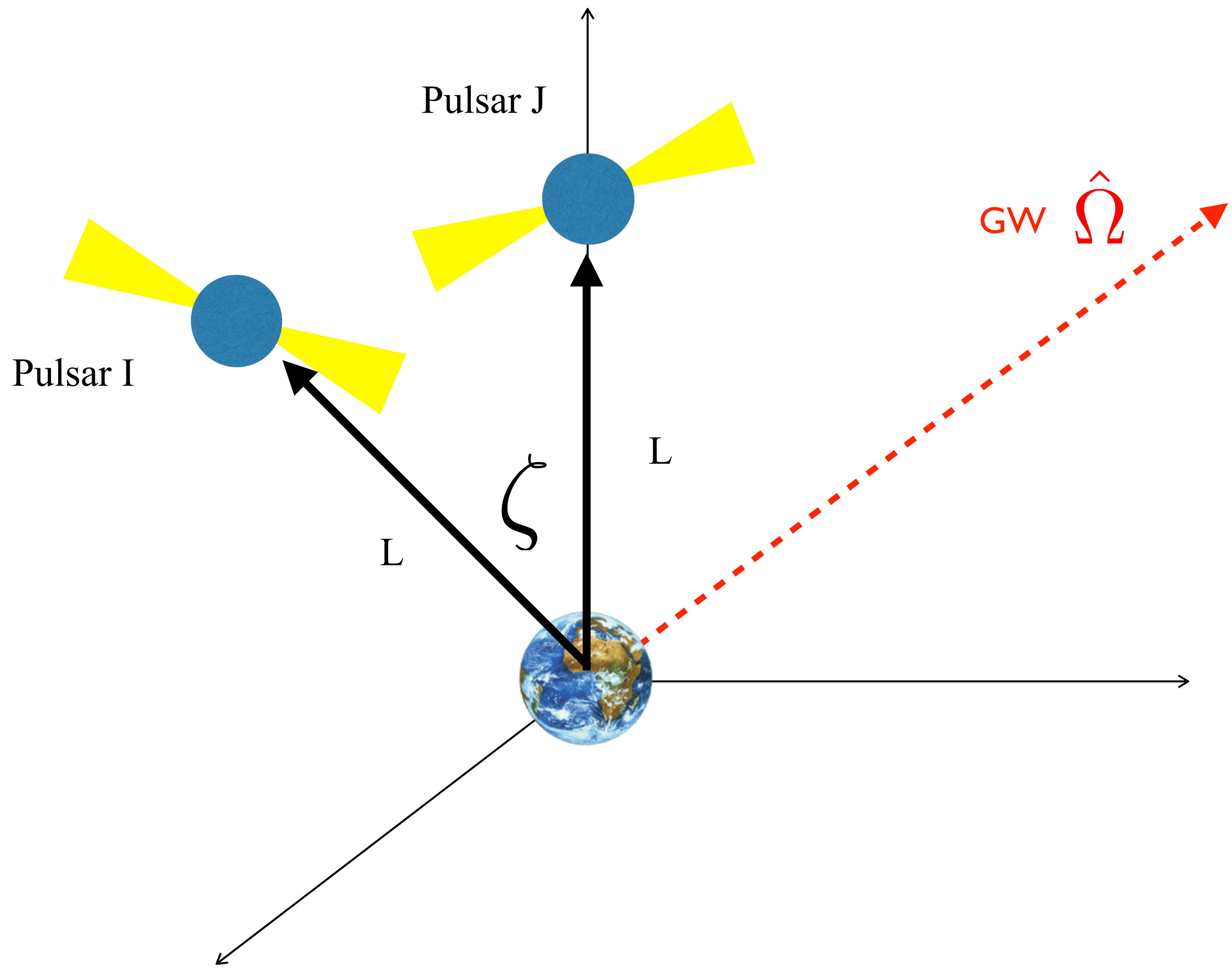
- Increasing redshift term for non-transverse GW polarizations suggests possible enhanced detector response to those polarizations



Let's consider how this effect manifests with a PTA!

What does this mean for PTA observations?

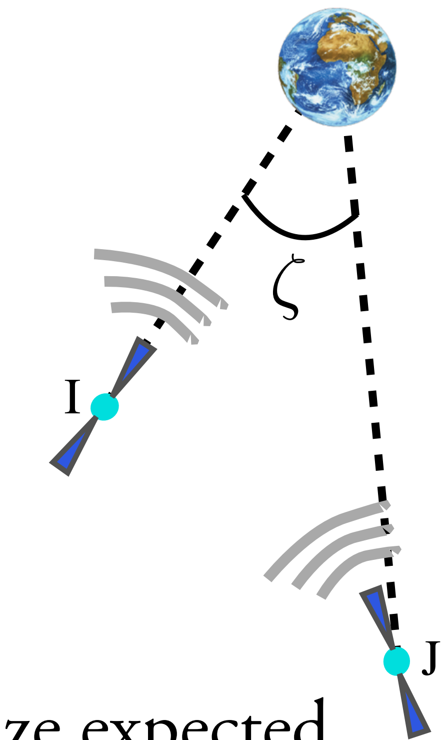
* Increasing redshift term for non-transverse GW polarizations suggests possible enhanced detector response to those polarizations **when** ζ **is small**



For simplicity, assume equidistant pulsars, i.e. $L_I = L_J = L$

Overlap reduction functions

* Recall from earlier in talk:



$$\langle \tilde{r}_I^*(f)\tilde{r}_J(f')\rangle \propto \Omega_{gw}(f)\,\delta(f-f')\,\chi_{IJ}(\zeta)$$

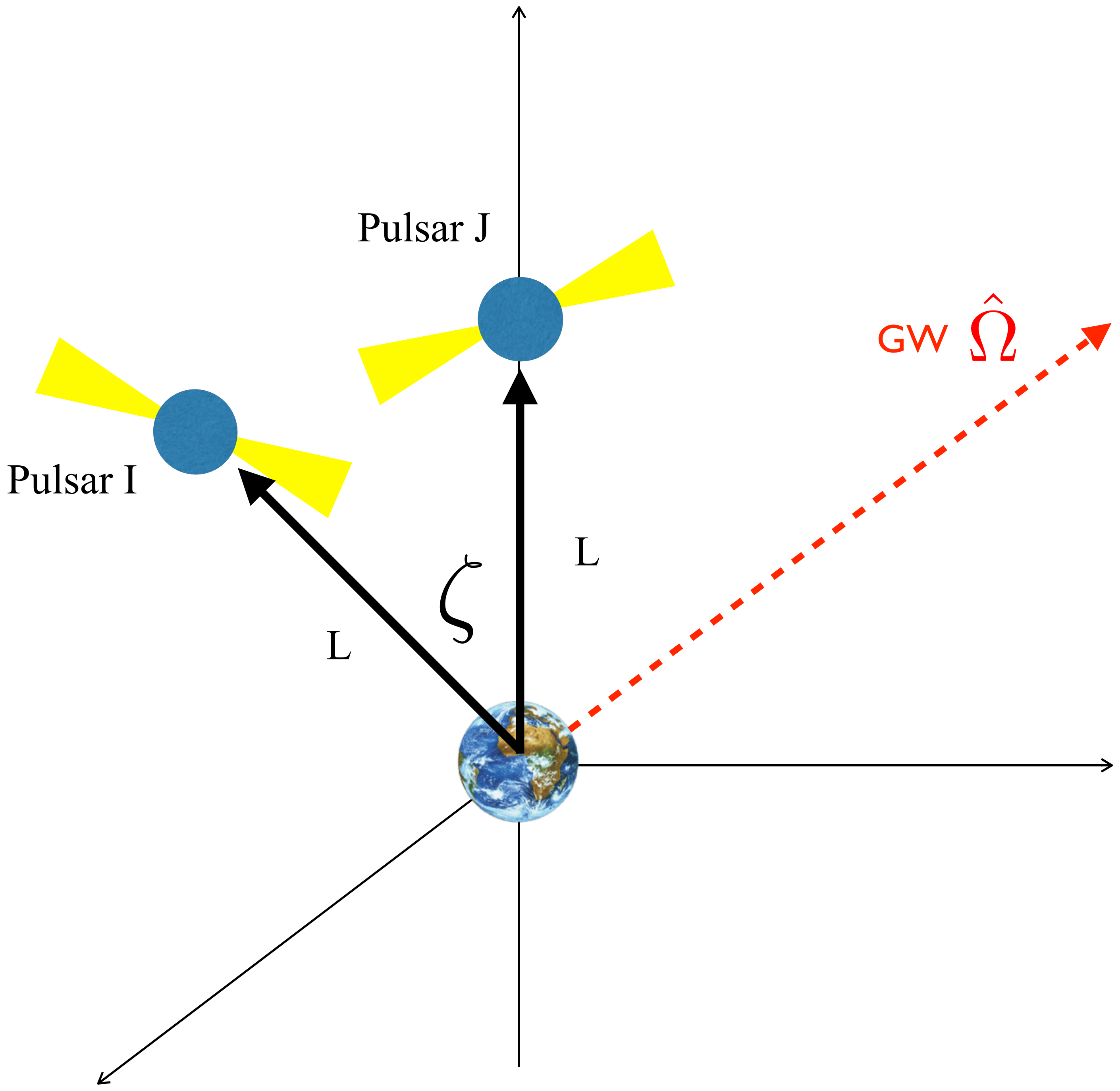
Correlated residuals — GW source spectrum — Hellings-Downs coefficient

* More generally, require **overlap reduction function** to characterize expected correlation:

General Relativity		Metric Theories
$\chi_{IJ}(\zeta)$	$\longrightarrow$	$\Gamma_{IJ}(f,\zeta) = F(f)\chi_{IJ}(\zeta)$

* In general, the larger Γ_{IJ} is the more sensitive we are to the GW.

Overlap reduction functions



For PTA experiments,

smallest frequencies sensitive to

$$f \sim 0.1\mathrm{yr}^{-1}$$

nearest pulsar distances

$$L \sim 100\,\mathrm{ly}$$

$\therefore$ for PTA experiments,

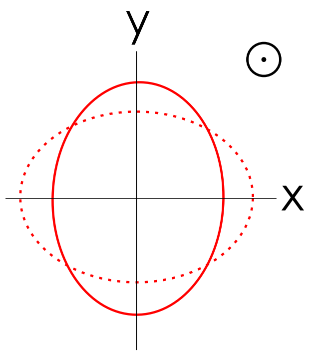
$$fL \gtrsim 10$$

Can plot overlap reduction functions to see what effects present when ζ small in this fL regime

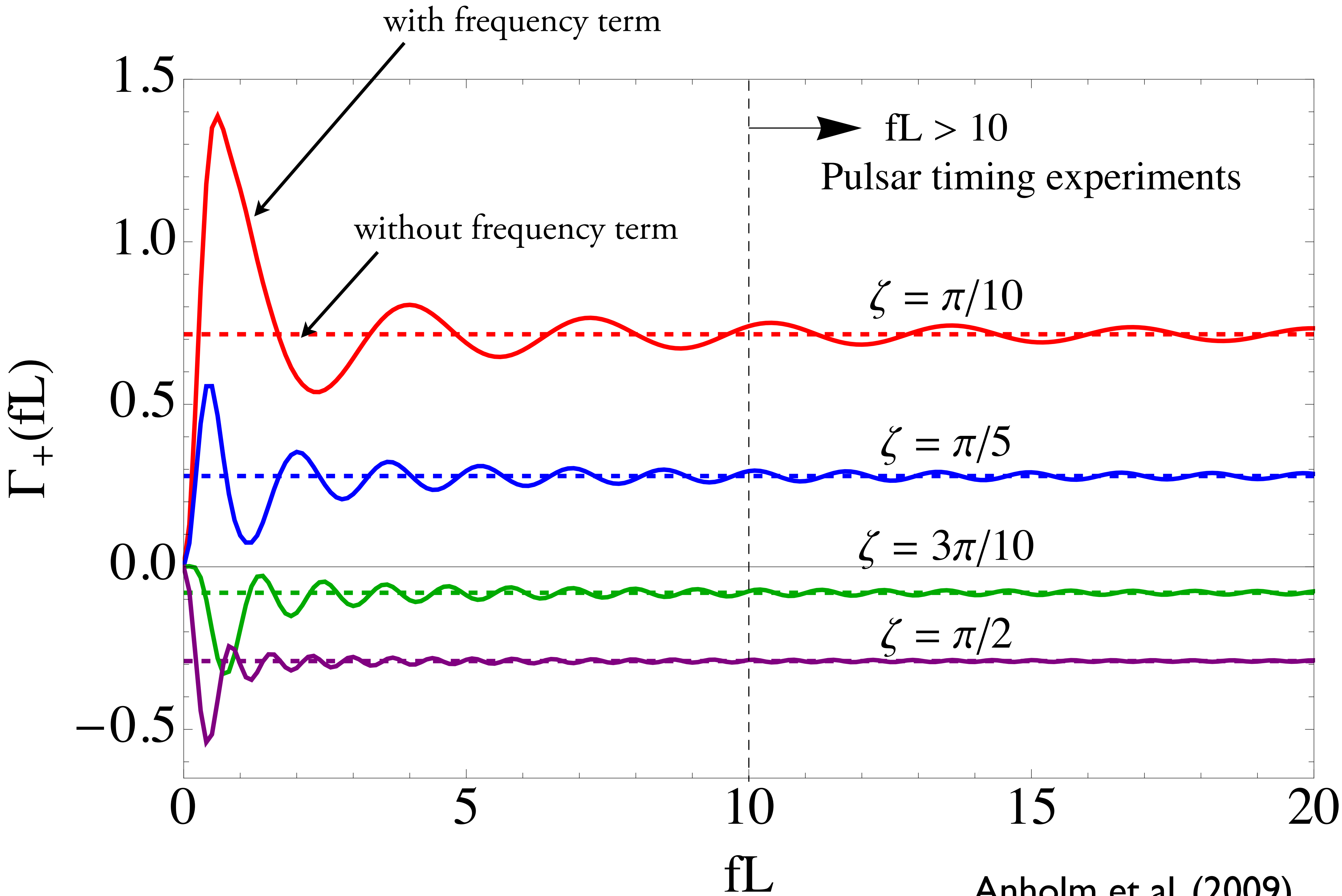
Overlap reduction functions for general relativity

$$\Gamma_{IJ}(fL,\zeta) = F(fL)\chi_{IJ}(\zeta)$$

can toss this out



(a) plus mode

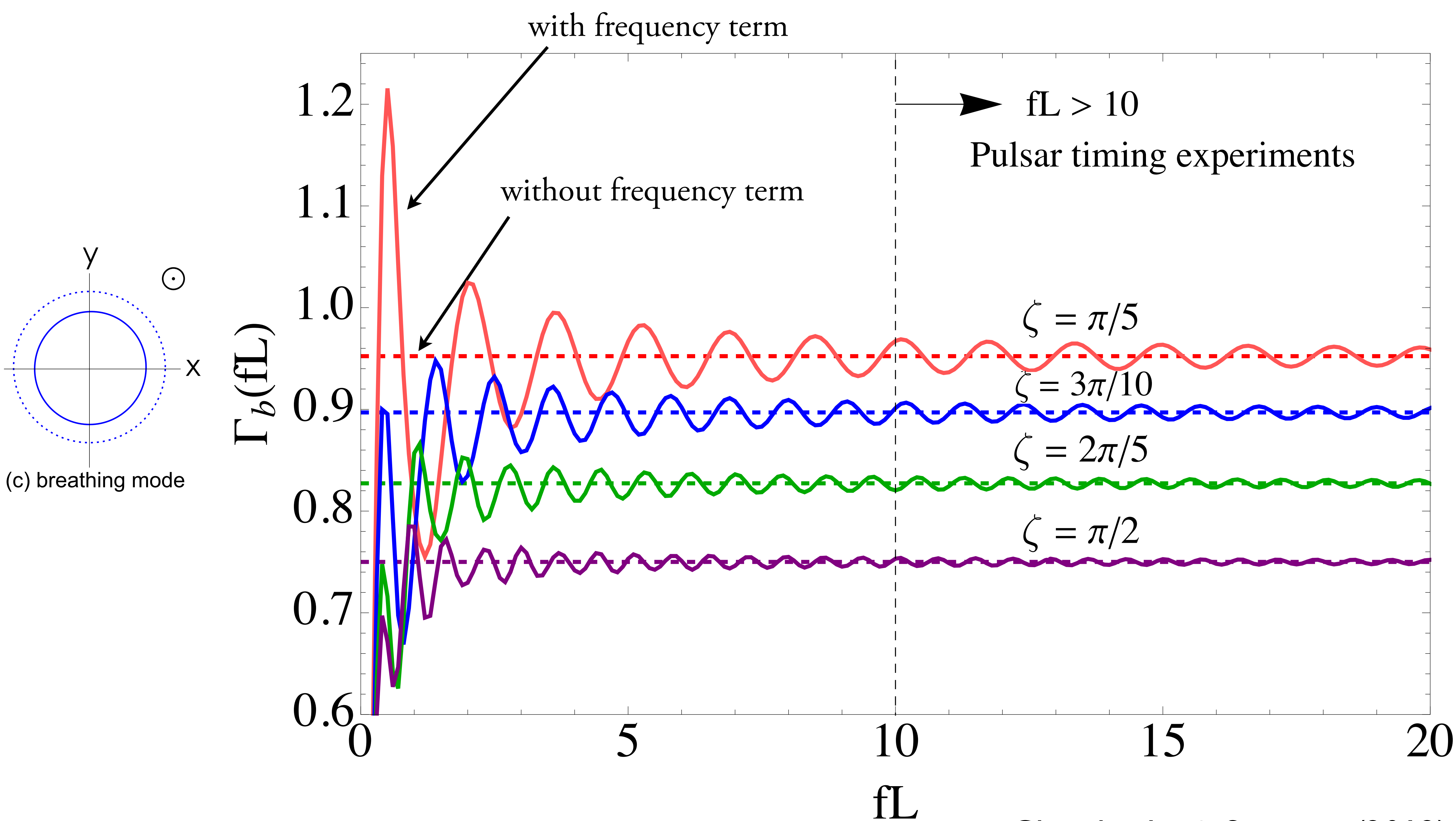


Anholm et al. (2009)

Overlap reduction functions for other GW polarizations

If $L_I = L_J = L$, can write $\Gamma_{IJ}(fL,\zeta) = F(fL)\chi_{IJ}(\zeta)$

can toss this out

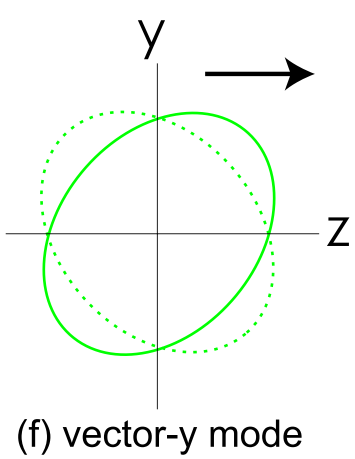


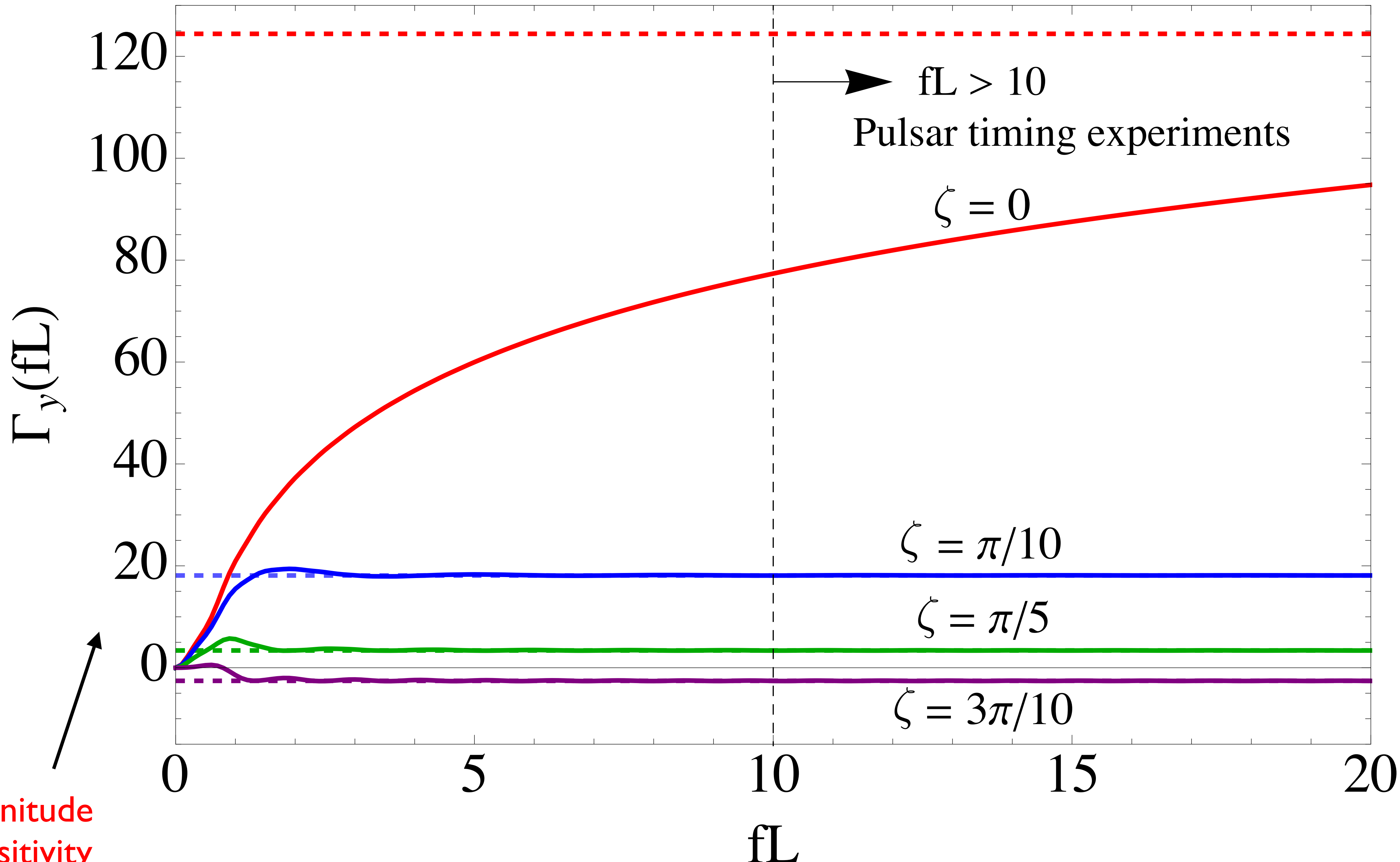
Chamberlin & Siemens (2012)

Overlap reduction functions for other GW polarizations

If $L_I = L_J = L$, can write $\Gamma_{IJ}(fL, \zeta) = F(fL)\chi_{IJ}(\zeta)$

need this term for pulsars with small angular separation





order of magnitude gained in sensitivity

Chamberlin & Siemens (2012)

Overlap reduction functions for other GW polarizations

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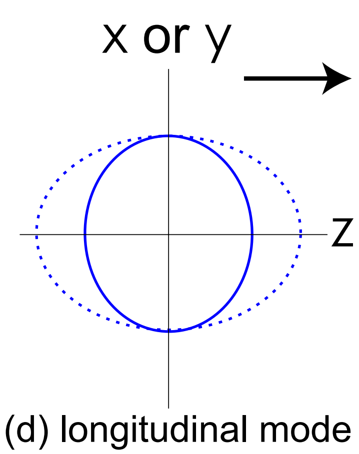
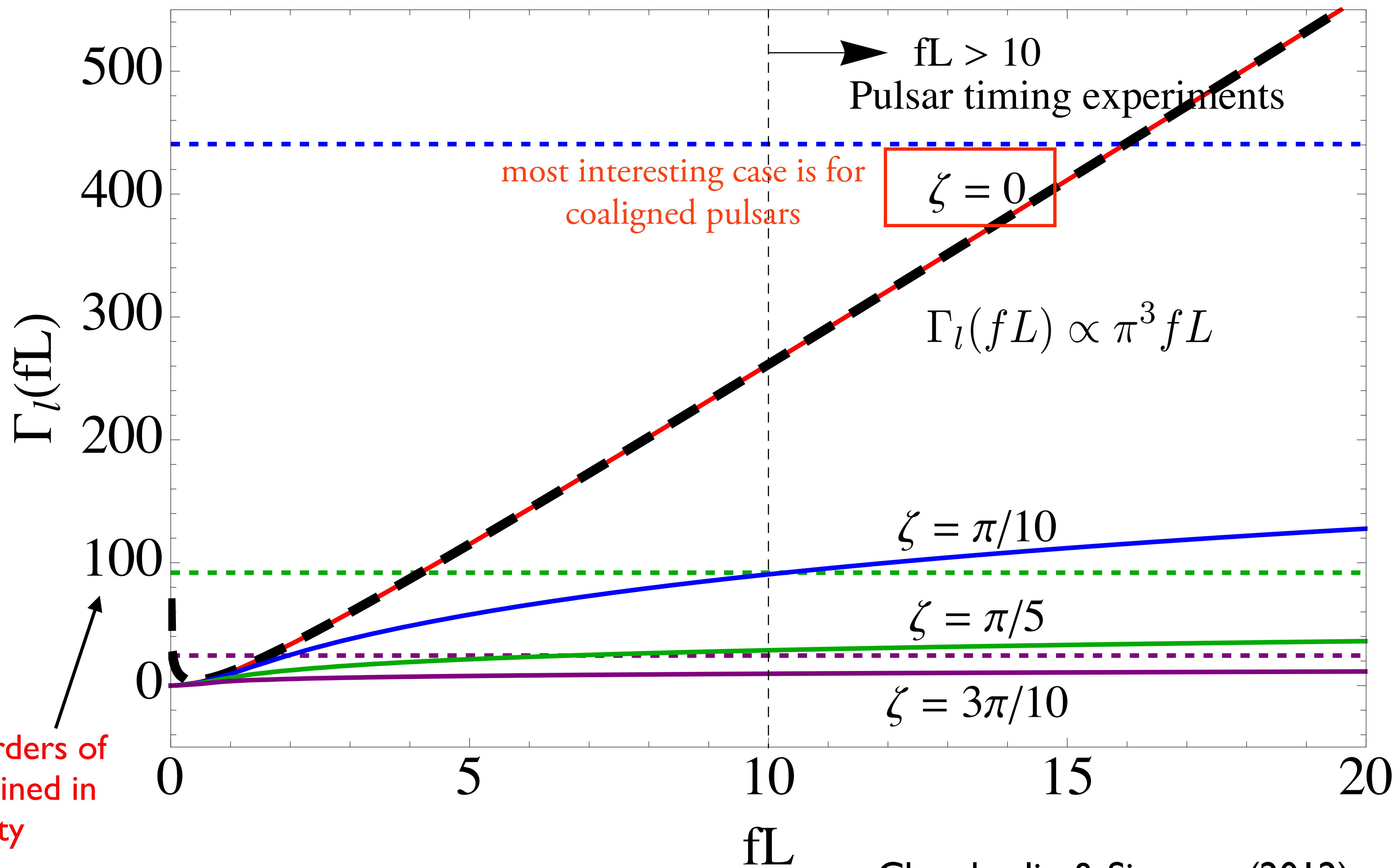
cannot remove frequency dependence





a couple of orders of magnitude gained in sensitivity

Chamberlin & Siemens (2012)

Implications and future work

* Frequency-dependence in the overlap reduction functions for non-transverse GW polarizations comes with an increase in sensitivity:

  for pulsars with small angular separations on the sky, PTAs will experience an enhanced response to non-transverse GWs (especially to scalar-longitudinal GWs)

* Effect can be observed by plotting overlap reduction functions for current NANOGrav pulsars

* Result in good agreement with other similar analyses in the literature:

  Lee et al. (2008) -- computed cross-correlation functions and observed frequency dependence in non-transverse modes

  Alves & Tinto (2011) -- computed antenna sensitivities for PTA, argue more sensitive to non-transverse modes

* Future work will determine how feasible it will be to extract polarization content from stochastic GW observations

Summary

Optimal Detection Statistic in Stochastic GWB Searches

* We have developed an optimal cross correlation that takes into account the GW spectrum and noise power spectra.
* There are many applications of this statistic, but it's better used for quick estimates; full Bayesian analysis still preferable.
* For details, see Chamberlin et al. (2014), accepted to Physical Review D (arXiv: 1410.8256)

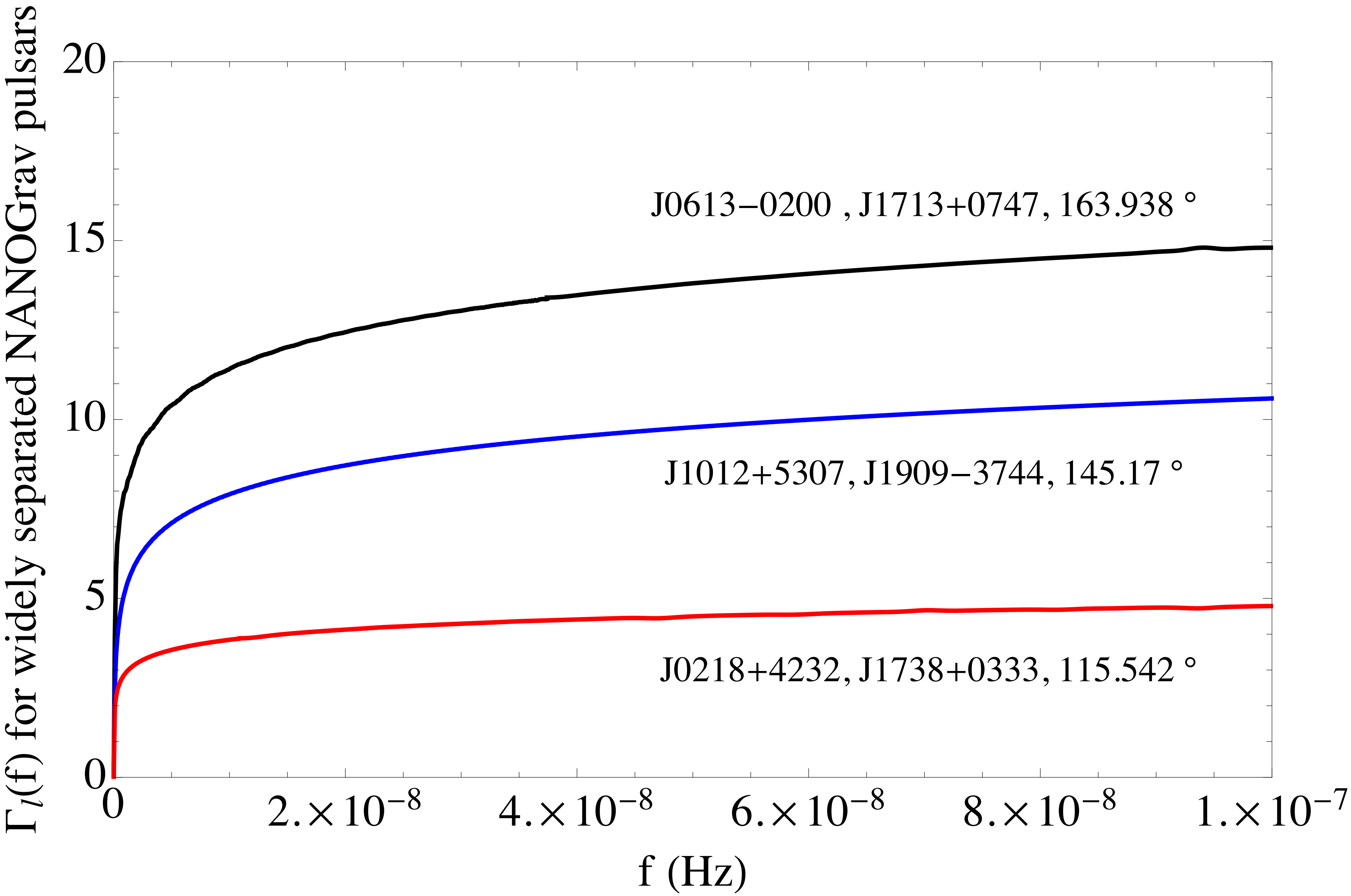
Tests of General Relativity with PTA Detection

* PTAs are potentially much more sensitive to scalar longitudinal and transverse GW polarization modes than to those of GR.
* Details in Chamberlin & Siemens (2012), Physical Review D **85** (082001)
* Future plans: disentangling GW polarizations from observed GWB

Thank you!

Application to NANOGrav pulsars

* Used pulsar distance data for 20 NANOGrav pulsars to calculate overlap reduction functions:



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